Selling to a Group

Nima Haghpanah (Penn State)

with Aditya Kuvalekar (Essex) and Elliot Lipnowski (Columbia)

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What mechanism is optimal (maximizes seller's profit)?
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Model

- Agents 1, ..., n
- Outcomes \( \{(x, m) | x \in [0, 1], m \in \mathbb{R}\} \)
Model

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- Outcomes \{ (x, m) | x \in [0, 1], m \in \mathbb{R} \}
- Agent i’s payoff \( v_i x - m \)
  - \( v_i \) (independently) from \( F_i \) (regular)
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  - \( v_i \) (independently) from \( F_i \) (regular)
- Seller's payoff \( m - c x \)
Model

- Agents 1, . . . , \(n\)
- Outcomes \(\{(x, m) | x \in [0, 1], m \in \mathbb{R}\}\)
- Agent \(i\)'s payoff \(v_i x - m\)
  - \(v_i\) (independently) from \(F_i\) (regular)
- Seller's payoff \(m - cx\)

Mechanism: \((x, m) : V_1 \times \ldots \times V_N \rightarrow [0, 1] \times \mathbb{R}\)
- IIR: \(\mathbb{E}_{v_i} [v_i x(v_i, v_{-i}) - m(v_i, v_{-i})] \geq 0, \forall i, v_i\)
- BIC standard
seller ($c = 0$) → software → company

CEO

$v_1 \sim U[0, 2]$

$v_2 \sim U[0, 3]$

CTO

company

money

sell at price $p$ if both agree

sell iff $w_1 v_1 + w_2 v_2 \geq \delta_{\text{max}}$

$p \approx 3.5$ at $p \approx 0.78$

$w_1 = \sqrt{3/7}$, $w_2 = 1 - w_1$

$\delta = 1 + w_2$
seller \( (c = 0) \)

company

software

company money

CTO

\[ v_2 \sim U[0, 3] \]

\[ w_1 \quad v_1 + w_2 v_2 \geq \delta \max \]

\[ P[v_1 \geq p] \approx 0.35 \at p \approx 0.78 \]

\[ w_1 = \sqrt{3/7}, \quad w_2 = 1 - w_1, \quad \delta = 1 + w_2^2 \frac{4}{17} \]
seller \( (c = 0) \) 

software 

company money 

CEO 

CTO 

\[ v_1 \sim U[0, 2] \] 

\[ v_2 \sim U[0, 3] \] 

Sell at price \( 32 \) if CTO agrees 

Not IR: 

\[ v_1 = 0 \] then Utility of CEO = \(-\frac{3}{2} \cdot \frac{1}{2} \) 

Sell at price \( p \) if both agree 

Sell iff \( w_1 v_1 + w_2 v_2 \geq \delta \) 

\[ p \approx 0.35 \text{ at } p \approx 0.78 \] 

\[ w_1 = \sqrt{\frac{3}{7}}, \quad w_2 = 1 - w_1, \quad \delta = 1 + w_2^2 \] 

\[ \frac{17}{4} \]
Sell at price \( \frac{3}{2} \) if CTO agrees

\( \nu_1 \sim U[0, 2] \)
\( \nu_2 \sim U[0, 3] \)

\( w_1 = \sqrt{\frac{3}{7}} \), \( w_2 = 1 - w_1 \), \( \delta = 1 + w_2^2 \)
Not IR: $v_1 = 0$ then Utility of CEO = $-\frac{3}{2} \cdot \frac{1}{2}$
Sell at price $p$ if both agree

Sell iff $w_1 v_1 + w_2 v_2 \geq \delta_{\text{max}}$

$P[v_1 \geq p] \approx 0.35$ at $p \approx 0.78$

$w_1 = \sqrt{\frac{3}{7}}, w_2 = 1 - w_1, \delta = 1 + w_2^2 / 17$
\[
\max_p \ p \mathbb{P}[v_1 \geq p] \mathbb{P}[v_2 \geq p] \approx 0.35 \text{ at } p \approx 0.78
\]
Sell iff $w_1 v_1 + w_2 v_2 \geq \delta$

\[ w_1 = \sqrt{\frac{3}{7}}, \quad w_2 = 1 - w_1, \quad \delta = 1 + \frac{w_2}{2} \]
Theorem

The following mechanism is optimal:

1. Allocation \textit{maximizes weighted sum of virtual values}

2. Weights \textit{minimize weighted virtual surplus}

3. \textit{Transfer rule is “defined appropriately”}
Theorem

The following mechanism is optimal:

