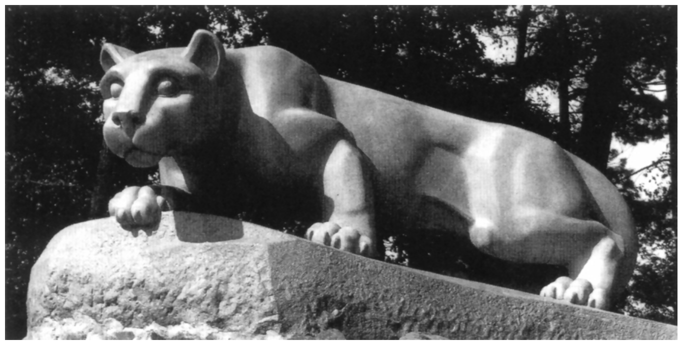
OAL13: Conference on Frames and Lattice-Ordered Groups



Penn State Erie - The Behrend College

June 24 - 26, 2013

Contents

1 Participants

Name	E-mail	Affiliation	Abstract
Gayle Apfel	gdmp27@gmail.com	University of Cape Town	Page 9
Rick Ball	rball@du.edu	University of Denver	Page 10
Papiya Bhattacharjee	pxb39@psu.edu	Penn State Erie	Page 11
Ricardo Carrera	ricardo@nova.edu	Nova Southeastern University	Page 12
Michael Darnel	mdarnel@iusb.edu	Indiana University South Bend	Page 12
Kevin Drees	kevin.drees@gmail.com		
Themba Dube	Dubeta@unisa.ac.za	University of South Africa	Page 13
John Frith	John.Frith@uct.ac.za	University of Cape Town	Page 13
Tony Hager	ahager@wesleyan.edu	Wesleyan University	Page 14
Charles Holland	Charles.Holland@colorado.edu	University of Colorado	Page 14
Tega Ighedo	Ighedo@unisa.ac.za	University of South Africa	Page 14
Michelle Knox	mathandlit@yahoo.com	Midwestern State University	Page 15
James Madden	jamesjmadden@gmail.com	Louisiana State University	Page 15
Jorge Martinez	gnocchiverdi@gmail.com	University of Florida	Page 16
Warren McGovern	Warren.McGovern@fau.edu	Florida Atlantic University	Page 16
Inderasan Naidoo	naidoi@unisa.ac.za	University of South Africa	Page 16
Anneliese Schauerte	Anneliese.Schauerte@uct.ac.za	University of Cape Town	Page 17
Philip Scowcroft	pscowcroft@wesleyan.edu	Wesleyan University	Page 18
Brian Wynne	bwynne@wesleyan.edu	Bard College at Simon's Rock	Page 18
Taewon Yang	yangjong@nmsu.edu	New Mexico State University	Page 19

2 Schedule of Talks

Monday, June 24, 2013

9:00 - 9:20 - Registration and Welcome

Morning Session
Session Chair: Kevin M. Drees

Time	Speaker	Title	Abstract
9:30 - 10:20	Charles Holland	The Relation of Scrimger ℓ-Groups to Various Classes of ℓ-Groups	Page 14
10:30 - 10:50	Michelle Knox	p-Extensions and $C(X,\mathbb{Z}) \leq C(X,\mathbb{Q}) \leq C(X)$	Page 15
11:00 - 11:20	Philip Scowcroft	Existentially Closed Abelian ℓ-groups	Page 18
11:30 - 11:50	James Madden	The Localic Yosida Theorem Without Weak Units	Page 15

12:00 - 1:20 - Break for lunch

Afternoon Session
Session Chair: Themba Dube

Time	Speaker	Title	Abstract
1:30 - 2:20	Rick Ball	Pointfree Integration	Page 10
2:30 - 2:50	John Frith	Coreflections for $\mathcal{S}$-Frames	Page 13
3:00 - 3:20	Anneliese Schauerte	Regularity and Normality for $\mathcal{S}$-Frames	Page 17
3:30 - 4:20	Warren McGovern	Rings of Continuous Functions with Countable Range	Page 16

