Pareto Improving Segmentation of Multi-product Markets

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joint with Ron Siegel

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Market Segmentation

Firms can segment the market based on available data

- age, sex, location, browsing history, ...
Market Segmentation

Firms can segment the market based on available data
  ▶ age, sex, location, browsing history, ...

Effects of segmentation on consumers?
  ▶ Segmentation can harm consumers
  ▶ Can segmentation benefit consumers?
Does a Pareto Improving Segmentation Exist?
Does a Pareto Improving Segmentation Exist?

Menu

<table>
<thead>
<tr>
<th>Product</th>
<th>Bundle$_1$</th>
<th>Bundle$_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5$</td>
<td>$10$</td>
<td>$8$</td>
</tr>
</tbody>
</table>

Un-segmented market:

CS$_c$(c)

Segmented market:

$\exists$ segmentation s.t.

CS$_c$(c, s) $\geq$ CS$_c$(c)

for all $c$, & $\exists$ c
Does a Pareto Improving Segmentation Exist?

Un-segmented market: $CS(c)$

Menu

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<td>...</td>
<td>...</td>
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</table>

consumer $c$
Does a Pareto Improving Segmentation Exist?

Un-segmented market: $CS(c)$
Does a Pareto Improving Segmentation Exist?

Un-segmented market: $CS(c)$

Segmented market: $CS(c, \text{segmentation})$

$\exists$ segmentation s.t. $CS(c, \text{segmentation}) \geq CS(c)$ for all $c$, & $> CS(c)$ for some $c$
Does a Pareto Improving Segmentation Exist?

Un-segmented market: \( CS(c) \)

Segmented market: \( CS(c, s) \)

\( \exists \text{ segmentation s.t. } CS(c, s) \geq CS(c) \text{ for all } c, \quad \& \quad > \text{ for some } c \)
A Single Product Example (with Unit Demands)

“Market”: \[ 1 - q \quad q \]
\[ \bullet \quad \bullet \]
Valuation \( v \): 1 2
A Pareto Improving (PI) segmentation exists if market $q$ is inefficient.

```
"Market": 1 - q  q
  ●  ●
Valuation $v$: 1  2
```
A Single Product Example (with Unit Demands)

A Pareto Improving (PI) segmentation exists if market $q$ is inefficient

- $q \in (0.5, 1)$

“Market”:

<table>
<thead>
<tr>
<th></th>
<th>1 - $q$</th>
<th>$q$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>●</td>
<td>●</td>
</tr>
</tbody>
</table>

Valuation $v$:

|        | 1       | 2   |

Holds with any number of valuations $1 - q_1 q_2$
A Single Product Example (with Unit Demands)

A Pareto Improving (PI) segmentation exists if market $q$ is inefficient

- $q \in (0.5, 1)$

```
“Market” : 1 - q    q
     ●   ●
Valuation \( v \) : 1    2
```

Optimal price:
```
\begin{align*}
\text{All markets:} & \quad 0 \quad 0.5 \quad 1 \\
\text{1} & \quad 2
\end{align*}
```

Surplus $q$ of $v = 0.5$ to $1$.
A Single Product Example (with Unit Demands)

A Pareto Improving (PI) segmentation exists if market $q$ is inefficient

- $q \in (0.5, 1)$

“Market”: $1 - q$  $q$

Valuation $v$: $1$  $2$

Optimal price: $1$  $2$

All markets:

Surplus of $v = 2$
A Single Product Example (with Unit Demands)

A Pareto Improving (PI) segmentation exists if market $q$ is inefficient

- $q \in (0.5, 1)$: Segment to $q' \leq 0.5$ and $q'' > q$

```
“Market”: 1 − q   q
Valuation v: 1   2
```

```
Optimal price:

All markets:

Surplus of $v = 2$
```

```
Surplus 1
```

```
0  0.5  1
q'  q  q''
```
A Single Product Example (with Unit Demands)

A Pareto Improving (PI) segmentation exists if market \( q \) is inefficient

- \( q \in (0.5, 1) \): Segment to \( q' \leq 0.5 \) and \( q'' > q \)
- Holds with any number of valuations

“Market”: \( 1 - q \quad q \)

Valuation \( v \): \( 1 \quad 2 \)

