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Pricing and hedging long-term options

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Abstract

Do long-term and short-term options contain differential information? If so, can long-term options better differentiate among alternative models? To answer these questions, we first demonstrate analytically that differences among alternative models usually may not surface when applied to short-term options, but do so when applied to long-term contracts. Using S&P 500 options and LEAPS, we find that short- and long-term contracts indeed contain different information. While the data suggest little gains from modeling stochastic interest rates or random jumps (beyond stochastic volatility) for pricing LEAPS, incorporating stochastic interest rates can nonetheless enhance hedging performance in certain cases involving long-term contracts. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

Option pricing has played a central role in the general theory of asset pricing. Its importance comes about because of the derivative nature of option contracts,

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that is, the value of an option is almost completely derived from, and hence closely tied to, the value of the underlying asset. Not surprisingly, option pricing has been proven to be the best place for exemplifying the power of the much celebrated arbitrage valuation approach. From an application perspective, however, option pricing formulas based on the arbitrage approach have not performed well empirically. Take the prominent Black and Scholes (1973) model as an example. When applied to European-style options, it produces pricing errors that are related to both moneyness and maturity in a U-shaped manner. Thus, the ‘implied volatility smiles’. The unsatisfactory performance by the Black–Scholes has led to a search for better alternatives that extend the classic model in one, or a combination, of three directions: (i) to allow for stochastic volatility; (ii) to allow for stochastic interest rates; and (iii) to allow for random jumps to occur in the underlying price process.¹ Each of the alternatives in principle offers some flexibility to correct for the biases of the Black–Scholes. For example, the stochastic-volatility (SV) models rely on the correlation coefficient between volatility and underlying price changes to internalize the level of skewness, and the variation-of-volatility coefficient to generate a desired kurtosis level, necessary to correct the volatility smiles. But, since existing SV models typically let volatility follow a diffusion process, the extent to which high levels of kurtosis in the return distribution can be internalized is limited. This points out a special role to be played by random jumps in the modeled price process. Thus, one should expect an option pricing model allowing for stochastic volatility and jumps (SVJ) to further enhance performance. In addition, even casual empiricism suggests that modeling stochastic interest rates in any pricing formula should be of practical significance as it ensures proper discounting of future payoffs. A model with stochastic volatility and stochastic interest rates (SVSI) should also have promise to further improve pricing and hedging performance.

While each generalization beyond the Black–Scholes may be sound and justifiably appealing on normative grounds, given the application-oriented nature of the problem at hand it is ultimately an empirical issue whether a given generalization and its consequential model complication are justified by the additional performance benefits (if any at all). Motivated by this, Bakshi et al.

¹ These include (i) the stochastic-interest-rate option pricing models of Merton (1973) and Amin and Jarrow (1992), (ii) the one-dimensional jump-diffusion/pure-jump models of Bates (1991), Madan and Chang (1996), and Merton (1976), (iii) the constant-elasticity-of-variance model of Cox and Ross (1976), (iv) the stochastic-volatility models of Heston (1993), Hull and White (1987), Melino and Turnbull (1990, 1995), Scott (1987), Stein and Stein (1991), and Wiggins (1987), (v) the stochastic-volatility and stochastic-interest-rates models of Amin and Ng (1993), Bailey and Stulz (1989), Bakshi and Chen (1997a,b), and Scott (1997), and (vi) the stochastic-volatility jump-diffusion models of Bakshi et al. (1997), Bates (1996a, 1999) and Scott (1997). See Ghysels et al. (1996) for a review of stochastic-volatility models.

(1997) conduct a comprehensive empirical study on the relative performance of the above alternative models, using regular S&P 500 index option prices. Their conclusions can be summarized as follows. First, while the SV model typically reduces the Black–Scholes pricing errors by about a half, adding jumps does not further improve the SV model's pricing performance, except for extremely short-term options; Neither does incorporating stochastic interest rates enhance the SV's pricing performance. Second, when hedging errors are used as a performance benchmark, the SV model still does better than the Black–Scholes, but the SVSI and the SVJ models do not show any improvement beyond the SV. These findings are somewhat surprising given that one would expect incorporating random jumps or stochastic interest rates to further enhance the performance.

Given that the options used in most existing studies are generally short-term (typically with less than one year to expiration), the purpose of this paper is to address two related questions. First, do long-term options contain different information than short-term options? Second, if so, can long-term options better differentiate among the Black–Scholes and its alternatives? Answering such questions is important not only because it will help better appreciate and learn more about existing parametric option pricing models, but also because it will enhance our understanding of long-term contracts in general. In recent years, such contracts as equity LEAPS (Long-term Equity Anticipation Securities) have attracted increasingly more attention in the investment community. Yet, at the same time the academic literature has paid, at most, limited attention to issues related to long-term contracts (see Ross, 1996, for a treatment on long-term versus short-term futures commitments). This paper thus serves to fill in this gap. Our study is also timely and feasible as closed-form option pricing formulas have recently become available even for the general cases. On the data front, we now have high-quality intradaily quote and transaction data available for such popular contracts as the S&P 500 LEAPS. Unlike the regular S&P 500 options, these LEAPS have up to three years to expiration and are hence ideal for the purpose of this study.

We begin our quest by demonstrating analytically that long-term options can distinguish among alternative models more dramatically than short-term options. Specifically, we first examine whether the Black–Scholes, the SV, the SVSI, and the SVJ models yield different hedge ratios, or option deltas, for a given option contract. When implemented using estimated parameters and implied volatility values, all the models assuming stochastic volatility produce stock price option deltas that are drastically different from those based on the Black–Scholes. This is true regardless of option maturity and whether it is a low-volatility day, an average-volatility day, or a high-volatility day. But, when the SV, the SVSI, and the SVJ are compared to each other, the hedge ratios for a given option are either similar or significantly different, depending on whether the option is short term or long term. For a 45-d put, for instance,

the three models yield virtually the same hedge ratios when the option is at the money, and are thus indistinguishable from each other. For a two-year put, however, the model-specific hedge ratios are generally far apart across the three models, even for a low-volatility day.

In the theory of asset pricing it is well understood that each parametric pricing model is associated with an Arrow–Debreu state-price density (SPD). This SPD function completely embodies the pricing structure implicit in the pricing model (see, e.g., Ait-Sahalia and Lo, 1998). Therefore, another way to examine alternative option pricing models is to compare their implicit SPDs, as their differences in pricing performance must come from differences in their SPDs. Again, when implemented using estimated parameters and implied volatilities, the SPDs of the SV, the SVSI, and the SVJ differ significantly from that of the Black–Scholes (regardless of time horizon): the former all assign more risk-neutral probability mass to the far lower tail and less mass to the upper tail of the underlying asset's return distribution, which effectively corrects and flattens out the volatility smiles associated with the Black–Scholes. For short-term options, the SPDs of the SV, the SVSI and the SVJ models coincide almost everywhere, except in the far left and far right tails. For long-term options, however, the difference among the SPDs of these models becomes especially pronounced when parameters implied by option prices taken from a relatively volatile day are used as input. Thus, long-term options should distinguish among alternative models more effectively than short-term options.

In the empirical exercise, we apply the method of simulated moments (MSM) to estimate each model's structural parameters out of the following considerations. First, the unobservability of the stock volatility process precludes the estimation by maximum likelihood. The unavailability of the moments of option prices in closed form also rules out the use of the generalized method of moments (GMM). With the MSM, on the other hand, we can jointly simulate the sample paths for the stock price and its return volatility, to construct a time series of simulated option prices of different strikes and maturities. The structural parameters estimated via the MSM will then reflect both the cross-sectional and the time-series information contained in option prices.

Empirically, we find that the S&P 500 LEAPS and regular options do provide distinct information. First, on a typical day, the two sets of options imply different volatility values (for any given option pricing model). Second, the implied-volatility time-series implied by regular options follows a drastically different path than that by LEAPS. Finally, the LEAPS-implied volatility exhibits a much higher level of long memory than the short term options implied, which suggests that volatility innovations will persist relatively longer (also see the recent work by Bollerslev and Mikkelsen (1996, 1999)).

In terms of out-of-sample pricing, the SVJ model performs the best among the four models in pricing short-term puts. In pricing medium-term options the SVSI model does better than the SV in certain categories, while the SV performs

better in pricing other moneyness-maturity puts. In pricing long-term puts, however, the SV model performs the best. Overall, even for pricing long-term options, adding the stochastic interest rates feature does not lead to consistent improvement in pricing errors. Overall, all four models are still misspecified statistically. For example, they each have moneyness- and maturity-related biases, though to varying degrees.

For the hedging exercise, we divide the discussion into two parts. In the first, the goal is to evaluate the relative effectiveness of (i) short-, medium- and long-term options (as hedging instruments) in hedging the underlying stock portfolio and (ii) the alternative option models in devising the desired hedge. The main results from this part can be summarized as follows. First, irrespective of the option model used, deep in-the-money LEAPS puts yield the lowest hedging errors on average, but short-term deep in-the-money puts generate the most stable hedging errors over time. Second, among the four option models, the BS-based hedge is always the least effective, regardless of the hedging instrument. The SV and the SVJ models lead to similar hedging errors, and both perform better than the SVSI.

In the second part of our hedging exercise, we let a LEAPS put be the hedging target and evaluate the relative effectiveness of (i) the underlying asset, (ii) a short-term put, and (iii) a medium-term put, as the hedging instrument. In a common practice, users and underwriters of long-term contracts often have no other choice but rely on exchange-traded, relatively short-term contracts to hedge their long-term commitments. At least, such short-term contracts have high liquidity and relatively low trading costs. Therefore, it is important to address the question of how effectively can short-term contracts hedge their long-term counterparts. On the other hand, this exercise also allows us to examine the relative hedging performance of the alternative models. The overall conclusion from this part is that medium-term options are generally the best instrument for hedging LEAPS, partly because they are more similar to the hedging target than either the underlying asset or a short-term option. Next, between the underlying asset and the short-term option as a hedging instrument, the former dominates the latter in hedging out-of-the-money LEAPS puts. Short-term contracts are good instruments only for hedging in-the-money LEAPS. In terms of model choice, the SVSI generally dominates the other models. Therefore, at least for devising hedges of long-term options, modeling stochastic interest rates does help improve empirical performance.

The paper is organized as follows. Section 2 develops an option formula that takes into account stochastic volatility, stochastic interest rate and random jumps. A description of the regular and LEAPS S&P 500 option data is provided in Section 3. Section 4 discusses the implementation of each option model and the MSM estimation of the structural parameters. Section 5 contrasts the information in short-term versus long-term options. The out-of-sample pricing exercise is conducted in Section 6. Section 7 addresses issues

related to hedging the underlying stock portfolio, and Section 8 evaluates the relative effectiveness of the underlying asset, short-term and medium-term options in hedging LEAPS. Concluding remarks are offered in Section 9. Proofs to all pricing formulas and hedging strategies can be found in the Appendix.

2. Valuation of European options

In this section we derive a closed-form option pricing model that incorporates stochastic volatility, stochastic interest rates, and random jumps. The model is sufficiently general to include as special cases all the models which we investigate in the empirical sections. As in Bakshi et al. (1997), we take a risk-neutral probability measure as given and specify from the outset risk-neutral dynamics for the spot interest rate, the spot stock price, and the stock return volatility.² Specifically, let the spot interest rate follow a square-root diffusion of the Cox et al. (1985) type:

$$dR(t) = [\theta_R - \kappa_R R(t)] dt + \sigma_R \sqrt{R(t)} d\omega_R(t), \quad (1)$$

where κ_R , θ_R/κ_R , and σ_R are respectively the speed of adjustment, the long-run mean, and the volatility coefficient of the $R(t)$ process; and $\omega_R(t)$ is a standard Brownian motion, uncorrelated with any other process in the economy. With the short rate in (1), the price of a zero-coupon bond that pays \$1 in τ periods from time t , denoted by $B(t, \tau)$, is

$$B(t, \tau) = \exp[-\varphi(\tau) - \varrho(\tau)R(t)], \quad (2)$$

where

$$\varphi(\tau) = \frac{\theta_R}{\sigma_R^2} \left\{ (\zeta - \kappa_R)\tau + 2 \ln \left[1 - \frac{(1 - e^{-\zeta\tau})(\zeta - \kappa_R)}{2\zeta} \right] \right\},$$

$$\varrho(\tau) = \frac{2(1 - e^{-\zeta\tau})}{2\zeta - [\zeta - \kappa_R](-e^{-\zeta\tau})}, \quad \zeta \equiv \sqrt{\kappa_R^2 + 2\sigma_R^2}.$$

The underlying stock is assumed to pay a constant dividend yield, denoted by δ , and its price $S(t)$ changes, under the risk-neutral measure, according to the

² It is noted that the exogenous valuation framework can be derived from a general equilibrium in which the volatility risk, interest risk and jump risk are priced. See Bakshi and Chen (1997a) and Bates (1996a, 1999) for details.

jump-diffusion process below:

$$\frac{dS(t)}{S(t)} = [R(t) - \delta - \lambda \mu_J] dt + \sqrt{V(t)} d\omega_S(t) + J(t) dq(t), \tag{3}$$

where unexpected percentage price changes have a diffusion component, $\sqrt{V(t)} d\omega_S(t)$, and a jump component, $J(t) dq(t)$. The size of the diffusion component is determined by $V(t)$, which represents, absent of any jump occurring, the level of (stochastic) return variance attributable to diffusion variations. For tractability, let $V(t)$ also follow a square-root process:

$$dV(t) = [\theta_v - \kappa_v V(t)]dt + \sigma_v \sqrt{V(t)} d\omega_v(t), \tag{4}$$

where κ_v , θ_v/κ_v , and σ_v respectively reflect the speed of adjustment, the long-run mean, and the variation coefficient of $V(t)$. The intensity of the jump component is measured by λ , whereas the size of percentage price jumps at time t is represented by $J(t)$ (which is lognormally, identically, and independently distributed over time with unconditional mean μ_J), that is,

$$\ln[1 + J(t)] \sim N(\ln[1 + \mu_J] - \frac{1}{2}\sigma_J^2, \sigma_J^2), \tag{5}$$

for some constant σ_J . In (3), $q(t)$ is a Poisson counter with $\Pr\{dq(t) = 1\} = \lambda dt$ and $\Pr\{dq(t) = 0\} = 1 - \lambda dt$. Finally, let $\text{Cov}_t[d\omega_S(t), d\omega_v(t)] \equiv \rho dt$, and assume that $q(t)$ and $J(t)$ are uncorrelated with each other or with $\omega_S(t)$ and $\omega_v(t)$. Under the assumed framework in (3)–(5), the total return variance consists of two components:

$$\frac{1}{dt} \text{Var}_t\left(\frac{dS(t)}{S(t)}\right) = V(t) + V_J(t), \tag{6}$$

where

$$V_J(t) \equiv \frac{1}{dt} \text{Var}_t[J(t) dq(t)] = \lambda[\mu_J^2 + (e^{\sigma_J^2} - 1)(1 + \mu_J)^2]$$

is the instantaneous variance of the jump component.