1. Allocation maximizes weighted sum of virtual values

   \[
   \text{Allocate} \iff \left( \sum_i w_i^* \phi_i(v_i) \right) - c \geq 0
   \]

   where \( \phi_i(v_i) = v_i - \frac{1 - F_i(v_i)}{f_i(v_i)} \)

2. Weights minimize weighted virtual surplus

3. Transfer rule is “defined appropriately”
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2. Weights minimize weighted virtual surplus

\[ w^* \in \arg \min_{w \in \Delta(\{1,\ldots,n\})} \mathbb{E} \left[ \max \left( \left( \sum_i w_i \phi_i(v_i) \right) - c, 0 \right) \right] \]

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   \text{Allocate} \iff \left( \sum_i w_i^* \phi_i(v_i) \right) - c \geq 0
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   \]

3. **Transfer rule is “defined appropriately”**

   so that payment identity is satisfied
Proof sketch

Implementability + Duality
Step 1: implementability

Our setting

Lemma

\[ \exists m \text{ s.t. } (x, m) \text{ is IC iff } \]

Among these mechanisms, 

\[ \text{opt revenue } = \]

With individual transfers

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\[ \text{Among these mechanisms, } \]

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Step 1: implementability

Our setting

Lemma

\( \exists m \text{ s.t. } (x, m) \text{ is IC iff } X_i \text{ is monotone } \forall i \)

Among these mechanisms,

\[
\text{opt revenue} = \min_i \mathbb{E}[X_i(v_i)\phi_i(v_i)]
\]
Step 1: implementability

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X_i is monotone ∀i

Among these mechanisms,

\[ \text{opt revenue} = \min_i \mathbb{E}[X_i(v_i)\phi_i(v_i)] \]

\[ M_i(v_i) = v_iX_i(v_i) - \int_{v_i} X_i(z)dz - U_i \]

With individual transfers

Lemma

∃m s.t. (x, m) is IC iff
X_i is monotone ∀i

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\[ \text{opt revenue} = \sum_i \mathbb{E}[X_i(v_i)\phi_i(v_i)] \]
Step 1: implementability

Our setting

Lemma

∃m that induces \( M_1, \ldots, M_n \) iff

\[ \exists m \text{ s.t. } (x, m) \text{ is IC iff } X_i \text{ is monotone } \forall i \]

Among these mechanisms,

\[ \text{opt revenue} = \min_i \mathbb{E}[X_i(v_i)\phi_i(v_i)] \]

\[ M_i(v_i) = v_iX_i(v_i) - \int_{v_i} X_i(z)dz - U_i \]

With individual transfers

Lemma

∃m s.t. \((x, m)\) is IC iff

\( X_i \) is monotone \( \forall i \)

Among these mechanisms,

\[ \text{opt revenue} = \sum_i \mathbb{E}[X_i(v_i)\phi_i(v_i)] \]
Step 1: implementability

Our setting

Lemma
\[ \exists m \text{ that induces } M_1, \ldots, M_n \iff \forall i, j \] \[ E_{v_i}[M_i(v_i)] = E_{v_j}[M_j(v_j)], \forall i, j \]

\[ \exists m \text{ s.t. } (x, m) \text{ is IC iff} \] \[ X_i \text{ is monotone } \forall i \]

Among these mechanisms,
\[ \text{opt revenue} = \min_i E[X_i(v_i)\phi_i(v_i)] \]

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Lemma
\[ \exists m \text{ s.t. } (x, m) \text{ is IC iff} \] \[ X_i \text{ is monotone } \forall i \]

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\[ \text{opt revenue} = \sum_i E[X_i(v_i)\phi_i(v_i)] \]

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Step 1: Implementability

Our setting

Lemma

\( \exists m \text{ that induces } M_1, \ldots, M_n \iff \)

\( E_{v_i}[M_i(v_i)] = E_{v_j}[M_j(v_j)], \forall i, j \)

\( \exists m \text{ s.t. } (x, m) \text{ is IC iff } \)

\( X_i \text{ is monotone } \forall i \)

Among these mechanisms,

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Lemma

\( \exists m \text{ s.t. } (x, m) \text{ is IC iff } \)

\( X_i \text{ is monotone } \forall i \)