Optimal price:

All markets:

Surplus of \( v = 2 \)
Screening Example: Qualities $L$ and $H$

“Market”:

<table>
<thead>
<tr>
<th>$1 - q$</th>
<th>$q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>●</td>
<td>●</td>
</tr>
</tbody>
</table>

$v_H$: 1 2

$v_L$: 0.75 1
Screening Example: Qualities \( L \) and \( H \)

PI segmentation \( \not\exists \) for inefficient market \( q = 0.75 \)

```
"Market": 1 - q  q
  ●  ●
\( v_H: \)  1  2
\( v_L: \)  0.75  1
```
Screening Example: Qualities $L$ and $H$

Pl segmentation $\not\subset$ for inefficient market $q = 0.75$

"Market":

$\nu_H$: 1 2

$\nu_L$: 0.75 1

$p(H) = \begin{cases} 1 & 1.75 \\ 0.75 & 2 \end{cases}$

$p(L) = \begin{cases} 0 & 0.25 \\ 0.75 & 1 \end{cases}$

$q$
Screening Example: Qualities $L$ and $H$

PI segmentation $\not\exists$ for inefficient market $q = 0.75$

"Market": \begin{align*}
1 - q & \quad q \\
\bullet & \quad \bullet
\end{align*}

$v_H$: \begin{align*}
1 & \quad 2 \\
\circ & \quad \circ
\end{align*}

$v_L$: \begin{align*}
0.75 & \quad 1
\end{align*}

\begin{align*}
p(H) &= \frac{1}{1.75} \quad \frac{1.75}{2} \\
p(L) &= \frac{0.75}{1.75} \quad \frac{1}{2}
\end{align*}

Surplus of high type:
- $0.25$
- $1$

$q$: \begin{align*}
0 & \quad 0.25 & \quad 0.75 & \quad 1 \\
\circ & \quad \circ & \quad \circ
\end{align*}
Screening Example: Qualities $L$ and $H$

PI segmentation $\not\exists$ for inefficient market $q = 0.75$

```
"Market": 1 - q  
q  
\[ \bullet \]  
\[ \bullet \]  
\[ v_H: \]
1 2
\[ v_L: \]
0.75 1
```

```
\[
p(H) = \begin{cases} 
1 & q = 0.75 \\
1.75 & q = 0.25 \\
2 & q = 0.25 \\
0.75 & q = 0.75 \\
\end{cases}
\]
```

```
\[
p(L) = \begin{cases} 
0.75 & q = 0.75 \\
0.75 & q = 0.25 \\
1 & q = 0.25 \\
1.75 & q = 0.75 \\
\end{cases}
\]
```

Surplus of high type

```
\[
\begin{cases} 
1 & q = 0.75 \\
0.25 & q = 0.25 \\
\end{cases}
\]
```
Screening Example: Qualities $L$ and $H$

PI segmentation $\not\exists$ for inefficient market $q = 0.75$

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<tbody>
<tr>
<td>$v_H$</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$v_L$</td>
<td>0.75</td>
<td>1</td>
</tr>
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</table>

$p(H) = \frac{1}{1.75}$ for $q \in [0, 0.25)$ and $p(H) = 1$ for $q \in [0.75, 1]$

$p(L) = \frac{1}{0.25 - 0.75}$ for $q \in [0.25, 0.75)$ and $p(L) = 0.75$ for $q \in [0.75, 1]$

Surplus of high type:
- $q' = 0.25$ with surplus $1$
- $q'' = 0.75$ with surplus $0.25$
Screening Example: Qualities $L$ and $H$