Tuesday, June 25, 2013

Morning Session
Session Chair: Charles Holland

Time	Speaker	Title	Abstract
9:00 - 9:50	Jorge Martinez	Hölder Categories	Page 16
10:00 - 10:20	Inderasan Naidoo	On Convergence in Frames	Page 16
10:30 - 10:50	Themba Dube	Realcompact Supports in Frames	Page 13
11:00 - 11:50	Tony Hager	On Monomorphisms in ℓ-Groups and Frames	Page 14

12:00 - 1:20 - Break for lunch

Afternoon Session
Session Chair: Michelle Knox

Time	Speaker	Title	Abstract
1:30 - 1:50	Papiya Bhattacharjee	p-Extensions in Frames	Page 11
2:00 - 2:20	Tega Ighedo	Covering Maximal Ideals with Their Minimal Primes	Page 14
2:30 - 2:50	Gayle Apfel	Compactifications, Completions, and Other Strict Extensions of Frames	Page 9
3:00 - 3:20	Taewon Yang	Some Notes on Idempotents of Rings	Page 19

6:00 - 9:00 - Dinner

Wednesday, June 26, 2013

Morning Session
Session Chair: Jorge Martinez

Time	Speaker	Title	Abstract
9:00 - 9:50	Michael Darnel	Minimal Non-Metabelian Varieties Which Contain No Nonabelian o-Groups	Page 12
10:00 - 10:20	Brian Wynne	Lex-Products of Existentially Closed Abelian ℓ-Groups	Page 18
10:30 - 10:50	Ricardo Carrera	Hull Classes in Frames	Page 12

11:00 - 1:30 - Break for lunch

Closing

3 Abstracts

Compactifications, Completions, and Other Strict Extensions of Frames

Gayle Apfel

University of Cape Town
e-mail: gdmp27@gmail.com

Abstract

Extensions of spaces have been constructed and used since the 19th century, for example, to form the complex sphere from the complex plain by adding a point at infinity. Once topological spaces were invented in the 20th century, completions and compactifications became important examples of extensions. Bernhard Banaschewski wrote that extension problems have a "philosophical charm" in that they seem to ask the question: "What possibilities in the unknown are determined by the known?"[1] He went on to study all the extensions of a given space, and showed that the coarsest possible topology on an extension space gives the so-called *strict extension*.

In this talk, we discuss strict extensions in the pointfree setting: here, a strict extension is an onto frame homomorphism whose right adjoint generates the domain. In [2], Sung Sa Hong showed how to construct a strict extension of a frame using a set of filters. In the pointfree setting, compactifications and completions are always strict extensions, and can be constructed using Hong's construction. Strong filters are used to construct a compactification of a frame, while regular Cauchy filters of a nearness frame are used for a completion.

This is work in progress towards an M.Sc.

References

[1] B. BANASCHEWSKI, *Extensions of topological spaces*, Canadian Mathematical Bulletin, 7 (1964), pp. 1–22.

[2] S. S. HONG, *Convergence in frames*, Kyungpook Mathematical Journal, 35 (1995), pp. 85–93.

Time of talk: Tuesday, June 25, 2013 at 2:30 - 2:50

Pointfree Pointwise Convergence

Rick Ball*

University of Denver
e-mail: rball@du.edu

Abstract

A family $\{g_i : i \in I\}$ of continuous real-valued functions on a Tychonoff space X has a a supremum $g_0 = \bigvee_I g_i$. Question: How can we tell if the join is pointwise, i.e., if $g_0(x) = \bigvee_I g_i(x)$ for all $x \in X$? Answer: iff

$$\bigvee_I g_i^{-1}(r,\infty) = g_0^{-1}(r,\infty),\ r \in \mathbb{R}.$$

This definition carries over directly to the pointfree context. (Recall the pointfree counterpart of CX is $\mathcal{R}L$, the family of frame maps $\mathcal{O}\mathbb{R} \to L$. L is assumed to be completely regular throughout.)