Among these mechanisms,

\( \text{opt revenue} = \sum_i E[X_i(v_i)\phi_i(v_i)] \)

\[ M_i(v_i) = v_iX_i(v_i) - \int_{0}^{v_i} X_i(z)dz - U_i \]

\[ m(v) = \frac{\prod_i M_i(v_i)}{E_{v_i}[M_i(v_i)]^{n-1}} \]
Step 2: duality

\[
\max_{\text{allocation}} \min_{i} \mathbb{E}[X_i(v_i)\phi_i(v_i)]
\]
Step 2: duality

\[
\max_{\text{allocation}} \min_i E[X_i(v_i)\phi_i(v_i)]
\]

\[
\min_{w \in \Delta(N)} \max_{\text{allocation}} \sum_i w_i E[X_i(v_i)\phi_i(v_i)] = \min_{w \in \Delta(N)} \max_{\text{allocation}} \sum_i w_i E[X_i(v_i)\phi_i(v_i)] = \min_{w \in \Delta(N)} E[\max(\sum_i w_i \phi_i(v_i), 0)]
\]
Step 2: duality

\[ \max_{\text{allocation}} \min_{w \in \Delta(N)} \sum_i w_i \mathbb{E}[X_i(v_i)\phi_i(v_i)] = \min_{w \in \Delta(N)} \max_{\text{allocation}} \sum_i w_i \mathbb{E}[X_i(v_i)\phi_i(v_i)] = \mathbb{E}[\max(\sum_i w_i \phi_i(v_i), 0)] \]
Step 2: duality

\[
\begin{align*}
\max_{\text{allocation}} & \min_i \mathbb{E}[X_i(v_i)\phi_i(v_i)] \\
= & \max_{\text{allocation}} \min_w \sum_i w_i \mathbb{E}[X_i(v_i)\phi_i(v_i)] \\
= & \min_w \max_{\text{allocation}} \sum_i w_i \mathbb{E}[X_i(v_i)\phi_i(v_i)]
\end{align*}
\]
Step 2: duality

\[
\begin{align*}
\max_{\text{allocation}} \min_{i} & \quad \mathbb{E}[X_i(v_i)\phi_i(v_i)] \\
= \max_{\text{allocation}} \min_{w \in \Delta(N)} & \quad \sum_i w_i \mathbb{E}[X_i(v_i)\phi_i(v_i)] \\
= \min_{w \in \Delta(N)} \max_{\text{allocation}} & \quad \sum_i w_i \mathbb{E}[X_i(v_i)\phi_i(v_i)] \\
= \min_{w \in \Delta(N)} & \quad \mathbb{E}[\max(\sum_i w_i\phi_i(v_i), 0)]
\end{align*}
\]
Which agent has a higher weight?

Proposition

If $\phi_1$ is smaller than $\phi_2$ in the "hazard rate" order, then $w^*_1 \geq w^*_2$.

If $v_1$ and $\alpha v_2$ are identically distributed for some $\alpha \leq 1$, then $\alpha w^*_1 \geq w^*_2$. 

Sell at price 2 if CTO agrees

Sell at price 3 if CEO agrees

Revenue = 0

Revenue = $\frac{9}{17}$
Which agent has a higher weight? The “weaker” one

Proposition

If $\phi_1$ is smaller than $\phi_2$ in the “hazard rate” order, then $w^*_{\phi_1} \geq w^*_{\phi_2}$.

If $v_1$ and $\alpha v_2$ are identically distributed for some $\alpha \leq 1$, then $\alpha w^*_{v_1} \geq w^*_{v_2}$. 

Sell at price $3$ if CTO agrees, and pay $\frac{9}{17}$ to company. 

Revenue = $0$ if CEO agrees, and sell at price $1$. 

$9 / 17$
Which agent has a higher weight? The “weaker” one

Proposition

If $\phi_1$ is smaller than $\phi_2$ in the “hazard rate” order, then $w_1^* \geq w_2^*$. 
Which agent has a higher weight? The “weaker” one

Proposition

If $\phi_1$ is smaller than $\phi_2$ in the “hazard rate” order, then $w_1^* \geq w_2^*$. 

\[
\begin{align*}
\text{seller} & \quad \text{software} \quad \text{company} \\
\text{company} & \quad \text{money} \quad \text{CEO} \\
& \quad \text{CTO} \\
& \quad v_1 \sim U[0, 2] \\
& \quad v_2 \sim U[1, 3]
\end{align*}
\]
Which agent has a higher weight? The “weaker” one

Proposition

If $\phi_1$ is smaller than $\phi_2$ in the “hazard rate” order, then $w_1^* \geq w_2^*$. 

Sell at price $\frac{3}{2}$ if CTO agrees

CEO

CTO

Sell at price $\frac{3}{2}$ if CTO agrees

company

company money

software

$v_1 \sim U[0, 2]$

$v_2 \sim U[1, 3]$
Which agent has a higher weight? The “weaker” one

**Proposition**

If $\phi_1$ is smaller than $\phi_2$ in the “hazard rate” order, then $w_1^* \geq w_2^*$. 