PI segmentation $\not\exists$ for inefficient market $q = 0.75$

"Market": $1 - q \quad q$

$v_H$: $1 \quad 2$

$v_L$: $0.75 \quad 1$

$p(H) = \begin{cases} 1 & 1.75 & 2 \\ 0.75 & 0.25 & 0.75 \end{cases}$

$p(L) = \begin{cases} 0.75 & 0.25 & 0.75 \end{cases}$

Surplus of high type

$1 \quad 0.25$

$q' \quad 0.25 \quad q' \cdot 1$
Screening Example: Qualities $L$ and $H$

PI segmentation $\not\exists$ for inefficient market $q = 0.75$

- PI segmentation exists for all but one inefficient markets

```
Surplus of high type
```

```
\begin{align*}
\text{"Market" :} & \quad 1 - q \quad q \\
\nu_H : & \quad 1 \quad 2 \\
\nu_L : & \quad 0.75 \quad 1 \\
\end{align*}
```

```
\begin{align*}
p(H) &= \frac{1}{2} \\
p(L) &= \frac{1}{2} \\
\end{align*}
```

```
\begin{align*}
p(H) &= \begin{cases} 1 & q = 0.75 \\ 1.75 & \text{otherwise} \end{cases} \\
p(L) &= \begin{cases} 0.75 & q = 0.75 \\ 0.25 & \text{otherwise} \end{cases}
\end{align*}
```

```
\begin{align*}
1 - q & \quad q \\
\nu_H & \quad 1 \quad 2 \\
\nu_L & \quad 0.75 \quad 1 \\
p(H) & \quad 1 \quad 1.75 \\
p(L) & \quad 0.75 \quad 0.25 \\
\end{align*}
```

```
\begin{align*}
\frac{1}{2} \quad \frac{1}{2} \\
\frac{1}{2} \quad \frac{1}{2} \\
\end{align*}
```

```
\begin{align*}
\frac{1}{2} \quad \frac{1}{2} \\
\frac{1}{2} \quad \frac{1}{2} \\
\end{align*}
```

```
\begin{align*}
\frac{1}{2} \quad \frac{1}{2} \\
\frac{1}{2} \quad \frac{1}{2} \\
\end{align*}
```
Screening Example: Qualities $L$ and $H$

PI segmentation $\not\exists$ for inefficient market $q = 0.75$

- PI segmentation exists for all but one inefficient markets

Two types, any $\#$ of qualities (characterize optimal mechanisms)

- PI segmentation exists for all but finitely many inefficient markets

\[
\begin{align*}
\text{"Market"}: & \quad 1 - q & q \\
\nu_H: & \quad 1 & 2 \\
\nu_L: & \quad 0.75 & 1 \\
\end{align*}
\]

\[
\begin{align*}
p(H) &= \begin{cases} 
1 & q = 0.75 \\
1.75 & q = 2 \\
2 & q = 1 
\end{cases} \\
p(L) &= \begin{cases} 
0.75 & q = 0.25 \\
1 & q = 0 \\
1 & q = 0.75 \\
1 & q = 1 
\end{cases}
\end{align*}
\]

Surplus of high type

\[
\begin{align*}
\text{Surplus of high type:} & \\
q: & 0 & 0.25 & 0.75 & 1 \\
\end{align*}
\]
Model

- **Types:** \( t \in T \) (finite)
- **Alternatives:** \( a \in A \) (finite)
  - e.g., qualities, quantities, configurations
  - e.g., bundles: \( \{1\}, \{2\}, \{1, 2\} \)
- **Valuations:** \( \nu(t, a) \)
- **Costs:** \( c(a) \)
Model

- **Types:** \( t \in T \) (finite)
- **Alternatives:** \( a \in A \) (finite)
  - e.g., qualities, quantities, configurations
  - e.g., bundles: \( \{1\}, \{2\}, \{1, 2\} \)
- **Valuations:** \( \nu(t, a) \)
- **Costs:** \( c(a) \)

**Mechanism:** \( a : T \to \Delta(A), \ p : T \to R \)
- **IC:** \( E[\nu(t, a(t))] - p(t) \geq E[\nu(t, a(t'))] - p(t') \)
- **IR:** \( E[\nu(t, a(t))] - p(t) \geq 0 \)
Markets and Segmentations

Market $f \in \Delta(T)$

- Mechanism optimal if maximizes $E_f[p(t) - c(t)]$
- $CS(t, f)$: surplus of type $t$ in “the” optimal mechanism for market $f$
  - (fix arbitrary selection rule when multiple optimal mechanisms)
Markets and Segmentations

Market $f \in \Delta(T)$

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- $CS(t, f)$: surplus of type $t$ in “the” optimal mechanism for market $f$
  - (fix arbitrary selection rule when multiple optimal mechanisms)