These assumptions are fairly general and can capture many features of empirical return distributions. Note that all the structural parameters, such as θ_R , κ_R , θ_v , κ_v and μ_J , are given under the risk-neutral measure, not under the true probability measure. Consequently, they may differ from their true-probability counterparts because of the risk-premium adjustments respectively for interest rate risk, volatility risk and jump risk. In general, the smaller the risk premiums

for these risks, the closer these parameters to their true-probability counterparts. See, for example, Bates (1996a) for a related discussion.

2.1. The option pricing formula

With the stochastic setup in (3)–(5), consider a European put option written on the stock with strike price K and term-to-expiration τ . Then, its time- t price $P(t, \tau)$ must, by a standard argument, be

$$P(t, \tau) = E_t^Q \left\{ \exp \left(- \int_t^{t+\tau} R(u) du \right) \max(0, K - S(t + \tau)) \right\}, \tag{7}$$

where $E_t^Q(\cdot)$ is the expectations operator with respect to the risk-neutral measure. Then solving the conditional expectation, as in Bakshi et al. (1997), we arrive at the following put option formula:

$$P(t, \tau) = KB(t, \tau) \{1 - \Pi_2(t, \tau)\} - S(t) e^{-\delta \tau} \{1 - \Pi_1(t, \tau)\}, \tag{8}$$

where the risk-neutral probabilities, Π_1 and Π_2 , are recovered from inverting the respective characteristic functions:

$$\Pi_j(t, \tau; S, R, V) = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \operatorname{Re} \left[\frac{e^{-i\phi \ln[K]} f_j(t, \tau, S(t), R(t), V(t); \phi)}{i\phi} \right] d\phi, \tag{9}$$

where $\operatorname{Re}[\cdot]$ stands for the ‘real part of’ operator and for $j = 1, 2$, the characteristic functions $f_1(t, \tau)$ and $f_2(t, \tau)$ are displayed respectively in (A.1) and (A.2) of the Appendix.

The put option valuation formula in (8) contains most existing models as special cases. For example, we obtain (i) the BS model (with constant dividend yield) by setting $\lambda = 0$ and $\theta_R = \kappa_R = \sigma_R = \theta_v = \kappa_v = \sigma_v = 0$; (ii) the SV model by setting $\lambda = 0$ and $\theta_R = \kappa_R = \sigma_R = 0$; (iii) the SVSI model by setting $\lambda = 0$; and (iv) the SVJ model by letting $\theta_R = \kappa_R = \sigma_R = 0$. In deriving each special case from (8), one may need to apply L’Hopital’s rule.

2.2. Option deltas under alternative models

Before examining the relative performance of the alternative models, we first look at the extent to which the BS, the SV, the SVSI, and the SVJ models can yield different deltas (or sensitivities to sources of risk) for a given option. The goal is to demonstrate analytically that model differences can show up more

dramatically using long-term options than short-term options. For this purpose, use the general model in (8) as the point of discussion and note that there are three sources of stochastic variation over time: price risk $S(t)$, volatility risk $V(t)$ and interest rate risk $R(t)$. Thus, there are three put option deltas of interest:

$$\Delta_S(t, \tau; K) \equiv \frac{\partial P(t, \tau)}{\partial S} = -e^{-\delta \tau}(1 - \Pi_1) \leq 0, \tag{10}$$

$$\Delta_V(t, \tau; K) \equiv \frac{\partial P(t, \tau)}{\partial V} = S(t)e^{-\delta \tau} \frac{\partial \Pi_1}{\partial V} - KB(t, \tau) \frac{\partial \Pi_2}{\partial V}, \tag{11}$$

$$\Delta_R(t, \tau; K) \equiv \frac{\partial P(t, \tau)}{\partial R} = S(t)e^{-\delta \tau} \frac{\partial \Pi_1}{\partial R} - KB(t, \tau) \left\{ \frac{\partial \Pi_2}{\partial R} + \varrho(\tau)[1 - \Pi_2] \right\}, \tag{12}$$

where, for $h = V, R$ and $j = 1, 2$,

$$\frac{\partial \Pi_j}{\partial h} = \frac{1}{\pi} \int_0^\infty \text{Re} \left[\frac{1}{i\phi} e^{-i\phi \ln|K|} \frac{\partial f_j}{\partial g} \right] d\phi. \tag{13}$$

Specializing these expressions according to each respective model’s assumptions, we obtain the deltas for the BS, the SV, the SVSI, and the SVJ models. These analytical expressions are later applied to construct hedging strategies.

To compare across these models, we focus on the stock-price delta, $\Delta_S(t, K; K)$, as this is the most relevant hedge ratio used in our hedging exercise to be conducted shortly. The structural parameters are estimated by using the method of simulated moments and taken from Table 2, and the implied spot volatility for each day is estimated by using cross-sectional S&P 500 put option data (the estimation method will be described in Section 4). For the SV model, for example, the estimated parameters are $\kappa_v = 1.12$, $\theta_v = 0.03$, $\sigma_v = 0.19$, $\rho = -0.25$, and $\delta = 0.02$. Given that the spot volatility changes over time, we choose the volatility estimates from three representative days, i.e., a low-volatility day, an average-volatility day, and a high-volatility day during the studied sample period, all based on the BS model’s implied volatility. For the sample period, the first-, second- and third-quartile implied volatilities respectively correspond to 22 November 1993, 19 May 1994 and 22 April 1994. For the SV, the SVSI and the SVJ models, their respective spot volatilities are taken separately from these same dates. On 22 November 1993, the implied spot volatilities are 14.76%, 11.70%, 13.21% and 10.48%, respectively, for the BS, the SV, the SVSI, and the SVJ models. On 19 May 1994, they are 15.88%, 13.03%, 14.00%, and 11.64% for each of the four models. On 22 April 1994, the spot volatilities are 16.74%, 14.35%, 15.85% and 13.50%, respectively.

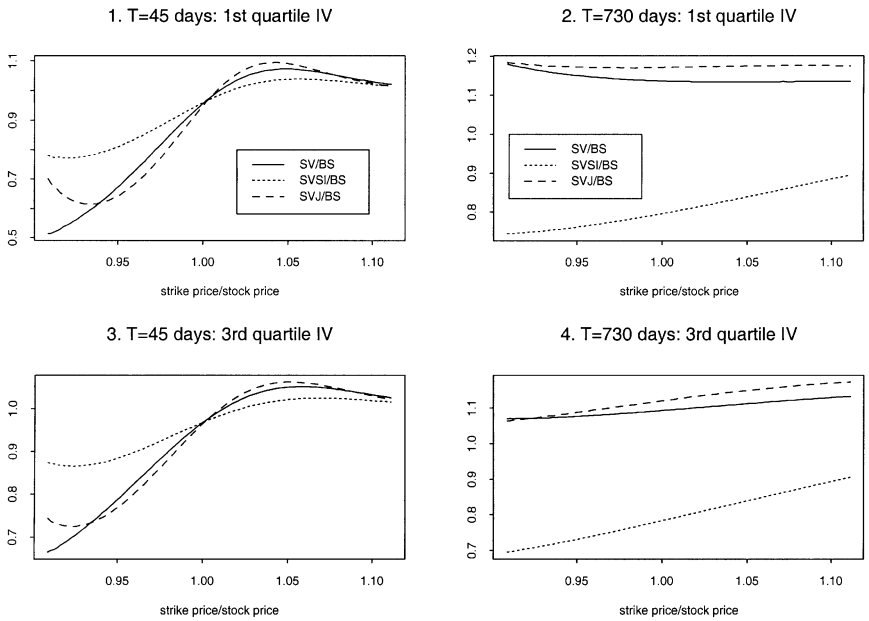


Fig. 1. The ratio of put option delta (Δ_S) with respect to underlying stock price between the SV (the SVSI, or the SVJ) and the BS models, corresponding to two terms to expiration (45 and 730 d). Structural parameters for each model are estimated by using the method of simulated moments, and taken from Table 2. For the BS model, we use the first- and third-quartile implied volatilities, which correspond to 22 November 1993 and 22 April 1994. For the SV, the SVSI and the SVJ models, the spot volatilities are taken from respective dates. The spot stock price is fixed at \$450 and the strike price varies from \$400 to \$500.

For the delta calculations, we use two terms to expiration: 45 days (short-term) and 730 days (long-term). We set $R(t) = 3.00\%$, $S(t) = 450$, and vary the strike price from \$400 to \$500 (which is at most 50 points away from the spot price of 450), to generate a delta graph for each model. For ease of comparison, we divide the SV model's delta for a given option by its corresponding BS model's delta. This normalization is also applied to the SVSI and the SVJ, and the resulting delta ratios for these models relative to the BS are displayed in Fig. 1.³

Fig. 1 shows that the delta for a given put option differs substantially across the models, especially between the BS model and the others. Take the low-volatility day as an example. For short-term options, the deltas based on the SV and the SVJ are fairly close, but they all differ from the corresponding BS delta.

³ Plots using the second-quartile volatility are similar and are omitted.

For short-term out-of-the-money (OTM) and in-the-money (ITM) puts, the SV model yields respectively lower and higher delta values than the BS does. The difference is particularly large for deep OTM puts. For short-term at-the-money puts, the difference in delta among the four models is the smallest.

In contrast, for long-term options, the difference among the BS, the SV, the SVSI and the SVJ models becomes more pronounced. Again, take the low-volatility day as an example. In Fig. 1, compared to the SV and SVJ models, the BS model understates every long-term put's delta (regardless of moneyness) by as much as 20%. The SV and the SVSI models yield significantly different delta values, with the difference ranging from 22% (for deep in-the-money puts) to 40% (for deep out-of-the-money puts).

2.3. State-price densities under alternative models

We can further understand model differences by comparing their respective state-price densities, or SPDs. As is known, the SPD summarizes and captures all the essential pricing structure of a candidate option pricing model. That is, if two models result in different pricing performance, it must be that their associated SPDs are accordingly different; conversely, if the SPDs implicit in two option pricing models possess distinct shapes, their pricing performance must differ accordingly (provided that a large enough sample of options is used in the estimations so as to fully reflect their SPD differences).⁴ By a result from Breeden and Litzenberger (1978), the SPD for the general model is

$$\text{SPD}(t, \tau) = \frac{1}{\pi} \int_0^{\infty} \text{Re}[\exp(-i\phi + 1) \ln[S(t + \tau)] + \ln[f_2(t, \tau)]] d\phi, \quad (14)$$

where $f_2(t, \tau)$ is the characteristic function given in (A.2). The respective SPDs for the BS, the SV, the SVSI, and the SVJ models each share the same functional form as given in (14), except that $f_2(t, \tau)$ need be adjusted for each model's assumptions.

To study the SPDs of the models under consideration, we utilize two separate sets of parameter estimates. For the first exercise, we pick a particular day (4 April 1994) and use all the put prices of that day to back out each given model's structural parameters. We choose 4 April 1994 for this purpose because this day's BS implied volatility is the highest (19.28%) during our September

⁴ Note that a snap-shot picture of a model's SPD captures mostly the model's cross-sectional pricing structure at a given point in time, and it may not tell one much about the model's dynamic fit of option prices. Thus, the cross-model SPD comparison conducted in this section is mostly related to the models' relative pricing, but not hedging, performance.

1993–August 1994 sample period. Using the parameters implied by a high-volatility day, we can see more clearly how the models perform relatively under different market conditions.⁵ Specifically, we take all available S&P 500 puts traded on 4 April 1994, and separately back out each model's structural parameters and spot volatility by minimizing the sum of squared fitting errors across the options. For the BS model, we have $\sqrt{V(t)} = 19.28\%$ and $\delta = 0.02$; For the SV, $\sqrt{V(t)} = 19.36\%$, $\kappa_v = 1.44$, $\theta_v = 0.05$, $\sigma_v = 0.49$, $\rho = -0.79$, and $\delta = 0.02$; For the SVJ, $\sqrt{V(t)} = 19.25\%$, $\kappa_v = 1.52$, $\theta_v = 0.05$, $\sigma_v = 0.49$, $\rho = -0.80$, $\lambda = 0.30$, $\mu_J = -0.12$, $\sigma_J = 0.03$, and $\delta = 0.02$; For the SVSI, $\sqrt{V(t)} = 19.27\%$, $\kappa_R = 0.62$, $\theta_R = 0.03$, $\sigma_R = 0.03$, $\kappa_v = 0.94$, $\theta_v = 0.04$, $\sigma_v = 0.48$, $\rho = -0.80$, and $\delta = 0.02$. Since the correlation coefficient ρ plays a crucial role in internalizing the skewness level of the stock's return distribution, we separately consider three cases: $\rho = -0.80$, $\rho = 0$, and $\rho = 0.80$. Corresponding to each ρ level (where applicable) and either one or two terms to expiration (45 d and 2 yr), Fig. 2 plots the SPD curves for each of the four models.

Let us first discuss the short-term SPD curves in the left column of Fig. 2. At both $\rho = -0.8$ and $\rho = 0$, the SV, the SVSI, and the SVJ models generate virtually identical SPDs for short-term options, with a slight departure occurring between the SVJ and the other two stochastic-volatility models in the upper tail (i.e. to the far right of the spot price point). This means that differential pricing and hedging performance between the SVJ and the other two is unlikely to occur when they are applied to price short-term OTM puts, and that only when they are applied to deep ITM puts (and deep OTM calls) can differences be observed between these models. Yet, compared to the BS model's SPD (which is symmetric around zero), the SPDs of the three stochastic-volatility models are distinctly different: at $\rho = -0.8$, for instance, they all assign more (risk-neutral) probability mass to the far left tail, and less mass to the far right tail, implying that these models' SPDs are more negatively skewed than the BS SPD. Therefore, the models with stochastic volatility can potentially correct the BS model's tendency to underprice deep OTM puts and overprice deep OTM calls. Even though short-term options may not help distinguish among the stochastic-volatility models, they can at least help separate these alternatives from the BS. At $\rho = 0.8$, the SPDs of the models with stochastic volatility are virtually the mirror image of their counterparts with $\rho = -0.8$.⁶

⁵ From September 1993 to August 1994, the average volatility of the S&P 500 is about 15%. But, more recently the market has been far more volatile. For example, the CBOE volatility index used to be in the low teens (before 1996); This index has stayed above 20% and sometimes gone above 30% since late 1996. It is therefore important to understand how the models may differ under more volatile market conditions.

⁶ For stocks, changes in volatility and the underlying asset price are usually negatively correlated. For foreign exchange rates, however, the two are positively correlated in general.

