Compactifications, Completions, and Other Strict Extensions of Frames

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Abstract

Extensions of spaces have been constructed and used since the 19th century, for example, to form the complex sphere from the complex plain by adding a point at infinity. Once topological spaces were invented in the 20th century, completions and compactifications became important examples of extensions. Bernhard Banaschewski wrote that extension problems have a "philosophical charm" in that they seem to ask the question: "What possibilities in the unknown are determined by the known?"[1] He went on to study all the extensions of a given space, and showed that the coarsest possible topology on an extension space gives the so-called *strict extension*.

In this talk, we discuss strict extensions in the pointfree setting: here, a strict extension is an onto frame homomorphism whose right adjoint generates the domain. In [2], Sung Sa Hong showed how to construct a strict extension of a frame using a set of filters. In the pointfree setting, compactifications and completions are always strict extensions, and can be constructed using Hong's construction. Strong filters are used to construct a compactification of a frame, while regular Cauchy filters of a nearness frame are used for a completion.

This is work in progress towards an M.Sc.

References

[1] B. BANASCHEWSKI, *Extensions of topological spaces*, Canadian Mathematical Bulletin, 7 (1964), pp. 1–22.

[2] S. S. HONG, *Convergence in frames*, Kyungpook Mathematical Journal, 35 (1995), pp. 85–93.

Pointfree Integration

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Abstract

The techniques of pointfree topology afford a development of integration which is rigorous, elegant, and revealing. We outline this development, emphasizing motivation. Along the way we classify some classical integrals in terms of their categorical properties.

* Joint work with Aleš Pultr

p-Extensions in Frames

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Abstract

Recently, p-extensions have been studied for commutative rings including the rings of continuous real-valued functions, $C(X)$. We extend this concept for the rings of continuous real functions on a frame L, $C(L)$, and compare the results with that of $C(X)$.

* Joint work with Drew Moshier and Joanne Walters-Wayland

Hull Classes in Frames

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Abstract

The subject of Hull Classes within the category of Archimedean lattice-ordered groups with designated weak unit and ℓ-homomorphisms that preserve the weak unit is one that has been extensive studied. Our goal is to introduce and present some preliminary results pertaining to hull classes within certain categories of frames.

Minimal Non-Metabelian Varieties Which Contain No Nonabelian o-Groups

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Abstract

In an early paper on varieties of lattice-ordered groups, Martinez suggested that the varieties $\mathcal{S}_n$ defined by what are now called Scrimger ℓ-groups S_n would be very small varieties which are metabelian. Scrimger then showed that if n is a positive prime integer, then $\mathcal{S}_n$ covers the abelian variety, which is the smallest nontrivial variety of ℓ-groups. Darnel, Gurchenkov, and Reilly then independently and nearly simultaneously proved that these are the only minimally nonabelian varieties which contain no nonabelian totally ordered groups.

Holland and Reilly then gave a complete description of the lattice of metabelian ℓ-group varieties which contain no nonabelian totally ordered groups.

This article now describes all of the minimally non-metabelian ł-group varieties which contain no nonabelian totally ordered groups. Smith's generalizations of the Scrimger ℓ-groups provide one countably infinite family of such varieties and the authors show that all others can be described by varieties generated by new ℓ-groups called M-groups $\mathcal{M}_{p,p,p^k}$, where p is a positive prime integer and k is a positive integer. Thus there are only countably infinitely many minimal non-metabelian varieties which contain no nonabelian o-groups.

Realcompact Supports in Frames

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Abstract

For a completely regular frame L we denote, as usual, by $\mathcal{R}L$ the ring of continuous real functions on L. The support of a function $\alpha \in \mathcal{R}L$ is the closed quotient $\uparrow(\operatorname{coz}\alpha)^*$. Put $\mathcal{R}_\varrho L = \{\alpha \in \mathcal{R}L \mid \downarrow(\operatorname{coz}\alpha)$ is realcompact$\}$ and $\mathcal{R}_\rho(L) = \{\alpha \in \mathcal{R}L \mid \uparrow(\operatorname{coz}\alpha)^*$ is realcompact$\}$. The first set is always an ideal of $\mathcal{R}L$; indeed, it is the intersection of the free real maximal ideals of $\mathcal{R}L$. If supports of L are coz-quotients, then $\mathcal{R}_\rho(L)$ is also an ideal. If L satisfies the stronger condition that supports are C-quotients, then $\mathcal{R}_\rho(L)$ is the intersection of pure parts of the free real maximal ideals of $\mathcal{R}L$. The ideal $\mathcal{R}_\varrho(L)$ is free iff L is locally realcompact iff L is (isomorphic to) an open quotient of υL. On the other hand, $\mathcal{R}_\varrho(L)$ is prime iff it is a free real maximal ideal iff $\upsilon L \to L$ is a one-point extension of L.