Segmentation $\mu$ of $f$: $\mu \in \Delta(\Delta(T))$ s.t. $E_\mu[f'(t)] = f(t), \forall t$

- e.g., two types, $E_\mu[q'] = q$
Markets and Segmentations

Market $f \in \Delta(T)$

- Mechanism optimal if maximizes $E_f[p(t) - c(t)]$
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Segmentation $\mu$ of $f$: $\mu \in \Delta(\Delta(T))$ s.t. $E_\mu[f'(t)] = f(t), \forall t$

- e.g., two types, $E_\mu[q'] = q$

Segmentation $\mu$ of $f$ is Pareto improving if

1. $\forall f' \in Supp(\mu): \forall t \in Supp(f'), CS(t, f') \geq CS(t, f)$
2. $\exists f' \in Supp(\mu): \exists t \in Supp(f'), >$
Markets and Segmentations

Market $f \in \Delta(T)$

- Mechanism optimal if maximizes $E_f[p(t) - c(t)]$
- $CS(t, f)$: surplus of type $t$ in “the” optimal mechanism for market $f$
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Segmentation $\mu$ of $f$ is Pareto improving if

1. $\forall f' \in Supp(\mu): \forall t \in Supp(f'), CS(t, f') \geq CS(t, f)$
2. $\exists f' \in Supp(\mu): \exists t \in Supp(f'), >$

($f' \succ CS f$ if 1 and 2)
Main Result: PI Segmentations Exist?

Recall: If \( \exists \) PI segmentation of \( f \) \( \Rightarrow \) \( f \) is inefficient

\[ (\exists t, a(t)/\in\arg\max a'(v(t), a'(t)) - c(a'(t))) \]

Theorem

For almost all inefficient markets, PI segmentations exist.

\( \{f: f \text{ inefficient } \&\, \neg\exists \text{ PI segmentation}\} \subseteq \text{a finite union of hyperplanes} \)

\[ H(a) = \{f | \sum_{t} a(t) \cdot f(t) = 0 \} \quad (a(t) \neq 0 \text{ for some } t) \]

Surplus of high type 0.25

\[ q_{0.25} = 0.75 \]

\[ q_{0.75} = 1 \]

\[ q_{1} = 1.25 \]

\[ q_{1.25} = 0.75 / 15 \]
Main Result: PI Segmentations Exist?

Theorem
For almost all inefficient markets, PI segmentations exist.

\[ \{ f : f \text{ inefficient } \& \neg \exists \text{ PI segmentation} \} \subseteq \bigcup \text{finite union of hyperplanes} \]

\[ H(a) = \{ f | \sum t a(t) \cdot f(t) = 0 \} \quad (a(t) \neq 0 \text{ for some } t) \]

Surplus \( q \) of high type 0.25 / 0.75 1 / 1.25 8 / 15
Main Result: PI Segmentations Exist?

Recall: If $\exists$ PI segmentation of $f \Rightarrow f$ is inefficient

- $(\exists t, a(t) \not\in \arg \max_{a'} v(t, a') - c(a'))$
Main Result: PI Segmentations Exist?

Recall: If $\exists$ PI segmentation of $f \Rightarrow f$ is inefficient

$\Rightarrow (\exists t, a(t) \notin \arg \max_{a'} v(t, a') - c(a'))$

Theorem

For almost all inefficient markets, PI segmentations exist.
Main Result: PI Segmentations Exist?

Recall: If $\exists$ PI segmentation of $f \Rightarrow f$ is inefficient
- $(\exists t, a(t) \not\in \arg \max_{a'} v(t, a') - c(a'))$

Theorem

For almost all inefficient markets, PI segmentations exist.

$\{f : f \text{ inefficient } \& \not\exists \text{ PI segmentation}\} \subseteq$ a finite union of hyperplanes
- $H(a) = \{f | \sum_t a(t) \cdot f(t) = 0\} (a(t) \neq 0$ for some $t)$
Main Result: PI Segmentations Exist?