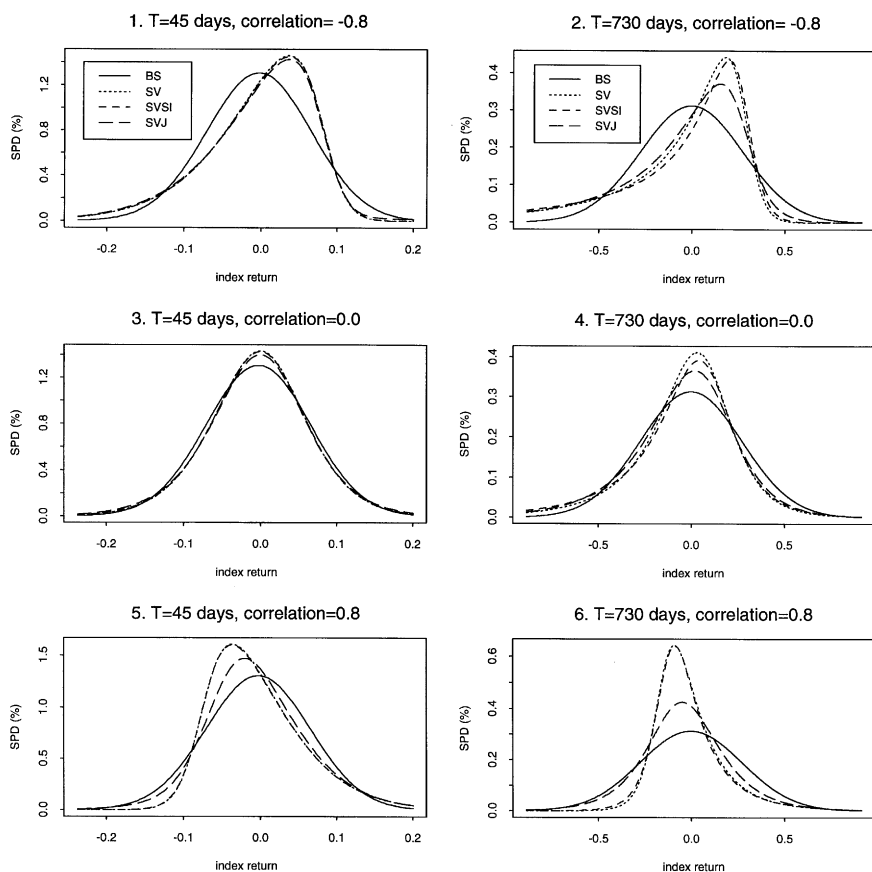


Fig. 2. The state-price densities of the BS, the SV, the SVSI and the SVJ models, corresponding to two time horizons (45 and 730 d) and three correlation levels ($\rho = -0.80, 0$, and 0.80). Other structural parameter values used in the computation are backed out from the put prices of 4 April 1994. For this date, the backed-out ρ value is -0.80 . For the 45-d horizon, the range of rates of stock return (during the 45-d period) is from -20% to 20% , whereas for the 730-d horizon the return range is from -80% to 80% .

The long-term SPD curves in Fig. 2 still show significantly different pricing structures between the BS and its stochastic-volatility alternatives. But, more importantly, the SPDs of the SV, the SVSI, and the SVJ also exhibit different shapes now. Focus for the moment on $\rho = -0.8$. The SVSI's SPD is almost a right-shifted version of the SV's SPD, which means that the SVSI discounts those payoffs that are within the -40% to 20% neighborhood of the current spot price, more than the SV model. But, the SVSI discounts those payoffs either in the far right or far left tail less than the SV does. The SVJ's SPD is, on the

other hand, distinct from those of the SV and the SVSI: it assigns more weight to both the entire lower tail and the far upper tail, but less weight to those payoffs that are between 0% and (about) 30% higher than the current spot price. Overall, long-term puts that are more than 50% out-of-the-money will be valued the most by the SVJ, the second most by the SVI and the SV, and the least by the BS. The disagreement among the SV, the SVSI, and the SVJ occurs mostly over the valuation of long-term ITM puts. Thus, having more ITM puts in an option sample should help more effectively differentiate among the alternative models.

Note that when $\rho = 0$, the SPDs of the models with stochastic volatility, regardless of the time horizon, all resemble the BS model's SPD and are symmetric around the current spot price, except that they have more weight concentrated in the middle. This indicates that when volatility and the underlying price changes are uncorrelated with each other, the alternative models to the BS will unlikely generate the levels of return skewness and kurtosis necessary to reconcile the BS implied-volatility smiles. Thus, stochastic-volatility models that assume $\rho = 0$ (e.g., Hull and White, 1987; Stein and Stein, 1991) should not be expected to perform much better than the BS.

In Fig. 3, we use a different set of structural parameter estimates to construct the SPD curves. For the BS model, its second-quartile implied volatility obtained from the September 1993–August 1994 sample period is used as the input for the BS SPD.⁷ For the other three models, their respective parameters are taken from Table 2 and based on the entire sample period (the estimation method to be explained shortly). These parameter values should thus reflect the 'average' return distributions implied by all daily put prices in our sample, rather than only by put prices on some special days. Consequently, these parameter estimates may not be as 'extreme'. Specifically, among the more important structural parameters, we have $\rho = -0.25$ and $\sigma_v = 0.19$ for the SV model; $\rho = -0.26$ and $\sigma_v = 0.20$ for the SVSI model; and $\rho = -0.21$ and $\sigma_v = 0.24$ for the SVJ. These parameter values are respectively much lower in magnitude than their counterparts used in constructing the SPDs in Fig. 2. Therefore, relative to Fig. 2, the SPD curves in Fig. 3 should be more 'smoothened' out. Indeed, the SPD curves in Fig. 3 share only one feature with those in Fig. 2 corresponding to $\rho = -0.80$: the SPDs of the SV, the SVSI, and the SVJ are all shifted rightward relative to the SPD of the BS model, and the former all assign more risk-neutral probability to the far-left returns and less to the far-right returns (helping flattening out the BS model's 'volatility smile'). But, with the method of simulated moments parameter estimates in Table 2,

⁷ SPD curves based on the first- and third-quartile BS implied volatilities are similar in shape, and are omitted.

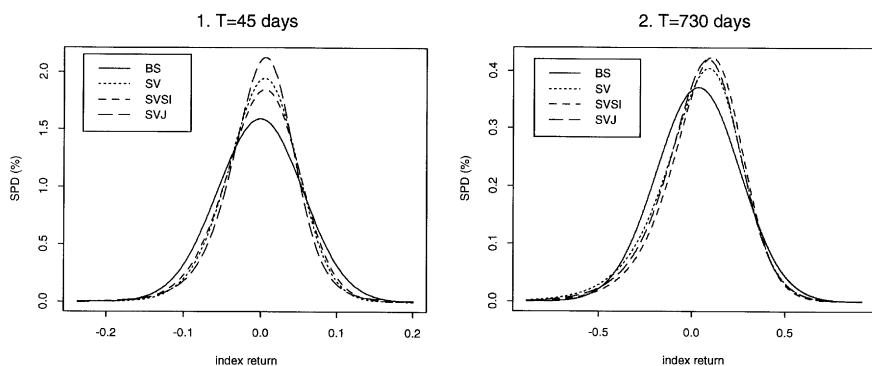


Fig. 3. The respective state-price densities of the BS, the SV, the SVSI and the SVJ models, corresponding to two time horizons (45 and 730 d). Structural parameters for each model are estimated by using the method of simulated moments, and taken from Table 2. For the BS model, we use the second-quartile implied volatility, which corresponds to 19 May 1994. For the SV, the SVSI and the SVJ models, the spot volatilities are taken from the same date. For the 45-d horizon, the range of rates of stock return (during the 45-d period) is from -20% to 20% , whereas for the 730-d horizon the return range is from -80% to 80% .

Fig. 3 shows that long-term options may no longer help better differentiate the models with stochastic volatility than short-term options.

Our exercise in this subsection has demonstrated that one can analytically examine the differential pricing (not necessarily hedging) performance among distinct models by comparing their respective SPDs. Any significant performance improvement by a candidate model must come from the model's SPD possessing a shape that is more consistent with the empirically observed option-price structure. The SPD curves in Figs. 2 and 3 show that while it is relatively easy to find a set of option prices to differentiate between the BS and its stochastic-volatility alternatives, it is more difficult to find the 'right' set of option contracts that allows one to distinguish among the three models with stochastic volatility. On more volatile days, long-term options can differentiate the models better than their short-term counterparts. But, on an 'average' day, long-term options may not offer much additional pricing information. Given that the models have been shown to have distinct delta values even on those 'average' days, hedging performance may thus be a more important yardstick for judging the models.

3. The S&P 500 options and LEAPS

Two sets of option contracts on the S&P 500 index are used in our empirical exercise.

- Regular S&P 500 index options (SPX). These options have up to one year to expiration.

- S&P 500 Long-term Equity Anticipation Securities (LEAPS), which usually expire two to three years from the date of listing.

Both types of option are European in nature and share the same trading hours and the same settlement arrangements. They are both listed on the Chicago Board Options Exchange and hence subject to the same regulations (e.g., minimum tick sizes) and margin requirements, and have the same market makers. The only real difference in contract design is that a LEAPS contract is one tenth of the size of a regular S&P 500 option contract. Consequently, when converted to a regular S&P 500 option contract size, LEAPS contracts usually have wider bid-ask spreads. Because of the contract size difference, S&P 500 LEAPS are not convertible to regular S&P 500 contracts even if a LEAPS contract has less than a year (or a month) to expiration. There are often LEAPS and regular S&P 500 options traded at the same time that have the same expiration date.

The sample period for our study extends from 1 September 1993 to 31 August 1994. The intradaily bid and ask quotes for the options are obtained from the Berkeley Option Database. To ease computational burden, for each day in the sample, only the last reported bid-ask quote (prior to 3:00 p.m. Central Daytime) of each option contract is employed in the study. Note that the recorded spot S&P 500 index values are not the daily closing index levels. Rather, they are the corresponding index levels at the moment when the option bid-ask quote was recorded. By far, most traded LEAPS contracts are puts, especially out-of-the-money puts (reflecting investors' desire for portfolio insurance). For example, in the original bid-ask quote (transaction) data the LEAPS sample contains 10,363 (5511) puts and 4558 (162) calls. To lessen the impact of illiquid LEAPS calls on the empirical results, we use in our empirical analysis only put options, both regular and LEAPS.

Following a standard practice, we use the average of a put's bid and ask price as a stand-in for the unique market value of the put. Daily Treasury bill and note rates with maturities up to three years are obtained from *DataStream International*. The 30-day Treasury-bill rate is used as the surrogate for the instantaneous interest rate $R(t)$.

In addition to eliminating option observations with obvious recording errors, we apply two exclusion filters to construct the final put sample. First, we retain quotes that have more than six days to expiration (to avoid the expiration-related price effects). Second, we eliminate price quotes lower than $\$ \frac{3}{8}$ (to avoid the impact of price discreteness). The remaining sample contains 4074 LEAPS put quotes and 8018 regular put quotes. We partition this final sample of 12,092 puts into three moneyness and three term-to-expiration classifications. A put option is said to be *at-the-money* (ATM) if its $K/S \in (0.97, 1.03)$; *out-of-the-money* (OTM) if $K/S \leq 0.97$; and *in-the-money* (ITM) if $K/S \geq 1.03$. A put option is said to be a *short-term* option if it has less than 60 d to expiration; *medium-term*

Table 1

Sample properties of S&P 500 index puts in each moneyness-maturity category, we report the average quoted bid-ask mid-point price, the average effective spread (ask price minus the bid-ask mid-point), and the total number of observations (in curly brackets)

| Moneyiness | SPX | | LEAPS | | |
|------------|--------------------|---------------------------|--------------------------|----------------------------|----------|
| | Days-to-expiration | | | | |
| K/S | < 60 | 180–365 | ≥ 365 | Subtotal | |
| OTM | < 0.94 | 1.18 (0.04) {344} | 7.17 (0.20) {690} | 11.86 (0.94) {1,440} | {2,474} |
| | 0.94–0.97 | 1.97 (0.07) {905} | 12.23 (0.27) {584} | 21.33 (1.05) {582} | {2,071} |
| | 0.97–1.00 | 3.75 (0.11) {1,281} | 16.11 (0.29) {700} | 23.08 (1.05) {687} | {2,668} |
| ATM | 1.00–1.03 | 9.62 (0.23) {1,215} | 21.34 (0.34) {532} | 29.15 (1.25) {707} | {2,454} |
| | 1.03–1.06 | 20.11 (0.38) {588} | 27.82 (0.39) {412} | 33.08 (1.43) {328} | {1,328} |
| ITM | ≥ 1.06 | 50.33 (0.48) {439} | 36.76 (0.44) {328} | 40.07 (1.43) {330} | {1,097} |
| | Subtotal | {4,772} | {3,246} | {4,074} | {12,092} |

Note: The sample period extends from 1 September 1993 through 31 August 1994 for a total of 12,092 puts. S denotes the spot S&P 500 index level and K is the exercise price. OTM, ATM and ITM denote out-of-the money, at-the-money, and in-the-money options, respectively.

option if it has between 180 and 365 d to expiration; and *long-term* option if it has more than a year to expiration.

In Table 1, we report three summary statistics: (a) the average bid–ask mid-point price; (b) the average effective bid–ask spread (i.e., the ask price minus the bid–ask midpoint); and (c) the total number of observations. A representative short-term deep OTM put costs \$1.18, versus \$11.86 for an average deep OTM LEAPS put. As expected, the average put price increases with the time to expiration. The effective bid–ask spread also increases with the time to expiration. For example, for ATM puts the average effective bid–ask spread is \$0.11

for short-term options, versus \$1.05 for LEAPS. OTM LEAPS (regular SPX) account for 49% (31%), and ATM LEAPS (regular) 34% (46%) of the total sample.

4. Estimating structural parameters and spot volatility

In implementing each candidate model, we estimate its relevant structural parameters and the spot volatility in two separate steps. First, we estimate the structural parameters by using the method of simulated moments (MSM) (e.g., Duffie and Singleton, 1993; Gouriéroux and Monfort, 1996).⁸ The MSM is chosen over the maximum likelihood method or the GMM because (i) volatility is unobservable, which hinders the use of the likelihood method, and (ii) the GMM requires a closed-form expression for each relevant moment of option prices, which is not readily available for option pricing models with stochastic volatility.⁹ After obtaining the structural parameters, in the second step we back out each day's spot volatility from the day's observed option prices by minimizing the sum of squared in-sample pricing errors. This step generates a time series of implied spot volatility for each given model.