Coreflections for $\mathcal{S}$-Frames

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Abstract

Madden, in [2], states “It will be possible, I believe, to formulate a useful notion of a partial frame. This would be a meet-semilattice in which certain distinguished subsets would all have suprema and in which meets would distribute over joins of such subsets.” Since then, further work has been done on this topic by, for instance, Paseka [3], Zhao [5], Zenk [4] and Banaschewski and Pultr [1].

We present a theory of such partial frames, which we call $\mathcal{S}$-frames, with the specific aim of discussing covering structures, both nearness and uniform. We show how to construct certain coreflective subcategories in the category of nearness $\mathcal{S}$-frames.

[1] Banaschewski, B. and Pultr, A. “A General View of Approximation,” Appl. Categ. Struct. 14 (2006) 165 – 190.

[2] Madden, J. “κ-frames,” J.P.A.A. 70 (1991) 107 – 127.

[3] Paseka, J. “Covers in generalized frames,” in General Algebra and Ordered Sets (Horni Lipova 1994) Palacky Univ. Olomouc, Olomouc, pp. 84 – 99.

[4] Zenk, E.R. “Categories of partial frames,” Algebra Univ. 54 (2005) 213 – 235.

[5] Zhao, D. “On projective Z-frames,” Canad. Math. Bull. 40 (1997) 39 – 46.

* Joint work with Anneliese Schauerte

On Monomorphisms in ℓ-Groups and Frames

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Abstract

The assignment to a **W**-object (archimedean ℓ-group with weak unit) the **LFrm**-object (Lindelof completely regular frame) of its **W**-kernels is a functor k from **W** onto **LFrm**. This k preserves and reflects monics; in **W** monic means one-to-one,while in **LFrm** it means dense. We call a **W**-map f kernel-injective (**KI**) if kf is one-to-one,and describe: The **KI**-maps in **W**;the **W**-objects all embeddings of which are **KI**,thus the **LFrm**-objects all monics from which are one-to-one;all the **KI**-epicompletions of $C(X)$.

The Relation of Scrimger ℓ-Groups to Various Classes of ℓ-Groups

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Abstract

An ℓ-group G is a group and a lattice such that for all $x, y, z \in G$,
$x(y \vee z) = (xy) \vee (xz)$ and $(y \vee z)x = (yx) \vee (zx)$.

I will describe the Scrimger ℓ-groups together with some generalizations of them, and then how they are related to certain classes of ℓ-groups.

In the first case, we will consider various generalizations of commutativity of ℓ-groups.

And in the second case, we will investigate the *amalgamation* of ℓ-groups. An ℓ-group A is said to be an *amalgamation base* of ℓ-groups if whenever A is embedded in ℓ-groups G_1 and G_2, there is an ℓ-group L such that G_1 and G_2 are each embedded in L so that both of the resulting embeddings of A in L are the same. The totally ordered group of integers $\mathbb{Z}$ is an amalgamation base of ℓ-groups. We will consider the other cardinal products $\mathbb{Z}^n$.

Covering Maximal Ideals with Their Minimal Primes

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Abstract

Let L be a completely regular frame, and let $\mathcal{R}L$ be the ring of continuous real functions of L. In this talk I will report on how nearly round quotients of the Stone-Čech compactification of a completely regular frame are defined, and use them to characterise those L for which every maximal ideal of $\mathcal{R}L$ is covered by its minimal prime ideals. We will also pursue this for maximal ideals of bounded distributive lattices. This is joint work with Themba Dube.

p-Extensions and $C(X, \mathbb{Z}) \leq C(X, \mathbb{Q}) \leq C(X)$

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Abstract

Let R be a commutative ring with identity. A ring extension $R \hookrightarrow S$ is called a p-extension (or R is a p-embedded in S) if for every $s \in S$ there is an $r \in R$ such that $sS = rS$, i.e., every principally generated ideal of S is generated by an element of R. We will discuss some theory of p-extensions in the context of $C(X)$.

The Localic Yosida Theorem Without Weak Units

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Abstract

In this talk, I will present a version of localic Yosida representation for archimedean ℓ-groups that are not required to have a weak order unit. The idea is to form all the localic representations that can be made as we allow the unit of the representation to vary over the positive cone A^+ of of the ℓ-group A that we wish to represent, while keeping track of the relationships between the various representations that arise using a pre-sheaf on A^+. We investigate the functorial properties of this construction and discuss what it means for the pre-sheaf we obtain to be a sheaf with respect to various natural Grothendieck topologies. This provides a unifying framework for some well-known constructions.