Definition 1 *A supremum $g_0 = \bigvee_I g_i$ in $\mathcal{R}L$ is said to* pointwise *if $\bigvee_I g_i(r,\infty) = g_0(r,\infty)$ for all $r \in \mathbb{R}$. We write $g_0 = \cdot\bigvee_I g_i$.*

A pointwise supremum in CX is robust in the following sense. If $\theta : CX \to CY$ is any **W**-morphism, and if $g_0 = \bigvee_I g_i$ is pointwise in CX, then $\theta(g_0) = \bigvee_I \theta(g_i)$ is pointwise in CY. That is because θ is realized by some continuous function $h : Y \to X$ by the rule $\theta(g) = g \circ h$ for all $g \in CX$. Therefore

$$\bigvee_I \theta(g_i)(y) = \bigvee_I g_i(h(y)) = g_0(h(y)) = \theta(g)(y),\quad y \in Y.$$

Essentially the same proof shows that pointwise suprema in $\mathcal{R}L$ are robust in this sense. What is surprising is that the converse holds as well.

Theorem 1 *A supremum $g_0 = \bigvee_I g_i$ in $\mathcal{R}L$ is pointwise iff it is preserved by every morphism, i.e., iff $\theta(g_0) = \bigvee_I \theta(g_i)$ for all* **W***-morphisms θ out of $\mathcal{R}L$.*

There is a close relationship between the completeness properties of $\mathcal{R}L$ and the separation properties of L. The relationship is usually given by some sort of Nakano-Stone Theorem. Let us say that a **W**-object G is *pointwise complete* (*pointwise σ-complete*) if every bounded (countable) subset has a pointwise join.

Theorem 2 *$\mathcal{R}L$ is pointwise complete (pointwise σ-complete) iff L is Boolean (a P-frame).*

Pointwise suprema reduce to countable joins.

Proposition 1 *If $g_0 = \cdot\bigvee_I g_i$ then there is some countable subset $I_0 \subseteq I$ such that $g_0 = \cdot\bigvee_{I_0} g_i$.*

Let us say that an increasing (decreasing) sequence $\{g_n\}$ in $\mathcal{R}L$ *converges pointwise upwards* (*downwards*) to g_0, and write $g_n \nearrow g_0$ ($g_n \searrow g_0$), if $g_n \leq g_{n+1}$ ($g_n \geq g_{n+1}$) for all n and $g_0 = \cdot\bigvee_I g_i$ ($g_0 = \cdot\bigwedge_I g_i$). We use the same terminology in an abstract **W**-object G: $g_n \nearrow g_0$ provided this is true in the Madden representation of G. So we can use the mechanism of this representation to characterize this convergence.

Lemma 1 *$g_n \nearrow g_0$ iff $\left[(g_n - r)^+ : n \in \mathbb{N}\right] = \left[(g - r)^+\right]$ for all $r \in \mathbb{R}$. (Here $[A]$ stands for the W-kernel generated by A.)*

Using the familiar lim sup = lim inf idea, we can use upward and downward pointwise convergence to define ordinary pointwise converge. Let $\{g_n\}$ be a bounded sequence and g_0 an element in a **W**-object G. Let L be the Madden frame of G and let $\mathcal{P}L$ be its P-frame reflection. The reflector map $L \to \mathcal{P}L$ provides an injection $\mathcal{R}L \to \mathcal{RP}L$, so we concatenate this with the Madden representation $G \to \mathcal{R}L$ to get a representation $G \to \mathcal{RP}L$.

Definition 2 *Assuming the preceding terminology, we say that* $\{g_n\}$ converges pointwise to *g_0, and write $g_n \dot{\to} g_0$, provided that*

$$\left(\cdot \bigvee_{n\geq m} g_n\right) \searrow g_0 \text{ and } \left(\cdot \bigwedge_{n\geq m} g_n\right) \nearrow g_0,$$

where the meets and joins are computed in $\mathcal{RP}L$.