4.1. Procedure for estimating structural parameters

Take the SV model as an example to illustrate the MSM procedures. Recall that our option sample is partitioned into 9 categories: short-term OTM, ATM, and ITM options, median-term OTM, ATM, and ITM options, and long-term OTM, ATM, and ITM options. We label them as categories $l = 1, 2, \dots, 9$. For each day in the sample, we take one observed option price from each of the 9 moneyness-maturity categories. Thus, there are 9 option-price time series, each consisting of options with similar moneyness and similar time to expiration. Every series has $T = 252$ observations.

Let $\hat{P}^j(t, \tau_{tj}, K_{tj})$ be the observed put price of category j on day t , and $P^j(t, \tau_{tj}, K_{tj}; \Phi)$ the theoretical price for a given $\Phi = (\kappa_v, \theta_v, \sigma_v, \rho, \delta)$, where τ_{tj} and K_{tj} are respectively the term-to-expiration and strike price of the

⁸ We would like to thank Eric Ghysels for suggesting the MSM estimation approach.

⁹ In an earlier version, we backed out the structural parameters together with the spot volatility from each day's option prices, one estimation per day as it is done in Bakshi et al. (1997) and Bates (1996a,b). That method effectively allows the parameters to vary from day to day, which is inconsistent with the model assumptions. The MSM, on the other hand, avoids this inconsistency by requiring the parameters to be constant over time.

observed put option in category j on day t . Consider the following disturbance terms:

$$g_t(\Phi) \equiv \begin{pmatrix} \frac{\hat{P}^1(t, \tau_{t1}, K_{t1})}{K_{t1}} - \frac{E(P^1(t, \tau_{t1}, K_{t1}; \Phi))}{K_{t1}} \\ \vdots \\ \frac{\hat{P}^9(t, \tau_{t9}, K_{t9})}{K_{t9}} - \frac{E(P^9(t, \tau_{t9}, K_{t9}; \Phi))}{K_{t9}} \end{pmatrix},$$

and the corresponding moment restrictions, $E[g_t(\Phi)] = 0$, where each option price is normalized by the strike price in order to at least partially neutralize the non-stationarity induced by changes in the underlying index. The MSM estimator of Φ is obtained by minimizing the quadratic form:

$$J_{T,M} \equiv \underset{\Phi}{\text{Argmin}} G'_T W_T G_T, \tag{15}$$

where $G_T(\Phi) = (1/T)\sum_{t=1}^T g_t(\Phi)$, W_T is the weighting matrix, M is the number of simulations, and $E[P^j(t, \tau_{tj}, K_{tj}; \Phi)]$ is approximated via simulation. Specifically, for each given value of $\Phi = (\kappa_v, \theta_v, \sigma_v, \rho, \delta)$, we conduct the simulation as follows:

1. Discretize the stock return and volatility processes of the SV model as

$$S(t + \Delta t) - S(t) = [R - \delta]S(t)\Delta t + \sqrt{V(t)}S(t)\varepsilon_s(t)\sqrt{\Delta t} \tag{16}$$

$$V(t + \Delta t) - V(t) = [\theta_v - \kappa_v V(t)]\Delta t + \sigma_v \sqrt{V(t)}\varepsilon_v(t)\sqrt{\Delta t} \tag{17}$$

2. Simulate a time series of two independent, standard normal processes

$$\begin{pmatrix} \varepsilon_s^*(t) \\ \varepsilon_v^*(t) \end{pmatrix},$$

where $t = 1, 2, \dots, T$.

3. Define

$$A = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix},$$

and generate a new vector:

$$\begin{pmatrix} \varepsilon_s(t) \\ \varepsilon_v(t) \end{pmatrix} = A^{1/2} \begin{pmatrix} \varepsilon_s^*(t) \\ \varepsilon_v^*(t) \end{pmatrix}.$$

The transformed vector is a bivariate normal process with zero mean and a variance–covariance matrix of A , where $\varepsilon_s(t)$ and $\varepsilon_v(t)$ have a correlation of ρ .

4. Construct the time series of $S(t)$ and $V(t)$, $t = 1, 2, \dots, T$, based on Eqs. (16) and (17) and using the simulated $\varepsilon_s(t)$ and $\varepsilon_v(t)$. The initial stock price is set to be the observed index level on the first day of the sample, and the initial value of volatility is chosen to be the long-term mean of volatility.¹⁰ The risk-free rate is the average spot rate during the sample period, and Δt equals one day.
5. Repeat Steps 2 and 4 for M times to obtain M sample paths of $\{S(t), V(t)\}$. Fixing a sample path of $\{S(t), V(t)\}$, calculate the date- t model price of the put option in the j th moneyness-maturity category, for each t and every j ($= 1, \dots, 9$). Thus, we obtain M different put-price paths for each of the 9 moneyness-maturity categories. For each t and j , the average of the M simulated prices is used to approximate the theoretical model price $P^j(t, \tau_{tj}, K_{tj}; \Phi)$. The average option price, $E[P^j(t, \tau_{tj}, K_{tj}; \Phi)]$, is then obtained by taking the time series average of simulated theoretical model price.

In the actual implementation, we set $M = 10$. As shown by Gouriéroux and Monfort (1996, p. 29), even a small number of simulations can achieve a practically sufficient level of efficiency. For example, with $M = 10$, the asymptotic relative efficiency is 90%.¹¹ The above simulation and estimation procedure is applied to each candidate model, adjusting for the model’s assumptions.

4.2. Estimating spot volatility

After obtaining the MSM parameter estimate, we back out the spot volatility for each model and for each day, by using all available put prices on that day. That is, let N be the observed number of put prices, and $\hat{P}_n(t, \tau_n, K_n)$ and $P_n(t, \tau_n, K_n, \hat{\Phi}; V(t))$ be respectively the observed and the model price of the n th put option. Then, using the MSM parameter estimate as input, we find day t ’s $V(t)$ by minimizing the sum of squared in-sample pricing errors:

$$SSE(t) \equiv \min_{V(t)} \sum_{n=1}^N |\hat{P}_n(t, \tau_n, K_n) - P_n(t, \tau_n, K_n, \hat{\Phi}; V(t))|^2, \tag{18}$$

¹⁰ We also consider alternative initial values and simulate $T + 500$ observations for each time series and throw away the first 500 observations. Overall, the corresponding results are qualitatively similar.

¹¹ We set $M = 10,000$ when estimate the optimal weighting matrix.

where $SSE(t)$ represents a goodness-of-fit statistic of day t 's put prices by the candidate model. Repeat this for each day of the sample to produce a time series of spot volatility.

Three sets of put options are separately used as the basis for implementing (18):

- *All options*: SPX regular and LEAPS puts of all strikes and terms to expiration. The corresponding average-spot-volatility estimates for the models are summarized in Table 2 under the heading 'All options'.
- *Short-term options*: All regular SPX options with less than 60 d to expiration. The daily-averaged estimates are reported in Table 2 under 'Short-Term options'.
- *Long-term options*: All S&P 500 LEAPS with over a year to expiration. The results are shown under 'Long-Term options'.

The three sets of spot volatility each serve a distinct purpose. The one based on 'All options' is consistent with the understanding that if a pricing formula holds empirically, then the same value of the spot volatility should fit all options of the same day well, regardless of moneyness and term to expiration. It is therefore of fundamental interest to see how well each model, when implemented with the same spot volatility for 'All options', performs in both pricing options out of sample and hedging options over time. But, there are also several considerations that justify treating short-term and LEAPS options separately. First, as regular and LEAPS options may possess differential information, it should be important to re-implement each model by separately using regular-options- and LEAPS-implied volatilities. Second, the models may show different abilities to price and hedge short-term options versus LEAPS. Thus, treating these options separately should help understand each model's potential more effectively. Finally, since LEAPS puts are usually more expensive than their short-term counterparts, the objective function in (18) tends to favor LEAPS at the expense of short-term options. Treating the two groups separately should correct, at least partly, such a bias.

4.3. MSM parameter estimates

Table 2 reports MSM parameter estimates,¹² average daily implied volatility, and the sum of squared fitting errors (SSE). First, note that the estimated

¹² We use the Newey and West (1987) method to adjust for heteroskedasticity and autocorrelations. The reported standard errors in parentheses are based on the upper bound of the asymptotic variance-covariance matrix, which is given by $(1 + 1/M)(D_0 W_0 D_0)^{-1}$, where $(D_0 W_0 D_0)^{-1}$ is the asymptotic variance-covariance matrix of the GMM estimator, with $D_0 = E[\partial G_T / \partial \Phi]$ and $W_0^{-1} = (\sum_{i=-\infty}^{\infty} E[g_i \cdot g_{i-1}'])$. For the BS model, δ is estimated via the MSM under the assumption that the return volatility is constant.

Table 2
Method of simulated moments (MSM) estimates of structural parameters and in-sample fit

| Model | Structural parameters | | | | | | | | | | | All options | | | Short-term options | | Long-term options | |
|-------|-----------------------|----------------|----------------|----------------|-----------------|----------------|----------------|----------------|----------------|-----------------|----------------|--------------------|-----------------|--------|--------------------|-------|-------------------|-------|
| | δ | κ_v | θ_v | σ_v | ρ | κ_R | θ_R | σ_R | λ | μ_J | σ_J | J_T [p-value] | IV (%) | SSE | IV (%) | SSE | IV (%) | SSE |
| BS | 0.02 (0.00) | | | | | | | | | | | 50.45 [< 0.01] | 15.76 (0.06) | 194.62 | 12.02 (0.11) | 10.43 | 16.47 (0.07) | 52.95 |
| SV | 0.02 (0.00) | 1.12 (0.08) | 0.03 (0.00) | 0.19 (0.03) | -0.25 (0.03) | | | | | | | 26.76 [< 0.01] | 13.04 (0.12) | 91.18 | 11.20 (0.11) | 8.41 | 13.81 (0.14) | 31.28 |
| SVSI | 0.02 (0.00) | 1.58 (0.11) | 0.03 (0.00) | 0.20 (0.03) | -0.26 (0.03) | 0.26 (0.09) | 0.04 (0.00) | 0.08 (0.01) | | | | 11.98 [< 0.01] | 13.87 (0.14) | 87.51 | 11.32 (0.11) | 8.13 | 15.15 (0.22) | 23.32 |
| SVJ | 0.02 (0.00) | 1.60 (0.15) | 0.03 (0.00) | 0.24 (0.06) | -0.21 (0.04) | | | | 0.78 (0.12) | -0.04 (0.00) | 0.06 (0.01) | 11.55 [< 0.01] | 13.40 (0.15) | 85.11 | 11.58 (0.11) | 4.42 | 14.56 (0.21) | 28.90 |

Note: The structural parameters for each option model are estimated by the method of simulated moments. The Newey–West’s heteroskedasticity-and- autocorrelation-consistent standard errors are reported in parentheses. Also reported is the J_T statistic (and the associated p-value) which is χ^2 -distributed with degrees of freedom df, where df for the BS, the SV, the SVSI and the SVJ models are 7, 4, 1, and 1, respectively. The implied volatility for each model is obtained by presetting the structural parameters and then minimizing the sum of squared in-sample fitting option errors. The reported implied volatility under the heading ‘IV’ denotes the average daily implied volatility and SSE the daily average sum of squared in-sample fitting errors. The implied volatility in the groups under ‘all options’, ‘short-term options’, and ‘long-term options’ are respectively obtained by using (i) all the available options of any maturity, (ii) only short-term options (with maturity less than 60 days), and (iii) only LEAPS options (with maturity longer than 365 days) in a given day as input into the estimation.

dividend yield δ is about 2% for all four models, which is consistent with the recent actual dividend yield levels for the S&P 500. The implied underlying asset price process, however, differs across the models. For example, the mean-reversion parameter κ_v ranges from the SV's 1.12, the SVSI's 1.58 to the SVJ's 1.60, while θ_v is 0.03 for all the three models with stochastic volatility, implying a long-run volatility level of 16.36% for the SV, 13.77% for the SVSI, and 13.69% for the SVJ. The volatility process is the least volatile (as measured by σ_v) according to the SV model, and the most volatile according to the SVJ. In addition, the SVJ yields an average jump frequency of 0.78 times per year, with an average jump size of -4% . The correlation parameter ρ is persistently negative for all three models, -0.21 for the SVJ and -0.25 for the SV model. Note that our MSM estimates of ρ and σ_v are respectively much smaller in magnitude than their daily cross-sectional-options-implied counterparts in Bakshi et al. (1997) and Bates (1996a, b). In these earlier studies, the estimate for ρ is typically around -0.70 , and that for σ_v about 0.40. Therefore, the MSM estimates-implied return distributions by the models are respectively far less skewed, and with lower levels of kurtosis, than the daily cross-sectional prices based distributions. A possible reason for this divergence is that the information contained in the joint time series of option prices and stock returns differs from that in the daily cross-sectional option prices.

For the SVSI model, the MSM estimates of κ_R and θ_R are 0.26 and 0.04, respectively, with the long-run interest rate estimated at 15.38%. The variation coefficient of the interest rate is 0.08.

In Table 2, we report the minimized J_T -statistic, which is distributed χ^2 with degrees of freedom df (the number of moment conditions minus the number of parameters in Φ). The J_T -statistic is 50.49 for the BS model; 26.76 for the SV model; 11.98 for the SVSI model; and 11.55 for the SVJ model, indicating that the SVJ achieves the best fit, followed by the SVSI, the SV and the BS. Thus, each additional feature lead to some pricing improvement. But, based on the associated p-values all the four models are rejected at the 1% confidence level, suggesting that the models are all misspecified statistically.

From the columns under 'all options', the volatility level implicit in the puts differs significantly across the models, ranging from the SV's 13.04% to the BS' 15.76%. This means that the BS mainly relies on a high volatility value to achieve its in-sample fit. According to the SSE values, all three models with stochastic volatility provide a much better fit than the BS. For example, the SV and the SVJ respectively reduce the BS model's SSE by 53% and 56%.