Recall: If \( \exists \) PI segmentation of \( f \) \( \Rightarrow \) \( f \) is inefficient

\[ (\exists t, a(t) \notin \text{arg max}_{a'} \, v(t, a') - c(a')) \]

Theorem

For almost all inefficient markets, PI segmentations exist.

\[ \{ f : f \text{ inefficient} \& \not\exists \text{ PI segmentation} \} \subseteq \text{a finite union of hyperplanes} \]

\[ H(a) = \{ f | \sum_t a(t) \cdot f(t) = 0 \} \quad (a(t) \neq 0 \text{ for some } t) \]
Main Result: PI Segmentations Exist?

Recall: If $\exists$ PI segmentation of $f \Rightarrow f$ is inefficient

$\exists t, a(t) \not\in \arg\max_{a'} v(t, a') - c(a')$

Theorem

For almost all inefficient markets, PI segmentations exist.

$f : f$ inefficient & $\not\exists$ PI segmentation $\subseteq$ a finite union of hyperplanes

$H(a) = \{f | \sum_t a(t) \cdot f(t) = 0\}$ ($a(t) \neq 0$ for some $t$)
Main Result: PI Segmentations Exist?

Recall: If \( \exists \) PI segmentation of \( f \) \( \Rightarrow \) \( f \) is inefficient

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Theorem

For almost all inefficient markets, PI segmentations exist.

\( \{ f : f \text{ inefficient } \& \not\exists \text{ PI segmentation} \} \subseteq \text{a finite union of hyperplanes} \)

- \( H(a) = \{ f | \sum_t a(t) \cdot f(t) = 0 \} \) (\( a(t) \neq 0 \) for some \( t \))
Proof Outline

Given inefficient market $f$

1. $\exists f_1$ s.t. $f_1 \succ_{CS} f$
   - $\forall t \in \text{Supp}(f_1) : CS(t, f') \geq CS(t, f), (\exists t, >)$

Define $f_2$ s.t. $f = \epsilon f_1 + (1 - \epsilon) f_2$

2. Small $\epsilon$, generic $f$: $\text{OptMech}(f) = \text{OptMech}(f_2)$
Step 1: Every Inefficient Market Is Dominated

Lemma

If $f$ is inefficient, then there exists an efficient $f_1$ s.t. $f_1 \succ_{CS} f$. 

Intuition: allocation of $t$ is distorted to extract rents from $v(t, \bar{a})$.
Step 1: Every Inefficient Market Is Dominated

Lemma

If $f$ is inefficient, then there exists an efficient $f_1$ s.t. $f_1 \succ_{CS} f$.

Assume: 1. $c = 0$ 2. $\exists t \forall a, v(t, a) < v(t, a)$ 3. $\exists \bar{a} \forall t, v(t, a) < v(t, \bar{a})$
Step 1: Every Inefficient Market Is Dominated

**Lemma**

*If f is inefficient, then there exists an efficient f₁ s.t. f₁ ≻_{CS} f.*

Assume: 1. \( c = 0 \) 2. \( \exists t \, \forall a, v(t, a) < v(t, a) \) 3. \( \exists \bar{a} \, \forall t, v(t, a) < v(t, \bar{a}) \)
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If $f$ is inefficient, then there exists an efficient $f_1$ s.t. $f_1 \succ_{CS} f$.

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If $f$ is inefficient, then there exists an efficient $f_1$ s.t. $f_1 \succ_{CS} f$.

Assume: 1. $c = 0$ 2. $\exists t \forall a, v(t, a) < v(t, \bar{a})$ 3. $\exists \bar{a} \forall t, v(t, a) < v(t, \bar{a})$

1. Some type $t$ is assigned $\bar{a}$
2. Type $t$ pays strictly more than $v(t, \bar{a})$
3. $f_1(t) = 1 - \delta, f_1(t) = \delta$ for small enough $\delta$
   - optimal to sell $\bar{a}$ at price $v(t, \bar{a})$
Step 1: Every Inefficient Market Is Dominated

Lemma

If \( f \) is inefficient, then there exists an efficient \( f_1 \) s.t. \( f_1 \succ_{CS} f \).