Pointwise convergence is a Hausdorff ℓ-convergence, i.e., all the ℓ-group operations are continuous with respect to it. This begs several important questions.

Questions 1

1. *Does pointwise density "capture" epis? A pointwise dense* **W***-extension is an epimorphic embedding. Is the converse true? We doubt this, but have yet to find an example which settles the issue. And there is another, quite different, ℓ-convergence which does capture epis.*
2. *$G \leq \beta G$, the embedding of G in its functorial epicompletion, is pointwise dense. Does this mean that βG can be constructed from G by means of pointwise Cauchy filter construction? If so, it would provide a nice characterization of epicompleteness, namely pointwise Cauchy completeness (every pointwise Cauchy filter converges). We believe this to be correct.*

* Joint work with Joanne Walters-Wayland

Time of talk: Monday, June 24, 2013 at 1:30 - 2:20

---

p-Extensions in Frames

Papiya Bhattacharjee*

Penn State Erie, The Behrend College
e-mail: pxb39@psu.edu

Abstract

Recently, p-extensions have been studied for commutative rings including the rings of continuous real-valued functions, $C(X)$. We extend this concept for the rings of continuous real functions on a frame L, $C(L)$, and compare the results with that of $C(X)$.

* Joint work with Drew Moshier and Joanne Walters-Wayland

Time of talk: Tuesday, June 25, 2013 at 1:30 - 1:50

Hull Classes in Frames

Ricardo Carrera

Nova Southeastern University
e-mail: richardo@nova.edu

Abstract

The subject of Hull Classes within the category of Archimedean lattice-ordered groups with designated weak unit and ℓ-homomorphisms that preserve the weak unit is one that has been extensive studied. Our goal is to introduce and present some preliminary results pertaining to hull classes within certain categories of frames.

Time of talk: Wednesday, June 26, 2013 at 10:30 - 10:50

Minimal Non-Metabelian Varieties Which Contain No Nonabelian o-Groups

Michael Darnel (with W. C. Holland)

Indiana University South Bend
e-mail: mdarnel@iusb.edu

Abstract

In an early paper on varieties of lattice-ordered groups, Martinez suggested that the varieties $\mathcal{S}_n$ defined by what are now called Scrimger ℓ-groups S_n would be very small varieties which are metabelian. Scrimger then showed that if n is a positive prime integer, then $\mathcal{S}_n$ covers the abelian variety, which is the smallest nontrivial variety of ℓ-groups. Darnel, Gurchenkov, and Reilly then independently and nearly simultaneously proved that these are the only minimally nonabelian varieties which contain no nonabelian totally ordered groups.

Holland and Reilly then gave a complete description of the lattice of metabelian ℓ-group varieties which contain no nonabelian totally ordered groups.

This article now describes all of the minimally non-metabelian ł-group varieties which contain no nonabelian totally ordered groups. Smith's generalizations of the Scrimger ℓ-groups provide one countably infinite family of such varieties and the authors show that all others can be described by varieties generated by new ℓ-groups called M-groups $\mathcal{M}_{p,p,p^k}$, where p is a positive prime integer and k is a positive integer. Thus there are only countably infinitely many minimal non-metabelian varieties which contain no nonabelian o-groups.