5. Differential information in regular options versus LEAPS

The purpose of this section is to investigate the differential information embedded in short-term options versus LEAPS. We start with Table 3, which

Table 3
Implied volatility based on the Black–Scholes model

| Moneyness | SPX | | LEAPS |
|-----------|--------------------|---------|-------|
| | Days-to-expiration | | |
| K/S | < 60 | 180–365 | ≥ 365 |
| < 0.94 | 17.48 | 16.07 | 17.17 |
| 0.94–0.97 | 15.13 | 15.88 | 16.60 |
| 0.97–1.00 | 12.72 | 15.33 | 16.22 |
| 1.00–1.03 | 10.85 | 14.13 | 15.72 |
| 1.03–1.06 | 13.43 | 13.75 | 16.09 |
| ≥ 1.06 | 25.90 | 13.41 | 15.19 |

Note: Reported in each moneyness-maturity category is the average Black–Scholes implied volatility. For each put option contract, the implied volatility is obtained by setting $\delta = 2\%$ (from Table 2) and then inverting the Black-Scholes model. The moneyness of the put option contract is determined by K/S , where S denotes the spot S&P 500 index level and K is the exercise price. The sample period extends from September 1, 1993 through August 31, 1994 for a total of 12,092 puts.

lists the average daily BS implied volatility across moneyness and maturity. Two patterns can be observed in the table. First, for short-term puts the implied volatility is U-shaped, whereas for medium- and long-term puts the implied volatility is declining as the put goes from OTM to ITM. Among the three maturity groups, the LEAPS' implied volatility exhibits the least variation with moneyness. Next, at different moneyness levels the term structure of implied volatilities can be U-shaped, flat, or upward sloping, depending on whether it is deep OTM (or deep ITM), near the money, or ATM. Overall, these patterns are consistent with the existing findings based on relatively short-term options (e.g., Bakshi et al., 1999; Bates, 1996b; Rubinstein, 1994).

Similar difference in information between short term and LEAPS options can be found in Table 2, under Short-Term and Long-Term options. Regardless of the model used, the option-implied volatility differs significantly between the two maturity types, even though the models yield distinct levels of volatility difference. For example, based on the SV, the volatility difference between LEAPS and short-term puts is 2.61%, whereas based on the BS it is 4.45%. In addition, for a given model the in-sample pricing fit as measured by the SSE also differs between the two maturity types. In absolute terms, each model seems to fit short-term puts better than LEAPS. Across the models, the SVJ fits short-term puts the best while the SVSI fits the LEAPS puts the best.

To further examine the difference between the maturity types, we plot in Fig. 4 the daily implied-volatility paths respectively for short term and LEAPS puts, based on both the BS and the SV models. From Fig. 4 it is clear that the two maturity types imply significantly different information about the underlying

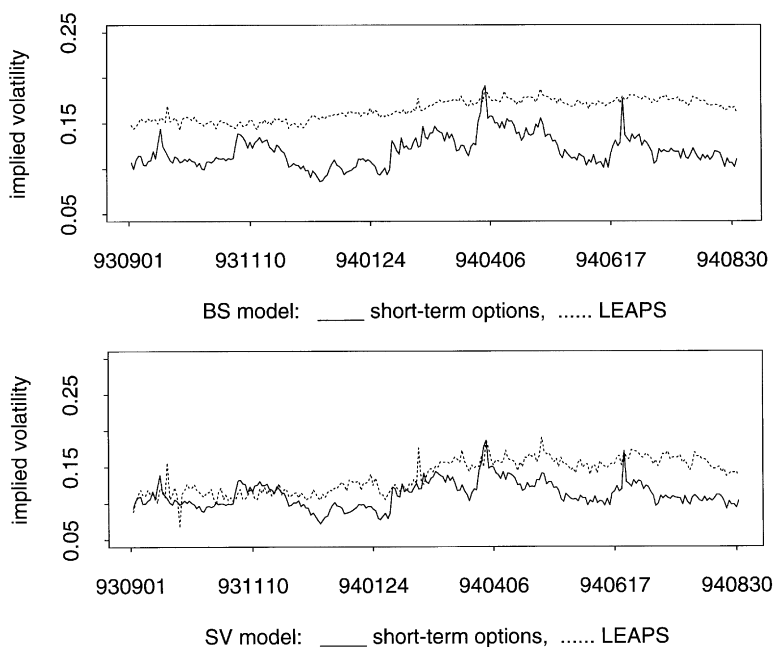


Fig. 4. Daily implied-volatility time series based on (i) puts with less than 60 d to expiration and (ii) LEAPS puts with longer than 365 d to expiration. The top graph is based on the BS model, while the bottom graph on the SV model. The sample period extends from 1 September 1993 through 31 August 1994.

price dynamics. Based on the BS model, for instance, the short-term implied volatility fluctuates more than the LEAPS implied volatility. Especially, the short-term volatility appears to jump up and down from time to time. Clearly, the short-term volatility is more sensitive to changes in market condition. When the SV model is applied, both volatility paths are still apart from each other, but they resemble each other more than when the BS model is applied. Therefore, the information embedded in LEAPS helps further illuminate the improvement by the SV beyond the BS model.

One noticeable feature of the LEAPS implied volatility is its long-term dependence over time. In Fig. 5, we plot the autocorrelation functions (ACF) for short-term and LEAPS implied volatility, together with their Bartlett confidence bands for no serial dependence. For this exercise, the implied volatilities are based on the BS model. The top graph shows that the ACF of the LEAPS implied volatility exhibits a geometric decay pattern. It decays slowly and disappears at the 70-d lag. The Ljung and Box portmanteau-test statistic is 6928.33 (which tests the joint significance of the ACFs from lag 1 to lag 70), and

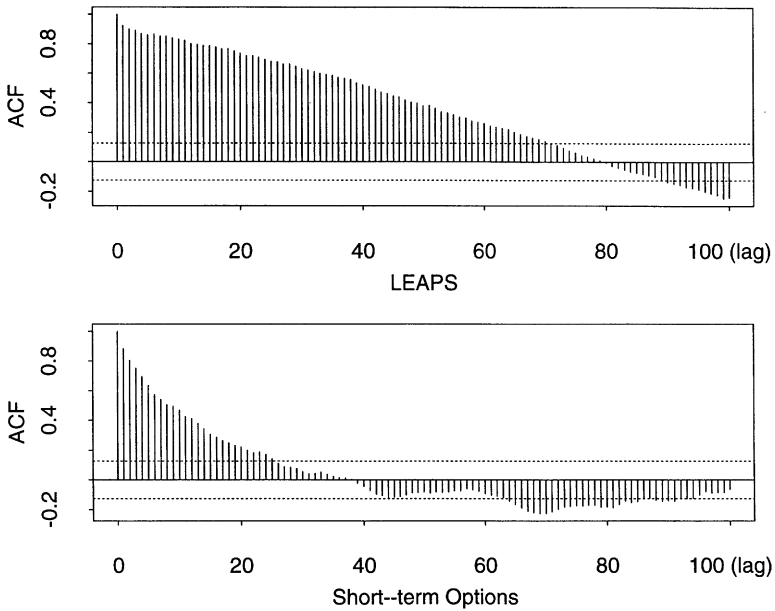


Fig. 5. The autocorrelation function (ACF) of the BS model implied volatility from (i) LEAPS puts with longer than 365 d and (ii) puts with less than 60 d to expiration. The dotted lines are the Bartlett 95% confidence bands. The sample period extends from 1 September 1993 through 31 August 1994.

it is significant at 1% level. In contrast, the ACF for the short-term implied volatility shows a hyperbolic decay pattern, and it exhibits no clear serial dependence for lags beyond 25 d. This long-memory property in LEAPS-implied volatility is consistent with the recent finding by Bollerslev and Mikkelsen (1996, 1999), Ding et al. (1993), and Ding and Granger (1996).¹³ Therefore, short-term options and LEAPS imply substantially different characteristics for the modeled volatility process: the former requires the volatility process to be much less persistent. This can be interpreted as evidence of the BS model's misspecification. Alternatively, it can be viewed as a challenge for the generalized models.

¹³ Bollerslev and Mikkelsen (1996, 1999) develop a new class of fractionally integrated GARCH (FIGARCH) and EGARCH (FIEGARCH) models to characterize the long-term dependence in the stock market volatility. Based on simulated GARCH-class processes, they find the FIEGARCH model results in the lowest pricing errors for LEAPS relative to other GARCH family models. See also Comte and Renault (1996) for long-memory continuous time models.

6. Out-of-sample pricing of regular and LEAPS puts

The preceding evidence suggests that each generalization to the BS model helps to improve the in-sample fit of option prices. Still, the question is: how large is the improvement in terms of *out-of-sample pricing and hedging*? Note that any model with good in-sample fit does not necessarily perform well out of sample. Our first step in answering this question is to examine the relative magnitude of out-of-sample option pricing errors. As before, our emphasis is not only on investigating the similarities and differences between short-term options and LEAPS, but also on evaluating the alternative models.

In conducting out-of-sample pricing test, we apply the MSM parameter estimates from Table 2 for every day and every option (regardless of moneyness and maturity) in the sample. But, in pricing the current day's puts, we use (i) the volatility implied by the *previous day's* put prices as the corresponding model's spot volatility and (ii) the current day's index level and yield curve. Option errors are then measured by (i) the *average dollar pricing error*, which is the average option pricing error in a given moneyness/maturity category, and (ii) the *mean absolute deviation*. To arrive at the latter measure, we first subtract the average dollar pricing error (within the same category) from a given option's pricing error, and then take its absolute value, the average of which across all puts of the given moneyness-maturity category is reported. Both error measures capture distinct aspects of the pricing-error distribution, with the mean absolute deviation reflecting the mean dispersion from the average error.

Table 4 reports two sets of out-of-sample pricing results: 'all-options-based' and 'maturity-based', where 'all-options-based' pricing errors are obtained using the spot volatility implied by *all of the previous day's puts* regardless of maturity, while those under 'maturity-based' using the spot volatility implied by the *previous-day puts in the same maturity category (short-term, medium-term, or LEAPS) as the option being priced*. We can summarize the results from Table 4 as follows.

First, as expected, the BS model performs the worst in pricing any put, according to both the average pricing errors (Panel A of Table 4) and the mean absolute deviation (Panel B). This is true using either 'all-options-based' or 'maturity-based' volatility input. The BS dollar pricing errors are as high in magnitude as $-\$2.48$ for short-term puts and $\$2.58$ for LEAPS. The BS undervalues OTM puts, especially OTM LEAPS puts, with large errors. In all cases, the mean absolute deviations are monotonically increasing in time to expiration. Adopting the 'maturity-based' treatment does make the pricing errors more stable across options, as the mean absolute deviations become lower.

Second, the SV model's pricing errors are often about one half (or less) of the corresponding BS errors. The term structure of mean absolute deviations is,

while still upward-sloping in most cases, much flatter for the SV, the SVSI, and the SVJ than for the BS, suggesting more stable pricing errors across options. All the models still undervalue OTM puts and overvalue ITM puts of almost any maturity.

Third, according to the dollar pricing errors, there is not a persistent pattern on the relative pricing performance between the SV and the SVSI. The SVSI does better than the SV for pricing short-term deep ITM puts, while the reverse is true for pricing short-term ATM puts. However, the difference between the two models becomes apparent according to the mean absolute deviation measure. In pricing short-term puts, the two models result in similar mean absolute deviations. In pricing both medium-term and LEAPS puts, the SV is far more consistent than the SVSI. This is true using both ‘all-options-based’

Table 4
Out-of-sample pricing errors

| Moneyness K/S | Model | All-options-based | | | Maturity-based | | |
|---|-------|--------------------|---------|-------|--------------------|---------|-------|
| | | Days-to-expiration | | | Days-to-expiration | | |
| | | < 60 | 180–365 | ≥ 365 | < 60 | 180–365 | ≥ 365 |
| <i>Panel A: Mean dollar pricing error</i> | | | | | | | |
| < 0.94 | BS | 0.77 | 1.58 | 2.58 | 1.01 | 1.98 | 1.66 |
| | SV | 0.43 | 0.94 | 1.83 | 0.87 | 1.66 | 1.45 |
| | SVSI | 0.44 | 1.03 | 0.26 | 0.86 | 1.01 | -0.31 |
| | SVJ | 0.42 | 1.26 | 1.88 | 0.56 | 1.46 | 1.52 |
| 0.94–0.97 | BS | 0.43 | -0.22 | 1.86 | 0.98 | 1.18 | 0.51 |
| | SV | 0.40 | 1.01 | 0.95 | 0.85 | 1.16 | 0.41 |
| | SVSI | 0.24 | 0.96 | 0.31 | 0.86 | 1.02 | -0.19 |
| | SVJ | 0.16 | 0.82 | 1.38 | 0.55 | 1.09 | 0.67 |
| 0.97–1.00 | BS | -1.55 | -1.05 | 1.37 | 0.28 | 0.59 | 0.54 |
| | SV | -0.42 | 0.37 | 0.86 | 0.26 | 0.54 | 0.16 |
| | SVSI | -0.79 | 0.44 | 1.11 | 0.25 | 0.51 | 0.18 |
| | SVJ | -0.43 | 0.31 | 1.20 | 0.23 | 0.60 | 0.41 |
| 1.00–1.03 | BS | -2.48 | -2.51 | 0.44 | -0.58 | -0.95 | -1.40 |
| | SV | -1.29 | -0.86 | -0.27 | -0.50 | -0.61 | -0.89 |
| | SVSI | -1.65 | -0.52 | 0.95 | -0.49 | -0.47 | 0.23 |
| | SVJ | -1.06 | -0.78 | 0.23 | -0.25 | -0.40 | 0.54 |
| 1.03–1.06 | BS | -1.45 | -3.38 | -1.01 | -0.47 | -1.91 | -1.72 |
| | SV | -0.75 | -1.91 | -0.38 | -0.27 | -1.71 | -1.38 |
| | SVSI | -1.01 | -1.46 | 1.25 | -0.23 | -1.33 | -0.32 |
| | SVJ | -0.63 | -1.70 | 0.15 | -0.13 | -1.34 | -1.08 |
| ≥ 1.06 | BS | 0.88 | -3.30 | -1.64 | 1.13 | -1.90 | -2.75 |
| | SV | 0.60 | -2.29 | -1.55 | 0.70 | -1.68 | -2.15 |
| | SVSI | 0.53 | -1.42 | 0.78 | 0.71 | -1.39 | -0.30 |
| | SVJ | 0.61 | -2.02 | -0.98 | 0.69 | -1.67 | -1.75 |