Assume:
1. \( c = 0 \)
2. \( \exists t \forall a, v(t, a) < v(t, \bar{a}) \)
3. \( \exists \bar{a} \forall t, v(t, a) < v(t, \bar{a}) \)

1. Some type \( t \) is assigned \( \bar{a} \)
2. Type \( t \) pays strictly more than \( v(t, \bar{a}) \)
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   - optimal to sell \( \bar{a} \) at price \( v(t, \bar{a}) \)

\[
\begin{array}{c}
\text{v(\cdot, a)} \\
\text{v(\cdot, \bar{a})}
\end{array}
\]
Step 1: Every Inefficient Market Is Dominated

Lemma

If \( f \) is inefficient, then there exists an efficient \( f_1 \) s.t. \( f_1 \succeq_{CS} f \).

Assume: 1. \( c = 0 \) 2. \( \exists t \forall a, v(t, a) < v(t, \bar{a}) \) 3. \( \exists \bar{a} \forall t, v(t, a) < v(t, \bar{a}) \)

1. Some type \( t \) is assigned \( \bar{a} \)
2. Type \( t \) pays strictly more than \( v(t, \bar{a}) \)
3. \( f_1(t) = 1 - \delta, f_1(t) = \delta \) for small enough \( \delta \)
   - optimal to sell \( \bar{a} \) at price \( v(t, \bar{a}) \)

Intuition: allocation of \( t \) distorted to extract rents from \( t \)
Step 1, General Proof Idea

In addition to types 2 and 3, include all other types in the "chain of information rents".
Step 1, General Proof Idea

\[ p(a_1) \]

\[ p(a_2) \]

market \( f \):

\[ v(\cdot, a_2) \]

\[ v(\cdot, a_1) \]

\[ 3 \cdot a_2 \]

\[ 2 \cdot a_1 \]

\[ 1 \cdot a_1 \]

In addition to types 2 and 3 include all other types in the "chain of information rents".

Final step: ensure mechanism is optimal for some market. 
Step 1, General Proof Idea

![Diagram showing a market with points labeled 1, 2, and 3, representing a general proof idea involving sets $p(a_1)$ and $p(a_2)$, and $v(\cdot, a_1)$ and $v(\cdot, a_2)$.]
Step 1, General Proof Idea

\[ \exists \, \text{market } f: \]

\[ v(\cdot, a_2) \]

\[ p(a_2) \]

\[ v(\cdot, a_1) \]

\[ p(a_1) \]

In addition to types 2 and 3 include all other types in the "chain of information rents".

Final step: ensure mechanism is optimal for some market.
Step 1, General Proof Idea

In addition to types 2 and 3 include all other types in the "chain of information rents".

Final step: ensure mechanism is optimal for some market $f$. 

$\exists ?$ market $f_1$: 

$\nu(\cdot, a_2)$ 

$\nu(\cdot, a_2)$ 

$\nu(\cdot, a_1)$ 

$\nu(\cdot, a_1)$ 

$p(a_2)$ 

$p(a_2)$ 

$p(a_1)$ 

$p(a_1)$
Step 1, General Proof Idea

In addition to types 2 and 3

- include all other types in the "chain of information rents"
Step 1, General Proof Idea

In addition to types 2 and 3

- include all other types in the “chain of information rents”

Final step: ensure mechanism is optimal for some market $f_1$
Recall Proof Outline

Given inefficient market $f$

1. $\exists f_1 \text{ s.t. } f_1 \succ_{CS} f \checkmark$
   - $\forall t \in \text{Supp}(f_1): CS(t, f') \geq CS(t, f), (\exists t, >)$

Define $f_2$ s.t. $f = \epsilon f_1 + (1 - \epsilon)f_2$

2. Small $\epsilon$, generic $f$: OptMech($f$) = OptMech($f_2$)
Step 2: Small Perturbation Preserves Optimal Mechanisms

\[ f = (1 - \epsilon)f_1 + \epsilon f_2 \]
Step 2: Small Perturbation Preserves Optimal Mechanisms

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**Lemma**

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Recall Proof Outline

Given inefficient market $f$

1. $\exists f_1 \text{ s.t. } f_1 \succ_{CS} f \checkmark$
   ▶ $\forall t \in \text{Supp}(f_1) : CS(t, f') \geq CS(t, f), (\exists t, >)$

Define $f_2$ s.t. $f = \epsilon f_1 + (1-\epsilon)f_2$

2. Small $\epsilon$, generic $f$: OptMech($f$) = OptMech($f_2$) $\checkmark$
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- (do not exist for some markets)
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Thanks!