Time of talk: Wednesday, June 26, 2013 at 9:00 - 9:50

Realcompact Supports in Frames

Themba Dude

University of South Africa
e-mail: Dubeta@unisa.ac.za

Abstract

For a completely regular frame L we denote, as usual, by $\mathcal{R}L$ the ring of continuous real functions on L. The support of a function $\alpha \in \mathcal{R}L$ is the closed quotient $\uparrow(\operatorname{coz}\alpha)^*$. Put $\mathcal{R}_\varrho L = \{\alpha \in \mathcal{R}L \mid \downarrow(\operatorname{coz}\alpha) \text{ is realcompact}\}$ and $\mathcal{R}_\rho(L) = \{\alpha \in \mathcal{R}L \mid \uparrow(\operatorname{coz}\alpha)^* \text{ is realcompact}\}$. The first set is always an ideal of $\mathcal{R}L$; indeed, it is the intersection of the free real maximal ideals of $\mathcal{R}L$. If supports of L are coz-quotients, then $\mathcal{R}_\rho(L)$ is also an ideal. If L satisfies the stronger condition that supports are C-quotients, then $\mathcal{R}_\rho(L)$ is the intersection of pure parts of the free real maximal ideals of $\mathcal{R}L$. The ideal $\mathcal{R}_\varrho(L)$ is free iff L is locally realcompact iff L is (isomorphic to) an open quotient of υL. On the other hand, $\mathcal{R}_\varrho(L)$ is prime iff it is a free real maximal ideal iff $\upsilon L \to L$ is a one-point extension of L.

Time of talk: Tuesday, June 25, 2013 at 10:30 - 10:50

Coreflections for $\mathcal{S}$-Frames

John Frith*

University of Cape Town
e-mail: John.Frith@uct.ac.za

Abstract

Madden, in [2], states "It will be possible, I believe, to formulate a useful notion of a partial frame. This would be a meet-semilattice in which certain distinguished subsets would all have suprema and in which meets would distribute over joins of such subsets." Since then, further work has been done on this topic by, for instance, Paseka [3], Zhao [5], Zenk [4] and Banaschewski and Pultr [1].

We present a theory of such partial frames, which we call $\mathcal{S}$-frames, with the specific aim of discussing covering structures, both nearness and uniform. We show how to construct certain coreflective subcategories in the category of nearness $\mathcal{S}$-frames.

[1] Banaschewski, B. and Pultr, A. "A General View of Approximation," Appl. Categ. Struct. 14 (2006) 165 – 190.

[2] Madden, J. "κ-frames," J.P.A.A. 70 (1991) 107 – 127.

[3] Paseka, J. "Covers in generalized frames," in General Algebra and Ordered Sets (Horni Lipova 1994) Palacky Univ. Olomouc, Olomouc, pp. 84 – 99.

[4] Zenk, E.R. "Categories of partial frames," Algebra Univ. 54 (2005) 213 – 235.

[5] Zhao, D. "On projective Z-frames," Canad. Math. Bull. 40 (1997) 39 – 46.

* Joint work with Anneliese Schauerte

Time of talk: Monday, June 24, 2013 at 2:30 - 2:50

On Monomorphisms in ℓ-Groups and Frames

TONY HAGER

Wesleyan University
e-mail: ahager@wesleyan.edu

Abstract

The assignment to a **W**-object (archimedean ℓ-group with weak unit) the **LFrm**-object (Lindelof completely regular frame) of its **W**-kernels is a functor k from **W** onto **LFrm**. This k preserves and reflects monics; in **W** monic means one-to-one,while in **LFrm** it means dense. We call a **W**-map f kernel-injective (**KI**) if kf is one-to-one,and describe: The **KI**-maps in **W**;the **W**-objects all embeddings of which are **KI**,thus the **LFrm**-objects all monics from which are one-to-one;all the **KI**-epicompletions of $C(X)$.

Time of talk: Tuesday, June 25, 2013 at 11:00 - 11:50

The Relation of Scrimger ℓ-Groups to Various Classes of ℓ-Groups

CHARLES HOLLAND

University of Colorado
e-mail: Charles.Holland@colorado.edu

Abstract

An ℓ-group G is a group and a lattice such that for all $x, y, z \in G$,
$x(y \vee z) = (xy) \vee (xz)$ and $(y \vee z)x = (yx) \vee (zx)$.

I will describe the Scrimger ℓ-groups together with some generalizations of them, and then how they are related to certain classes of ℓ-groups.

In the first case, we will consider various generalizations of commutativity of ℓ-groups.

And in the second case, we will investigate the *amalgamation* of ℓ-groups. An ℓ-group A is said to be an *amalgamation base* of ℓ-groups if whenever A is embedded in ℓ-groups G_1 and G_2, there is an ℓ-group L such that G_1 and G_2 are each embedded in L so that both of the resulting embeddings of A in L are the same. The totally ordered group of integers $\mathbb{Z}$ is an amalgamation base of ℓ-groups. We will consider the other cardinal products $\mathbb{Z}^n$.