Table 4. (Continued)

| Moneyness K/S | Model | All-options-based | | | Maturity-based | | |
|---|-------|--------------------|---------|-------|--------------------|---------|-------|
| | | Days-to-expiration | | | Days-to-expiration | | |
| | | < 60 | 180–365 | ≥ 365 | < 60 | 180–365 | ≥ 365 |
| <i>Panel B: Mean absolute deviation</i> | | | | | | | |
| < 0.94 | BS | 0.29 | 0.49 | 0.90 | 0.26 | 0.40 | 0.82 |
| | SV | 0.22 | 0.35 | 0.64 | 0.22 | 0.31 | 0.63 |
| | SVSI | 0.26 | 0.40 | 0.84 | 0.22 | 0.38 | 0.81 |
| | SVJ | 0.25 | 0.37 | 0.72 | 0.21 | 0.33 | 0.67 |
| 0.94–0.97 | BS | 0.53 | 0.61 | 1.20 | 0.32 | 0.56 | 1.03 |
| | SV | 0.43 | 0.46 | 0.86 | 0.26 | 0.44 | 0.80 |
| | SVSI | 0.40 | 0.54 | 0.90 | 0.26 | 0.50 | 1.00 |
| | SVJ | 0.42 | 0.46 | 0.92 | 0.26 | 0.45 | 0.83 |
| 0.97–1.00 | BS | 0.81 | 0.73 | 1.26 | 0.43 | 0.66 | 1.12 |
| | SV | 0.68 | 0.55 | 0.88 | 0.40 | 0.53 | 0.82 |
| | SVSI | 0.77 | 0.62 | 1.04 | 0.40 | 0.57 | 0.83 |
| | SVJ | 0.68 | 0.55 | 1.02 | 0.34 | 0.52 | 0.89 |
| 1.00–1.03 | BS | 0.84 | 0.78 | 1.53 | 0.41 | 0.69 | 1.30 |
| | SV | 0.69 | 0.65 | 1.07 | 0.40 | 0.62 | 0.97 |
| | SVSI | 0.80 | 0.67 | 1.00 | 0.41 | 0.65 | 1.00 |
| | SVJ | 0.74 | 0.65 | 1.17 | 0.41 | 0.59 | 1.01 |
| 1.03–1.06 | BS | 1.10 | 0.92 | 1.49 | 0.64 | 0.75 | 1.33 |
| | SV | 0.82 | 0.71 | 1.02 | 0.62 | 0.68 | 0.97 |
| | SVSI | 0.94 | 0.89 | 1.05 | 0.60 | 0.73 | 1.06 |
| | SVJ | 0.79 | 0.68 | 1.17 | 0.55 | 0.66 | 1.11 |
| ≥ 1.06 | BS | 0.62 | 0.80 | 1.48 | 0.53 | 0.73 | 1.30 |
| | SV | 0.50 | 0.70 | 1.17 | 0.48 | 0.67 | 1.02 |
| | SVSI | 0.53 | 0.73 | 1.19 | 0.46 | 0.71 | 1.20 |
| | SVJ | 0.49 | 0.72 | 1.34 | 0.47 | 0.68 | 1.17 |

Note: The reported mean dollar pricing error is the sample average of the market price minus the model price of the option. The mean absolute deviation is constructed by demeaning each dollar pricing error (in a moneyness-maturity category) and then computing the average of the absolute value. The results under ‘all-options-based’ are obtained using the volatility implied by all previous day’s puts (and the structural parameters in Table 2); Those under ‘maturity-based’ using the volatility implied by the previous day’s options of a given maturity (short term, medium term, or long term) to price current day’s options of the same maturity. The sample period is 09:1993–08:1994, with a total of 12,092 put option prices.

and ‘maturity-based’ volatility input. Thus, generalizing the SV model by allowing interest rates to be stochastic actually lowers the pricing performance.

Fourth, in pricing short-term puts, the SVJ performs by far the best among all four models, especially according to the ‘maturity-based’ results. Both of its average pricing errors and mean absolute deviations are the lowest for each

given put option. A possible reason is that its random-jump component affords it with more flexibility in internalizing high levels of return kurtosis (and skewness) even at very short time horizons. On the other hand, the SVJ does much worse than the SV and the SVSI in pricing LEAPS puts. Therefore, depending on whether short-term or LEAPS options are being priced, performance rankings of the SV, the SVSI, and the SVJ can differ fundamentally.

In summary, incorporating both stochastic volatility and random jumps leads to the lowest errors for short-term options; but, for LEAPS modeling stochastic volatility alone results in reasonably good performance, while adding random jumps to the model only helps to worsen the performance. It may not be surprising that adding stochastic interest rates to the SV model does not improve its performance in pricing short-term puts. But, even in pricing LEAPS the SVSI model does not perform better than the SV, which is somewhat surprising given that one would expect LEAPS to be more sensitive to interest rates.

7. Hedging the underlying asset

Having studied the out-of-sample pricing performance, we now examine each model's hedging performance as hedging reflects a model's dynamic fit of option prices. For our first exercise, we assume that a manager wants to hedge a long position in the underlying asset (the S&P 500). In choosing an instrument for such a hedge, the manager can select from a large set of put options: short term, medium term, and long term. The goal in this section is to examine (i) the relative effectiveness of short-term versus long-term options in hedging the S&P 500 index; and (ii) the relative performance of the alternative models in devising the desired hedge.

Suppose that the manager can utilize only one hedging instrument, and let the SVSI-J model be the point of discussion. Then, if the candidate instrument is a put on the index with strike price K and with τ periods to expiration, a minimum-variance hedge of the underlying asset consists of (i) a long position in $X_p(t)$ units of the put and (ii) $X_0(t)$ dollars in the instantaneous riskfree bond, where

$$X_p(t) = \frac{-\rho\sigma_v VS\Delta_V - \Delta_S VS^2}{S^2 V\Delta_S^2 + 2\rho\sigma_v SV\Delta_S\Delta_V + \sigma_v^2 V\Delta_V^2 + \sigma_R^2 R\Delta_R^2} \quad (19)$$

where Δ_S , Δ_V , and Δ_R are respectively given in (10)–(12). To make the starting overall position self-financed, the cash position is: $X_0(t) = -X_p(t)P(t, \tau) - S(t)$. Optimal hedges under the other models can be derived from (19) by specializing the structural parameters to meet each model's assumptions.

Suppose that the hedge is established at date t as determined in (19) and using the spot volatility implied by all of date- $(t - 1)$ put prices. Then, after the next rebalancing interval of length Δt (either 7 days or 30 days), its hedging error from t to $(t + \Delta t)$ is computed as

$$H(t + \Delta t) = X_0 e^{R(t)\Delta t} + X_p(t)P(t + \Delta t, \tau - \Delta t) + S(t + \Delta t). \quad (20)$$

At the same time, reconstruct a date- $(t + \Delta t)$ self-financed portfolio and record the hedging error $H(t + 2\Delta t)$. Continue these steps for $M \equiv (\tau - t)/\Delta t$ times and for each of the puts in the sample. Finally, compute the average dollar hedging error and the mean absolute deviation, both as a function of rebalancing interval Δt .

Table 5 reports these hedging-error values for each model and every option category, at the 7-day and the 30-day rebalancing frequencies. The first pattern in Table 5 is that the dollar hedging errors are negative, regardless of the model used and for every moneyness-maturity category: each hedge so constructed *overhedges* the underlying asset.

Second, in hedging the S&P 500 index, short-term puts are more likely to overhedge than LEAPS (except for $K/S \in (1.03-1.06)$). In general, the dollar hedging error declines with the term to expiration, and LEAPS puts are the best to use for hedging the underlying asset. Based on the SV, for instance, a typical hedging error at the 7-day revision frequency is $-\$0.85$ using a short-term ATM put; $-\$0.38$ using a medium-term ATM put; and $-\$0.20$ using an ATM LEAPS put. When the hedging instrument is out of the money or at the money, the mean absolute deviation is related to the instrument's time to expiration in a U-shaped manner, that is, medium-term OTM and ATM puts lead to the most stable pricing errors. When the instrument is an ITM put, the mean absolute deviation increases with the time to expiration.

Of a given option maturity (especially short term), a put option's effectiveness in hedging the underlying asset increases as the instrument's moneyness increases, regardless of the option model used and hedge-revision frequency. For example, when choosing among short-term puts, a portfolio hedger should select one that is deep in the money. When the hedge is rebalanced once every 7 d using the BS model, the dollar hedging error and the mean absolute deviation are respectively $-\$2.58$ and $\$3.37$ if an OTM put with $K/S \leq 0.94$ is used in the hedge, and $-\$0.24$ and $\$0.61$ if a deep ITM put is adopted instead. The fact that the deeper in the money a put is, the better in hedging the underlying, may not come as a surprise because the deeper ITM puts resemble the underlying asset more (their deltas are closer to one while gammas closer to zero). It is seen from Table 5 that based on average hedging errors, the portfolio manager should choose a deep ITM LEAPS put to hedge the underlying, but based on the mean absolute deviations the manager should use a deep ITM short-term put.

Table 5

Errors from hedging the underlying asset

| Hedging instrument | Model | Mean dollar hedging error | | | | | | Mean absolute deviation | | | | | |
|--------------------|-------|---------------------------|---------|------------|-----------------|---------|------------|-------------------------|---------|------------|-----------------|---------|------------|
| | | 7-day revision | | | 30-day revision | | | 7-day revision | | | 30-day revision | | |
| | | SPX | SPX | LEAPS | SPX | SPX | LEAPS | SPX | SPX | LEAPS | SPX | SPX | LEAPS |
| K/S | | 60 | 180-365 | ≥ 365 | 60 | 180-365 | ≥ 365 | 60 | 180-365 | ≥ 365 | 60 | 180-365 | ≥ 365 |
| < 0.94 | BS | -2.58 | -0.34 | -0.46 | -11.00 | -2.17 | -2.29 | 3.37 | 1.60 | 2.81 | 4.08 | 2.37 | 3.60 |
| | SV | -1.53 | -0.13 | -0.26 | -10.80 | -1.66 | -1.45 | 4.41 | 1.35 | 2.62 | 3.65 | 2.19 | 3.43 |
| | SVSI | -1.41 | -0.24 | -0.29 | -10.73 | -1.90 | -1.62 | 4.40 | 1.50 | 2.96 | 3.99 | 2.27 | 3.89 |
| 0.94-0.97 | SVJ | -1.56 | -0.11 | -0.25 | -10.97 | -1.62 | -1.47 | 4.40 | 1.34 | 2.59 | 3.75 | 2.20 | 3.42 |
| | BS | -1.67 | -0.46 | -0.32 | -9.57 | -2.89 | -2.68 | 2.30 | 1.45 | 2.37 | 4.00 | 2.53 | 3.30 |
| | SV | -1.58 | -0.40 | -0.19 | -9.31 | -2.46 | -1.88 | 2.57 | 1.31 | 2.21 | 3.31 | 2.29 | 3.24 |
| 0.97-1.00 | SVSI | -1.58 | -0.36 | -0.26 | -9.27 | -2.41 | -1.37 | 2.54 | 1.35 | 2.70 | 3.43 | 2.32 | 3.37 |
| | SVJ | -1.60 | -0.39 | -0.18 | -9.40 | -2.45 | -1.88 | 3.01 | 1.31 | 2.23 | 2.30 | 2.30 | 3.24 |
| | BS | -0.89 | -0.63 | -0.41 | -5.30 | -3.05 | -2.00 | 1.74 | 1.26 | 2.25 | 3.09 | 2.17 | 2.76 |
| 1.00-1.03 | SV | -0.85 | -0.38 | -0.20 | -5.11 | -2.16 | -1.24 | 1.78 | 1.26 | 2.15 | 3.01 | 2.00 | 2.75 |
| | SVSI | -0.82 | -0.44 | -0.20 | -5.11 | -2.35 | -1.27 | 1.77 | 1.24 | 2.30 | 3.02 | 2.01 | 2.91 |
| | SVJ | -0.94 | -0.38 | -0.19 | -5.36 | -2.17 | -1.26 | 2.04 | 1.25 | 2.13 | 3.23 | 2.00 | 2.74 |
| 1.03-1.06 | BS | -0.46 | -0.45 | -0.26 | -2.94 | -2.35 | -2.27 | 1.33 | 1.21 | 2.36 | 2.75 | 1.98 | 3.16 |
| | SV | -0.37 | -0.30 | -0.11 | -2.48 | -1.83 | -1.50 | 1.14 | 1.15 | 2.24 | 2.36 | 1.83 | 3.35 |
| | SVSI | -0.40 | -0.32 | -0.04 | -2.61 | -1.90 | -1.62 | 1.16 | 1.15 | 2.43 | 2.43 | 1.79 | 3.10 |
| 1.03-1.06 | SVJ | -0.34 | -0.30 | -0.10 | -2.35 | -1.84 | -1.51 | 1.10 | 1.15 | 2.26 | 2.28 | 1.82 | 3.31 |
| | BS | -0.34 | -0.49 | -0.60 | -2.06 | -2.49 | -2.54 | 1.11 | 1.15 | 2.21 | 2.27 | 1.74 | 2.49 |
| | SV | -0.20 | -0.27 | -0.47 | -1.52 | -1.75 | -1.58 | 0.82 | 1.06 | 2.24 | 1.69 | 1.48 | 2.37 |
| ≥ 1.06 | SVSI | -0.25 | -0.30 | -0.46 | -1.70 | -1.87 | -1.85 | 0.89 | 1.05 | 2.19 | 1.84 | 1.47 | 2.43 |
| | SVJ | -0.14 | -0.26 | -0.47 | -1.30 | -1.73 | -1.61 | 0.73 | 1.06 | 2.25 | 1.55 | 1.47 | 2.37 |
| | BS | -0.24 | -0.52 | -0.32 | -1.20 | -2.28 | -1.86 | 0.61 | 1.06 | 1.84 | 1.00 | 1.63 | 2.48 |
| ≥ 1.06 | SV | -0.17 | -0.17 | -0.06 | -0.88 | -1.09 | -0.56 | 0.54 | 1.02 | 1.81 | 0.79 | 1.44 | 2.47 |
| | SVSI | -0.20 | -0.23 | -0.11 | -1.03 | -1.28 | -1.12 | 0.57 | 1.00 | 1.79 | 0.89 | 1.44 | 2.45 |
| | SVJ | -0.15 | -0.14 | -0.06 | -0.76 | -1.02 | -0.62 | 0.52 | 1.03 | 1.75 | 0.73 | 1.45 | 2.47 |

Note: All hedges in this table have the underlying asset as the target, and the hedging instrument is (i) a short-term SPX put, (ii) a medium-term SPX put, or (iii) a LEAPS put. Each hedge is constructed using the previous day's implied stock volatility and structural parameters in Table 2. The hedge is rebalanced every 7 d or 30 d. For each hedge, its error is, as of the revision day, the difference between the target's market price and the replicating portfolio value. As before, we report the mean dollar hedging error and the mean absolute deviation. The sample period is 09:1993-08:1994. In computing the 7-d and the 30-d revision frequency hedging errors, we respectively use 9745 and 6450 option observations.

Finally, across the option models, hedging performance differs substantially. The BS performs the worst by a significant margin, irrespective of the put instrument and hedge-revision frequency. The difference among the three models with stochastic volatility is not as clear cut, however. According to both the average hedging errors and the mean absolute deviations, the SV model performs slightly better in most cases than the SVSI and the SVJ when the put instrument is out of the money (with $K/S \leq 1$), whereas the SVJ does better than the other two models when the put instrument is in the money (with $K/S \geq 1$). For most option categories, the SVSI seems to give the worst hedging errors, except in some cases in which the instrument is an ITM LEAPS put. Thus, the stochastic interest rate feature also fails to add much dynamic hedging performance. Depending on whether the instrument is in the money or out of the money, a hedger may be better off by applying the SV or the SVJ to devise the desired hedge.