Hölder Categories

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Abstract

A *Hölder category* is a complete, well-powered category in which

1. every quasi-initial object is simple;
2. and which has a simple, quasi-initial coseparator R.

The initial and terminal objects in a category are well understood and well documented in the literature; the quasi-initial objects far less so, and the literature on simple objects is (as far as we know) non-existent. An object U is *quasi-initial* if $|\hom(U, B)| \leq 1$ for each object B. Call an object S is simple if for each morphism $e : A \longrightarrow B$ is either monic, or else the (extremal-epi, mono)-factorization of e is through the terminal object.

Several examples will be given, but only some of them well-understood. We will have achieved some measure of success if we are able to establish that the formalisms introduced here capture Hölder's Theorem. We describe the maximum monoreflection μ in a Hölder category in several ways. Under a mild hypothesis on pushouts we also show that the reflected objects under μ are epicomplete.

Rings of Continuous Functions with Countable Range

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Abstract

The ring of continuous real-valued functions with countable range is denoted by $C_c(X)$. We discuss what we have learned about different ring theoretic properties of $C_c(X)$. We shall include a discussion about rings of quotients of this ring.

On Convergence in Frames

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Abstract

The talk is joint work with Themba Dube in which we revisit the notion of convergence and clustering of filters in frames together with the various notions of completeness in uniform frames. We also introduce strong clustering of filters. Featured in the talk are the studies incorporated with (countable) paracompactness, uniform (countable) paracompactness and the notion of uniform para-Lindelofness.

Regularity and Normality for $\mathcal{S}$-Frames

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Abstract

In this talk, we look at examples of coreflective subcategories that arise from using the method outlined by John Frith in his talk "Coreflections for $\mathcal{S}$-frames." Further, the fact that a frame has a compatible nearness structure if and only if it is regular leads us to consider regularity for $\mathcal{S}$-frames. We consider several cases, corresponding to different choices of $\mathcal{S}$, and find that regularity can be characterized in terms of familiar properties. In conclusion, we introduce a notion of normality and show that the finitely fine nearness is uniform if and only if the underlying $\mathcal{S}$-frame is regular and normal.

* Joint work with John Frith

Existentially Closed Abelian ℓ-Groups

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Abstract

Among Abelian ℓ-groups, the existentially closed (e.c.) ℓ-groups play a role analogous to that of algebraically closed fields among fields or of real-closed ordered fields among ordered fields. After defining the e.c. ℓ-groups and mentioning the amalgamation property and Nullstellensatz to which they give rise, this talk will describe methods for building e.c. ℓ-groups and point out algebraic operations under which this class of ℓ-groups is closed.

Lex-Products of Existentially Closed Abelian ℓ-Groups

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Abstract

Given Abelian ℓ-groups A and B and a lattice homomorphism π from the principal convex ℓ-subgroups of A into the summands of B with $\pi(0) = 0$, a construction of Ball, Conrad, and Darnel provides an ℓ-group structure on the Cartesian product $A \times B$ in which "A lies above B." I refer to the resulting ℓ-group as a lex-product of A and B. I will discuss applications of the lex-product construction to the study of existentially closed Abelian ℓ-groups and related results.

Some Notes on Idempotents of Rings

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Abstract

It is well-known that the collection $\mathcal{E}(R)$ of all idempotents of the unital ring R form an orthomodular poset (Flachsmeyer [3] and Harding [4]). If the ring R in question is commutative, then $\mathcal{E}(R)$ coincides with the familiar Boolean lattice $Idp(R)$ of all idempotents of the ring R. In this talk, we consider the orthomodular poset $\mathcal{E}(R)$ of the endomorphism ring $R = End(\xi)$ of all bundle maps on some Hermitian vector bundle ξ (Crown [2]). In particular, we show that $\mathcal{E}(R)$ is not lattice-ordered.

References

[1] R. Börger, *The tensor product of orthomodular posets*, Categorical structures and their applications, World Sci. Publ., River Edge, NJ, 2004, 29-40.

[2] G. Crown, *On some orthomodular posets of vector bundles.* J. Natur. Sci. and Math. 15 (1975), no. 1-2, 11-25

[3] J. Flachsmeyer, *Note on orthocomplemented posets.* Proceedings of the Conference, Topology and Measure III. Greifswald, Part 1, 65-73, 1982.

[4] J. Harding, *Decompositions in quantum logic*, Trans. Amer. Math. Soc. 348 (1996), no. 5, 1839-1862.