Time of talk: Monday, June 25, 2013 at 9:30 - 9:20

Covering Maximal Ideals with Their Minimal Primes

Tega Ighedo

University of South Africa
e-mail: Ighedo@unisa.ac.za

Abstract

Let L be a completely regular frame, and let $\mathcal{R}L$ be the ring of continuous real functions of L. In this talk I will report on how nearly round quotients of the Stone-Čech compactification of a completely regular frame are defined, and use them to characterise those L for which every maximal ideal of $\mathcal{R}L$ is covered by its minimal prime ideals. We will also pursue this for maximal ideals of bounded distributive lattices. This is joint work with Themba Dube.

Time of talk: Tuesday, June 25, 2013 at 2:00 - 2:20

---

p-Extensions and $C(X,\mathbb{Z}) \leq C(X,\mathbb{Q}) \leq C(X)$

Michelle Knox

Midwestern State University
e-mail: mathandlit@yahoo.com

Abstract

Let R be a commutative ring with identity. A ring extension $R \hookrightarrow S$ is called a p-extension (or R is a p-embedded in S) if for every $s \in S$ there is an $r \in R$ such that $sS = rS$, i.e., every principally generated ideal of S is generated by an element of R. We will discuss some theory of p-extensions in the context of $C(X)$.

Time of talk: Monday, June 24, 2013 at 10:30 - 10:50

The Localic Yosida Theorem Without Weak Units

James Madden

Louisiana State University
e-mail: jamesjmadden@gmail.com

Abstract

In this talk, I will present a version of localic Yosida representation for archimedean ℓ-groups that are not required to have a weak order unit. The idea is to form all the localic representations that can be made as we allow the unit of the representation to vary over the positive cone A^+ of of the ℓ-group A that we wish to represent, while keeping track of the relationships between the various representations that arise using a pre-sheaf on A^+. We investigate the functorial properties of this construction and discuss what it means for the pre-sheaf we obtain to be a sheaf with respect to various natural Grothendieck topologies. This provides a unifying framework for some well-known constructions.

Time of talk: Monday, June 26, 2013 at 11:30 - 11:50

Hölder Categories

Jorge Martinez

University of Florida
e-mail: gnocchiverdi@gmail.com

Abstract

A *Hölder category* is a complete, well-powered category in which

1. every quasi-initial object is simple;
2. and which has a simple, quasi-initial coseparator R.

The initial and terminal objects in a category are well understood and well documented in the literature; the quasi-initial objects far less so, and the literature on simple objects is (as far as we know) non-existent. An object U is *quasi-initial* if $|\hom(U, B)| \leq 1$ for each object B. Call an object S is simple if for each morphism $e : A \longrightarrow B$ is either monic, or else the (extremal-epi, mono)-factorization of e is through the terminal object.

Several examples will be given, but only some of them well-understood. We will have achieved some measure of success if we are able to establish that the formalisms introduced here capture Hölder's Theorem. We describe the maximum monoreflection μ in a Hölder category in several ways. Under a mild hypothesis on pushouts we also show that the reflected objects under μ are epicomplete.

Time of talk: Monday, June 24, 2013 at 3:30 - 4:20

Rings of Continuous Functions with Countable Range

Warren McGovern

Florida Atlantic University
e-mail: Warren.McGovern@fau.edu

Abstract

The ring of continuous real-valued functions with countable range is denoted by $C_c(X)$. We discuss what we have learned about different ring theoretic properties of $C_c(X)$. We shall include a discussion about rings of quotients of this ring.