8. Hedging long-term options

As noted before, most exchange-traded derivatives have, even to this date, relatively short terms to expiration. Commitments taken over the counter, on the other hand, are often long term. As made clear by Ross (1996) using the Metallgesellschaft's oil-contracts case, users and underwriters of illiquid long-term contracts usually have no other choice but rely on the underlying asset or exchange-traded short-term contracts to hedge their commitments. At least, these short-term contracts are liquid and their trading costs are low. One is then tempted to ask: How effective can the underlying asset and short-term contracts be in hedging long-term commitments? Which model results in the best hedging performance of long-term contracts using the underlying asset or short-term contracts? In the present study, we have high-quality data for short-term, medium-term and long-term option contracts, which provides an ideal opportunity to answer the above questions. This is in contrast with Melino and Turnbull (1995), where simulated prices, rather than actual market prices, for long-term currency options are used to study the effectiveness of option pricing models in devising hedging strategies.

Suppose that the hedging target is a long-term option with strike price K and term-to-expiration τ , whose price is $P(t, \tau; K)$. Let $Y(t)$ be the time- t value of the candidate hedging instrument. Then, using the SVSI-J model as the point of discussion, we have the minimum-variance hedge consisting of $X(t)$ units of the instrument and $X_0(t)$ dollars in the instantaneous risk-free bond:

$$X(t) = \frac{\text{Cov}_t[dP(t, \tau; K), dY(t)]}{\text{Var}_t(dY(t))}, \quad (21)$$

$$X_0(t) = P(t, \tau, K) - X(t)Y(t). \quad (22)$$

In the case where a $\bar{\tau}$ -period put with strike price K is the hedging instrument, we replace $Y(t)$ by $P(t, \bar{\tau}, K)$ and substitute the following into the above solution:

$$\begin{aligned} \text{Cov}_t[dP(t, \tau, K), dY(t)] &= \Delta_R(t, \tau)\Delta_R(t, \bar{\tau})R\sigma_R^2 + \Delta_V(t, \tau, K)\Delta_V(t, \bar{\tau})V\sigma_v^2 \\ &\quad + \Delta_S(t, \tau)\Delta_S(t, \bar{\tau})S^2V + \rho\sigma_vSV\Delta_V(t, \tau)\Delta_S(t, \bar{\tau}) \\ &\quad + \rho\sigma_vSV\Delta_S(t, \tau)\Delta_V(t, \bar{\tau}), \end{aligned} \tag{23}$$

$$\begin{aligned} \text{Var}_t[dY(t)] &= \Delta_R^2(t, \bar{\tau})R\sigma_R^2 + \Delta_V^2(t, \bar{\tau})V\sigma_v^2 + \Delta_S^2(t, \bar{\tau})VS^2 \\ &\quad + 2\rho\sigma_vSV\Delta_V(t, \bar{\tau})\Delta_S(t, \bar{\tau}). \end{aligned} \tag{24}$$

In the case where the underlying asset is the instrument, we have $S(t)$ substituting for $Y(t)$ in (21) and obtain

$$\begin{aligned} X(t) &\equiv \frac{V}{V + V_J} \Delta_S(t, \tau) + \rho\sigma_v\Delta_V(t, \tau) \frac{V}{S(V + V_J)} \\ &\quad + \frac{\lambda}{S(V + V_J)} [A_1(t) - A_2(t) - \mu_J P(t, \tau, K) \\ &\quad + \mu_J KB(t, \tau) - S(1 + \mu_J)], \end{aligned} \tag{25}$$

where $A_1(t)$ and $A_2(t)$ are respectively given in (A.3) and (A.4) of the Appendix. We take the second case as an example to illustrate the intuition of the optimal hedge ratio. The first term on the right-hand side of (25) serves to control for the fluctuation of the target put’s price directly caused by the underlying price changes: the larger the diffusion component’s contribution to total underlying asset volatility (as measured by $V/(V + V_J)$), the more weight assigned to controlling for this direct impact. The second term is due to the fact that (i) volatility changes affect the target put’s price and (ii) the underlying price’s diffusion component is correlated with volatility. This term helps offset, to the extent possible, the effect of the underlying price-related volatility changes on the target’s price: the more correlated the diffusion volatility process with the underlying price (as reflected by ρ), the more a single position in the underlying can do to mitigate the effect of volatility changes on the value of the target. The last term in (25) serves to offset at least part of the jump risk’s impact on the target value. Under the standard BS setup, volatility is not stochastic (i.e.,

$\rho = \sigma_v = 0$) and no jump risk is present (i.e., $\lambda = 0$), which renders the second and the last terms of (25) equal zero and $X(t) = \Delta_S(t, \tau)$. Accounting for their modeling differences, we can similarly determine the exact minimum-variance hedges under the BS, the SV, the SVSI, and the SVJ models.

Following similar implementation steps as before, we obtain in Table 6 the average hedging error and the mean absolute deviation for each moneyness-maturity category and for each model. Note from Table 6 that in hedging LEAPS, the different instruments perform quite differently. Regardless of the option model employed and for every LEAPS target, medium-term options lead to the best hedging results. This is particularly true when the hedge is rebalanced every 30 d, and according to the mean absolute deviation measure. Between the other two candidate hedging instruments, the underlying asset dominates short-term options in hedging LEAPS, according to the mean absolute deviation measure. However, the choice between these two instruments is not as clear based on the dollar hedging errors. When an OTM LEAPS put is the hedging target, the underlying asset produces lower average hedging errors (in magnitude) than short-term options. On the other hand, when the target is an ITM LEAPS put, short-term puts are better hedging instruments for the underlying. These conclusions hold at both the 7-day and the 30-day revision frequencies.

It appears surprising that in many cases the underlying asset is a better hedging instrument for LEAPS than short-term options. To understand this point, let us first see an example using medium-term puts. Based on the SV model, for example, the delta and gamma positions of the resulting hedged portfolio are, respectively,

$$\Delta_{\text{port}} = \Delta_{\text{tgt}} - X\Delta_{\text{inst}} \quad (26)$$

$$\Gamma_{\text{port}} = \Gamma_{\text{tgt}} - X\Gamma_{\text{inst}}, \quad (27)$$

where Δ_{tgt} and Γ_{inst} , for example, are the delta of the LEAPS target and the gamma of the hedging instrument, and X is the position taken in the instrument. On the first day of our sample period, the observed spot index was 460.70. Using that day's LEAPS put with strike price 450 and with 469 d to expiration as the target, we obtain the following delta and gamma value for the overall hedged portfolio when the instrument is

1. The underlying asset:

$$\Delta_{\text{port}} = -0.36 - (-0.41) \times 1 = 0.05,$$

$$\Gamma_{\text{port}} = 0.006 - (-0.41) \times 0 = 0.006.$$

Table 6

Errors from hedging LEAPS

| Hedging Instrument | Model | Mean dollar hedging error | | | | | | Mean absolute deviation | | | | | |
|--------------------|-------|---------------------------|---------|-------|--------------------|-------|---------|-------------------------|---------|------|--------------------|------|---------|
| | | 7-day revision | | | 30-day revision | | | 7-day revision | | | 30-day revision | | |
| | | Index | SPX | SPX | Index | SPX | SPX | Index | SPX | SPX | Index | SPX | SPX |
| | | Days-to-expiration | | | Days-to-expiration | | | Days-to-expiration | | | Days-to-expiration | | |
| K/S | | 60 | 180-365 | 60 | 180-365 | 60 | 180-365 | 60 | 180-365 | 60 | 180-365 | 60 | 180-365 |
| < 0.94 | BS | -0.10 | 0.17 | -0.04 | -0.52 | 1.17 | 0.05 | 0.67 | 0.83 | 0.68 | 0.92 | 1.34 | 0.81 |
| | SV | -0.08 | 0.14 | -0.03 | -0.40 | 1.02 | 0.03 | 0.71 | 0.74 | 0.67 | 0.91 | 1.47 | 0.79 |
| | SVSI | -0.06 | 0.11 | -0.02 | -0.32 | 1.23 | -0.03 | 0.68 | 0.78 | 0.65 | 0.98 | 1.23 | 0.81 |
| 0.94-0.97 | SVJ | -0.08 | 0.12 | -0.03 | -0.41 | 1.00 | 0.04 | 0.67 | 0.83 | 0.67 | 0.91 | 1.52 | 0.79 |
| | BS | -0.08 | 0.50 | -0.04 | -0.87 | 1.17 | 0.22 | 0.89 | 1.04 | 0.79 | 1.20 | 1.73 | 0.96 |
| | SV | -0.04 | 0.38 | -0.01 | -0.66 | 1.08 | 0.11 | 0.88 | 0.88 | 0.79 | 1.17 | 1.74 | 0.85 |
| 0.97-1.00 | SVSI | -0.01 | 0.47 | 0.05 | -0.41 | 0.96 | 0.21 | 0.94 | 0.96 | 0.84 | 1.09 | 1.45 | 0.82 |
| | SVJ | -0.02 | 0.34 | 0.00 | -0.65 | 0.99 | 0.12 | 0.89 | 0.94 | 0.79 | 1.17 | 1.77 | 0.81 |
| | BS | -0.14 | -0.06 | -0.16 | -0.70 | 1.07 | 0.12 | 0.85 | 1.00 | 0.91 | 1.14 | 2.00 | 0.92 |
| 1.00-1.03 | SV | -0.07 | -0.05 | -0.14 | -0.47 | 1.18 | 0.10 | 0.87 | 1.04 | 0.87 | 1.13 | 2.01 | 0.87 |
| | SVSI | -0.07 | -0.02 | -0.09 | -0.42 | 0.90 | 0.05 | 0.81 | 0.89 | 0.81 | 1.23 | 1.59 | 0.90 |
| | SVJ | -0.09 | -0.02 | -0.13 | -0.48 | 1.09 | 0.09 | 0.84 | 1.02 | 0.86 | 1.13 | 2.04 | 0.87 |
| 1.03-1.06 | BS | -0.10 | -0.05 | -0.07 | -1.04 | 0.55 | -0.18 | 1.07 | 1.07 | 1.03 | 1.45 | 1.79 | 1.32 |
| | SV | -0.03 | -0.03 | -0.07 | -0.81 | 0.47 | -0.15 | 1.09 | 1.02 | 1.00 | 1.23 | 1.69 | 1.21 |
| | SVSI | -0.03 | -0.05 | -0.07 | -0.69 | 0.27 | -0.09 | 1.07 | 0.98 | 0.99 | 1.27 | 1.45 | 1.11 |
| ≥ 1.06 | SVJ | -0.05 | -0.06 | -0.07 | -0.80 | 0.43 | -0.15 | 1.07 | 1.01 | 1.00 | 1.22 | 1.64 | 1.18 |
| | BS | -0.26 | 0.19 | -0.10 | -1.20 | 0.34 | -0.23 | 1.09 | 1.25 | 1.25 | 1.19 | 1.75 | 1.06 |
| | SV | -0.17 | 0.09 | -0.09 | -0.85 | -0.08 | -0.10 | 1.18 | 1.18 | 1.20 | 1.19 | 1.61 | 1.01 |
| ≥ 1.06 | SVSI | -0.18 | -0.07 | -0.07 | -0.92 | -0.28 | -0.20 | 1.05 | 1.09 | 1.13 | 1.17 | 1.44 | 1.03 |
| | SVJ | -0.18 | 0.07 | -0.08 | -0.86 | -0.09 | -0.11 | 1.11 | 1.17 | 1.19 | 1.18 | 1.59 | 1.01 |
| | BS | -0.29 | 0.15 | 0.26 | -1.03 | 0.60 | 0.09 | 1.10 | 1.12 | 1.12 | 1.38 | 1.60 | 1.17 |
| ≥ 1.06 | SV | -0.07 | 0.09 | 0.19 | -0.58 | 0.44 | 0.91 | 1.17 | 1.07 | 1.07 | 1.44 | 1.51 | 1.15 |
| | SVSI | -0.22 | -0.02 | 0.09 | -0.86 | 0.33 | 0.66 | 1.07 | 1.02 | 1.02 | 1.34 | 1.45 | 1.21 |
| | SVJ | -0.17 | 0.09 | 0.17 | -0.61 | 0.46 | 0.87 | 1.12 | 1.08 | 1.06 | 1.42 | 1.52 | 1.16 |

Note: All hedges in this table have a LEAPS put as the target, and the hedging instrument is (i) the underlying asset, (ii) a short-term SPX put, or (iii) a medium-term SPX put. Each hedge is constructed using previous day's implied stock volatility and structural parameters in Table 2. The hedge is rebalanced every 7 d or 30 d. For each hedge, its error is, as of the revision day, the difference between the target's market price and the replicating portfolio value. As before, we report the mean dollar hedging error and the mean absolute deviation. The sample period is 09:1993-08:1994. In computing the 7-d and the 30-d revision frequency hedging errors, we respectively use 8464 and 5113 option observations.

2. A put with the same strike price but 42 d to expiration (short-term):

$$\Delta_{\text{port}} = -0.36 - 1.50(-0.22) = -0.03,$$

$$\Gamma_{\text{port}} = 0.006 - 1.50 \times 0.018 = -0.021.$$

3. A put with the same strike price but 287 d to expiration (medium-term):

$$\Delta_{\text{port}} = -0.36 - 0.98(-0.34) = -0.026,$$

$$\Gamma_{\text{port}} = 0.006 - 0.98 \times 0.008 = -0.002.$$

From this example, it is evident that the medium term puts are the most appropriate instrument for hedging LEAPS as they result in the lowest delta and gamma values (in magnitude) for the hedged portfolio. Intuitively, medium-term puts, among the three hedging instruments considered, should resemble the LEAPS targets the closest. With the other two types of instruments, one leads to a higher overall delta while the other leads to a higher overall gamma value, which explains why it is not always the case that the short-term puts achieve lower hedging errors than the underlying asset. The fact that short-term puts are not always good hedging instruments for LEAPS again demonstrates that LEAPS possess distinct properties that are not reflected in short-term options.

Finally, for the relative performance across the models, Table 6 suggests that models with stochastic volatility outperform the BS model by a significant margin. Among the three models with stochastic volatility, the SV and the SVSI in general have lower hedging errors and lower mean absolute deviations than the SVJ model. In most cases, the SVSI model performs better than the SV.

In summary, for hedging LEAPS, medium-term options are by far the best instrument to use in most cases. Between the underlying asset and the short-term option, the underlying asset is a better instrument for hedging OTM LEAPS, while a short-term option is better for hedging ITM LEAPS. This finding contradicts the conventional practice of rolling over short-term contracts to hedge long-term commitments without considering the moneyness of the hedging target. In terms of the model choice, the SVSI is in general most suitable for devising hedges of LEAPS contracts, followed sequentially by the SV, the SVJ, and the BS. Therefore, for designing hedges of LEAPS, modeling stochastic interest rates explicitly does tend to improve the hedging performance. The finding that the SVJ does not improve over the SV's hedging performance appears puzzling. But, given the fact that the estimated jump-intensity parameter λ is 0.78 times per year (see Table 2), it may not be surprising that modeling jump risk is not as important for hedging LEAPS, because it

takes on average more than a year for a jump of the average magnitude to occur and yet each hedge is rebalanced either once every 7 d or every 30 d. Clearly, during such rebalancing intervals the chance for a significant price jump (or fall) is quite small.

9. Conclusions

This paper has studied the differential information in, and the pricing and hedging of, short-term versus long-term equity options, by comparing four alternative option pricing models. Theoretically, we have shown that long-term options should be able to differentiate the alternative models more effectively than short-term options. This has been illustrated by the differences among the option hedge ratios and the state-price densities of the models. Empirically, short-term and long-term options do contain differential information, and they imply different price dynamics for the underlying asset and hence lead to different rankings of the alternative models.

Our work thus adds to the existing literature by studying long-term options. As mentioned earlier, Bakshi et al. (1997) examine the relative empirical performance of the BS, the SV, the SVSI, and the SVJ models. Nandi (1996) studies the empirical performance of the BS versus the SV model. Ait-Sahalia and Lo (1998), Broadie et al. (1999), and Jackwerth and Rubinstein (1997) examine the relative performance of the BS and non-parametric option pricing models. Rubinstein (1985, 1994) studies the pricing performance of the BS versus other parametric option pricing models. All these studies focus on regular S & P 500 or S & P 100 options with less than a year to expiration.

It is a common understanding in the literature that stochastic interest rates may not be important for the pricing and hedging of short-term options, but should be so for long-term options. Our study suggests that once the model has accounted for stochastically varying volatility, allowing interest rates to be stochastic does not improve pricing performance any further, even for long-term options. Only for devising a hedge of a LEAPS put does incorporating stochastic interest rates make a noticeable difference. The fact that modeling stochastic interest rates has not been found to be important for pricing performance may be explained by Fig. 3, where it is shown that even at the two-year horizon most of the difference among the SPDs of the SV, the SVSI, and the SVJ occurs between 0% and 50% for the underlying stock's rate of return. This means that most of these models' differences can only be reflected in the pricing of ITM (especially deep ITM) LEAPS puts. But, on a typical day, there are far more ATM and OTM LEAPS puts traded than ITM ones. In this sense, no matter how far one may stretch the option maturity horizon, observations on long-term ITM puts will still be limited. It is therefore unlikely to see a more important role played by stochastic interest rates in pricing equity options than found

in this study.¹⁴ Modeling stochastic volatility is perhaps of true first-order importance.

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Appendix

A.1. Characteristic functions for the put option pricing formula in (8)

From Bakshi et al. (1997), the characteristic functions are respectively given by

$$\begin{aligned}
 f_1(t, \tau) = \exp \left\{ -\frac{\theta_R}{\sigma_R^2} \left[2 \ln \left(1 - \frac{[\zeta_R - \kappa_R](1 - e^{-\zeta_R \tau})}{2\zeta_R} \right) + [\zeta_R - \kappa_R]\tau \right] \right. \\
 \left. - \frac{\theta_v}{\sigma_v^2} \left[2 \ln \left(1 - \frac{[\zeta_v - \kappa_v + (1 + i\phi)\rho\sigma_v](1 - e^{-\zeta_v \tau})}{2\zeta_v} \right) \right] \right\} \\
 - (1 + i\phi) - \delta\tau - \frac{\theta_v}{\sigma_v^2} [\zeta_v - \kappa_v + (1 + i\phi)\rho\sigma_v]\tau + i\phi \ln[S(t)] \\
 + \frac{2i\phi(1 - e^{-\zeta_R \tau})}{2\zeta_R - [\zeta_R - \kappa_R](1 - e^{-\zeta_R \tau})} R(t) \\
 + \lambda(1 + \mu_J)\tau[(1 + \mu_J)^{i\phi} e^{i\phi/2(1+i\phi)\sigma_v^2} - 1] - \lambda i\phi\mu_J\tau \\
 \left. + \frac{i\phi(i\phi + 1)(1 - e^{-\zeta_v \tau})}{2\zeta_v - [\zeta_v - \kappa_v + (1 + i\phi)\rho\sigma_v](1 - e^{-\zeta_v \tau})} V(t) \right\}, \quad (A.1)
 \end{aligned}$$

¹⁴ Foreign-exchange options may be an exception, where modeling stochastic interest rates is of fundamental importance. See Bakshi and Chen (1997b) for foreign-exchange option formulas under stochastic interest rates. It however remains to be seen whether the stochastic interest rates feature is indeed significant for pricing and hedging foreign-exchange options empirically.

and,

$$\begin{aligned}
 f_2(t, \tau) = \exp \left\{ -\frac{\theta_R}{\sigma_R^2} \left[2 \ln \left(1 - \frac{[\zeta_R^* - \kappa_R](1 - e^{-\zeta_R^* \tau})}{2\zeta_R^*} \right) + [\zeta_R^* - \kappa_R] \tau \right] \right. \\
 - \frac{\theta_v}{\sigma_v^2} \left[2 \ln \left(1 - \frac{[\zeta_v^* - \kappa_v + i\phi \rho \sigma_v](1 - e^{-\zeta_v^* \tau})}{2\zeta_v^*} \right) \right. \\
 + [\zeta_v^* - \kappa_v + i\phi \rho \sigma_v] \tau + i\phi \ln[S(t)] - \ln[B(t, \tau)] \\
 + \frac{2(i\phi - 1)(1 - e^{-\zeta_R^* \tau})}{2\zeta_R^* - [\zeta_R^* - \kappa_R](1 - e^{-\zeta_R^* \tau})} R(t) - i\phi \delta \tau \\
 + \lambda \tau [(1 + \mu_J)^{i\phi} e^{i\phi/2(i\phi - 1)\sigma_J^2} - 1] - \lambda i\phi \mu_J \tau \\
 \left. \left. + \frac{i\phi(i\phi - 1)(1 - e^{-\zeta_v^* \tau})}{2\zeta_v^* - [\zeta_v^* - \kappa_v + i\phi \rho \sigma_v](1 - e^{-\zeta_v^* \tau})} V(t) \right] \right\}, \tag{A.2}
 \end{aligned}$$

where $\zeta_R = \sqrt{\kappa_R^2 - 2\sigma_R^2 i\phi}$, $\zeta_v = \sqrt{[\kappa_v - (1 + i\phi)\rho \sigma_v]^2 - i\phi(i\phi + 1)\sigma_v^2}$,
 $\zeta_R^* = \sqrt{\kappa_R^2 - 2\sigma_R^2(i\phi - 1)}$, $\zeta_v^* = \sqrt{[\kappa_v - i\phi \rho \sigma_v]^2 - i\phi(i\phi - 1)\sigma_v^2}$.

A.2. Minimum variance hedge in (25)

Solving the conditional expectation $E_t[JP(S(1 + J), R, V)]$ results in (25) with

$$\begin{aligned}
 A_1(t) = \frac{S(t)}{2} [\mu_J + \mu_J^2 + (e^{\sigma_J^2} - 1)(1 + \mu_J)^2] \\
 + \frac{S(t)}{\pi} \int_0^\infty \text{Re} \left[e^{-i\phi \ln[K]} \frac{f_1(t, \tau) \bar{m}_1}{i\phi} \right] d\phi \tag{A.3}
 \end{aligned}$$

$$A_2(t) = \frac{KB(t, \tau) \mu_J}{2} + \frac{KB(t, \tau)}{\pi} \int_0^\infty \text{Re} \left[e^{-i\phi \ln[K]} \frac{f_2(t, \tau) \bar{m}_2}{i\phi} \right] d\phi \tag{A.4}$$

where

$$\begin{aligned}
 \bar{m}_1 &\equiv \exp \left[(2 + i\phi) \left(\ln[1 + \mu_J] - \frac{1}{2} \sigma_J^2 \right) + \frac{1}{2} (2 + i\phi)^2 \sigma_J^2 \right] \\
 &\quad - \exp \left[(1 + i\phi) \left(\ln[1 + \mu_J] - \frac{1}{2} \sigma_J^2 \right) + \frac{1}{2} (1 + i\phi)^2 \sigma_J^2 \right] \\
 \bar{m}_2 &\equiv \exp \left[(1 + i\phi) \left(\ln[1 + \mu_J] - \frac{1}{2} \sigma_J^2 \right) + \frac{1}{2} (1 + i\phi)^2 \sigma_J^2 \right] \\
 &\quad - \exp \left[i\phi \left(\ln[1 + \mu_J] - \frac{1}{2} \sigma_J^2 \right) - \frac{1}{2} \phi^2 \sigma_J^2 \right]
 \end{aligned}$$

References

- Ait-Sahalia, Y., Lo, A., 1998. Nonparametric estimation of state-price densities implicit in financial prices. *Journal of Finance*, 53, 499–548.
- Amin, K., Jarrow, R., 1992. Pricing options on risky assets in a stochastic interest rate economy. *Mathematical Finance* 2, 217–237.
- Amin, K., Ng, V., 1993. Option valuation with systematic stochastic volatility. *Journal of Finance* 48, 881–910.
- Bailey, W., Stulz, R., 1989. The pricing of stock index options in a general equilibrium model. *Journal of Financial and Quantitative Analysis* 24, 1–12.
- Bakshi, G., Cao, C., Chen, Z., 1997. Empirical performance of alternative option pricing models. *Journal of Finance* 52, 2003–2049.
- Bakshi, G., Chen, Z., 1997a. An alternative valuation model for contingent claims. *Journal of Financial Economics* 44 (1), 123–165.
- Bakshi, G., Chen, Z., 1997b. Equilibrium valuation of foreign exchange claims. *Journal of Finance* 52 (2), 799–826.
- Bates, D., 1991. The crash of 87: was it expected? The evidence from options markets. *Journal of Finance* 46, 1009–1044.
- Bates, D., 1996a. Jumps and stochastic volatility: exchange rate processes implicit in Deutschemark options. *Review of Financial Studies* 9 (1), 69–108.
- Bates, D., 1996b. Testing option pricing models. In: Maddala, G.S., Rao, C.R. (Eds.), *Handbook of Statistics*, Vol. 14: Statistical Methods in Finance, North-Holland, Amsterdam, pp. 567–611.
- Bates, D., 1999. Post-87 crash fears in S & P 500 futures options. *Journal of Econometrics*, this issue.
- Black, F., Scholes, M., 1973. The pricing of options and corporate liabilities. *Journal of Political Economy* 81, 637–659.
- Bollerslev, T., Mikkelsen, H., 1996. Modeling and pricing long memory in stock market volatility. *Journal of Econometrics* 73, 151–184.
- Bollerslev, T., Mikkelsen, H., 1999. Long-term equity anticipation securities and stock market volatility dynamics. *Journal of Econometrics*, this issue.
- Breeden, D., Litzenberger, R., 1978. Prices of state contingent claims implicit in option prices. *Journal of Business* 51, 621–652.
- Broadie, M., Detemple, J., Ghysels, E., Torres, O., 1999. American options with stochastic volatility and stochastic dividends: A nonparametric investigation. *Journal of Econometrics*, this issue.
- Comte, F., Renault, E., 1996. Long-memory continuous time models. *Journal of Econometrics* 73, 101–149.
- Cox, J., Ingersoll, J., Ross, S., 1985. A theory of the term structure of interest rates. *Econometrica* 53, 385–408.
- Cox, J., Ross, S., 1976. The valuation of options for alternative stochastic processes. *Journal of Financial Economics* 3, 145–166.
- Ding, Z., Granger, C., Engle, R., 1993. A long memory property of stock market returns and a new model. *Journal of Empirical Finance* 1, 83–106.
- Ding, Z., Granger, C., 1996. Modeling volatility persistence of speculative returns: a new approach. *Journal of Econometrics* 73, 185–215.
- Duffie, D., Singleton, K., 1993. Simulated moments estimation of markov models of asset prices. *Econometrica* 61, 929–952.
- Gouriéroux, C., Monfort, A., 1996. Simulation based econometric methods. Core Lecture Series.
- Ghysels, E., Harvey, A., Renault, E., 1996. Stochastic volatility. In: Maddala G.S., Rao, C.R. (Eds.), *Handbook of Statistics*, Vol. 14: Statistical Methods in Finance. North-Holland, Amsterdam.
- Heston, S., 1993. A closed-form solution for options with stochastic volatility with applications to bond and currency options. *Review of Financial Studies* 6, 327–343.

- Hull, J., White, A., 1987. The pricing of options with stochastic volatilities. *Journal of Finance* 42, 281–300.
- Jackwerth, J., Rubinstein, M., 1997. Recovering stochastic processes from option prices. Working paper, University of California, Berkeley.
- Madan, D., Chang, E., 1996. The variance gamma option pricing model. Working Paper, University of Maryland.
- Melino, A., Turnbull, S., 1990. Pricing foreign currency options with stochastic volatility. *Journal of Econometrics* 45, 239–265.
- Melino, A., Turnbull, S., 1995. Misspecification and the pricing and hedging of long-term foreign currency options. *Journal of International Money and Finance* 14, 373–393.
- Merton, R., 1973. Theory of rational option pricing. *Bell Journal of Economics* 4, 141–183.
- Merton, R., 1976. Option pricing when the underlying stock returns are discontinuous. *Journal of Financial Economics* 4, 125–144.
- Nandi, S., 1996. Pricing and hedging index options under stochastic volatility. Working Paper, Federal Reserve Bank of Atlanta.
- Ross, S., 1996. Hedging long-run commitments: exercises in incomplete market pricing. Working Paper, Yale School of Management.
- Rubinstein, M., 1985. Nonparametric tests of alternative option pricing models using all reported trades and quotes on the 30 most active CBOE options classes from August 23, 1976 through August 31, 1978. *Journal of Finance* 40, 455–480.
- Rubinstein, M., 1994. Implied binomial trees. *Journal of Finance* 49, 771–818.
- Scott, L., 1987. Option pricing when the variance changes randomly: theory, estimators, and applications. *Journal of Financial and Quantitative Analysis* 22, 419–438.
- Scott, L., 1997. Pricing stock options in a jump-diffusion model with stochastic volatility and interest rates: application of fourier inversion methods. *Mathematical Finance* 7, 413–426.
- Stein, E., Stein, J., 1991. Stock price distributions with stochastic volatility. *Review of Financial Studies* 4, 727–752.
- Wiggins, J., 1987. Option values under stochastic volatilities. *Journal of Financial Economics* 19, 351–372.