Time of talk: Monday, June 24, 2013 at 3:30 - 4:20

On Convergence in Frames

Inderasan Naidoo

University of South Africa
e-mail: naidoi@unisa.ac.za

Abstract

The talk is joint work with Themba Dube in which we revisit the notion of convergence and clustering of filters in frames together with the various notions of completeness in uniform frames. We also introduce strong clustering of filters. Featured in the talk are the studies incorporated with (countable) paracompactness, uniform (countable) paracompactness and the notion of uniform para-Lindelofness.

Time of talk: Tuesday, June 25, 2013 at 10:00 - 10:20

Regularity and Normality for $\mathcal{S}$-Frames

Anneliese Schauerte*

University of Cape Town
e-mail: Anneliese.Schauerte@uct.ac.za

Abstract

In this talk, we look at examples of coreflective subcategories that arise from using the method outlined by John Frith in his talk "Coreflections for $\mathcal{S}$-frames." Further, the fact that a frame has a compatible nearness structure if and only if it is regular leads us to consider regularity for $\mathcal{S}$-frames. We consider several cases, corresponding to different choices of $\mathcal{S}$, and find that regularity can be characterized in terms of familiar properties. In conclusion, we introduce a notion of normality and show that the finitely fine nearness is uniform if and only if the underlying $\mathcal{S}$-frame is regular and normal.

* Joint work with John Frith

Time of talk: Monday, June 24, 2013 at 3:00 - 3:20

Existentially Closed Abelian ℓ-Groups

Philip Scowcroft

Wesleyan University
e-mail: pscowcroft@wesleyan.edu

Abstract

Among Abelian ℓ-groups, the existentially closed (e.c.) ℓ-groups play a role analogous to that of algebraically closed fields among fields or of real-closed ordered fields among ordered fields. After defining the e.c. ℓ-groups and mentioning the amalgamation property and Nullstellensatz to which they give rise, this talk will describe methods for building e.c. ℓ-groups and point out algebraic operations under which this class of ℓ-groups is closed.

Time of talk: Monday, June 24, 2013 at 11:00 - 11:20

Lex-Products of Existentially Closed Abelian ℓ-Groups

Brian Wynne

e-mail: bwynne@wesleyan.edu

Abstract

Given Abelian ℓ-groups A and B and a lattice homomorphism π from the principal convex ℓ-subgroups of A into the summands of B with $\pi(0) = 0$, a construction of Ball, Conrad, and Darnel provides an ℓ-group structure on the Cartesian product $A \times B$ in which "A lies above B." I refer to the resulting ℓ-group as a lex-product of A and B. I will discuss applications of the lex-product construction to the study of existentially closed Abelian ℓ-groups and related results.

Time of talk: Wednesday, June 26, 2013 at 10:00 - 10:20

Some Notes on Idempotents of Rings

Taewon Yang

New Mexico State University
e-mail: yangjong@nmsu.edu

Abstract

It is well-known that the collection $\mathcal{E}(R)$ of all idempotents of the unital ring R form an orthomodular poset (Flachsmeyer [3] and Harding [4]). If the ring R in question is commutative, then $\mathcal{E}(R)$ coincides with the familiar Boolean lattice $Idp(R)$ of all idempotents of the ring R. In this talk, we consider the orthomodular poset $\mathcal{E}(R)$ of the endomorphism ring $R = End(\xi)$ of all bundle maps on some Hermitian vector bundle ξ (Crown [2]). In particular, we show that $\mathcal{E}(R)$ is not lattice-ordered.

References

[1] R. Börger, *The tensor product of orthomodular posets*, Categorical structures and their applications, World Sci. Publ., River Edge, NJ, 2004, 29-40.

[2] G. Crown, *On some orthomodular posets of vector bundles.* J. Natur. Sci. and Math. 15 (1975), no. 1-2, 11-25

[3] J. Flachsmeyer, *Note on orthocomplemented posets.* Proceedings of the Conference, Topology and Measure III. Greifswald, Part 1, 65-73, 1982.

[4] J. Harding, *Decompositions in quantum logic*, Trans. Amer. Math. Soc. 348 (1996), no. 5, 1839-1862.

Time of talk: Tuesday, June 25, 2013 at 3:00 - 3:20