- **CEO**
  - Sell at price $\frac{3}{2}$ if CTO agrees
  - Pay $\frac{9}{8}$ to company

- **CTO**
  - $v_1 \sim U[0, 2]$
  - $v_2 \sim U[1, 3]$

- **Software**

- **Company**
  - Revenue = 0
  - Sell at price $\frac{1}{2}$ if CEO agrees
  - Revenue = $\frac{9}{17}$
Which agent has a higher weight? The “weaker” one

**Proposition**

If $\phi_1$ is smaller than $\phi_2$ in the “hazard rate” order, then $w_1^* \geq w_2^*$. 

Diagram:

- **Seller**
  - Sell at price $\frac{3}{2}$ if CTO agrees
  - Pay $\frac{9}{8}$ to company
- **CEO**
  - Revenue = 0
- **CTO**
  - $v_1 \sim U[0, 2]$
  - $v_2 \sim U[1, 3]$
- **Company**
  - Money
Which agent has a higher weight? The “weaker” one

Proposition

If $\phi_1$ is smaller than $\phi_2$ in the “hazard rate” order, then $w_1^* \geq w_2^*$. 

![Diagram showing seller, software, company, CEO, CTO, and money distributions with marginal distributions $v_1 \sim U[0, 2]$ and $v_2 \sim U[1, 3]$.]
Which agent has a higher weight? The “weaker” one

Proposition

If \( \phi_1 \) is smaller than \( \phi_2 \) in the “hazard rate” order, then \( w_1^* \geq w_2^* \).

Sell at price 1 if CEO agrees

Revenue = \( \frac{1}{2} \)

Company

CEO

CTO

\( v_1 \sim U[0, 2] \)

\( v_2 \sim U[1, 3] \)
Which agent has a higher weight? The “weaker” one

**Proposition**

*If* $\phi_1$ *is smaller than* $\phi_2$ *in the “hazard rate” order, then* $w_1^* \geq w_2^*$. *

If $v_1$ and $\alpha v_2$ are identically distributed for some $\alpha \leq 1$, then $\alpha w_1^* \geq w_2^*$. *

---

**Diagram:**

- **Seller:** Sell at price 1 if CEO agrees.
- **CEO:**
  - $v_1 \sim U[0, 2]$
- **CTO:**
  - $v_2 \sim U[1, 3]$
- **Software Revenue:** $\text{Revenue} = \frac{1}{2}$

**Company:**

- Company money

---
Which agent has a higher weight? The “weaker” one

Proposition

If $\phi_1$ is smaller than $\phi_2$ in the “hazard rate” order, then $w_1^* \geq w_2^*$. 
Which agent has a higher weight? The “weaker” one

**Proposition**

If \( \phi_1 \) is smaller than \( \phi_2 \) in the “hazard rate” order, then \( w_1^* \geq w_2^* \).

1. Suppose \( w_1 < w_2 \) are optimal
2. \( w_1 \phi_1(v_1) + w_2 \phi_2(v_2) \) larger than \( w_2 \phi_1(v_1) + w_1 \phi_2(v_2) \) in hazard rate order
3. \( \mathbb{E}[\max(w_1 \phi_1(v_1) + w_2 \phi_2(v_2), 0)] \geq \mathbb{E}[\max(w_2 \phi_1(v_1) + w_1 \phi_2(v_2), 0)] \)
4. Contradicts uniqueness of optimal weights
Extensions

1. Pareto optimality
2. Fixed-share payment rules
3. Beyond veto bargaining
4. Sub-optimality of posted pricing
1. Pareto Optimality

Theorem

A mechanism is Pareto optimal iff

\[ \exists \gamma \in [0, 1], \lambda, w^* \in \Delta(\{1, \ldots, n\}) \text{ s.t.} \]

\[ \text{Allocation } x^* \text{ maximizes weighted sum of values & virtual values} \]

\[ \text{Allocate } \Leftrightarrow (1 - \gamma)(\sum_i w_i^* \phi_i(v_i)) + \gamma(\sum_i \lambda_i v_i) \geq c \]

where

\[ \phi_i(v_i) = v_i - 1 - F_i(v_i) f_i(v_i) \]

2. Weights

\[ w^* \in \arg \min_{w \in \Delta(\{1, \ldots, n\})} E[(\sum_i w_i \phi_i(v_i) - c) x^*(v_i)] \]

3. Transfer rule is “defined appropriately” so that payment identity is satisfied.
1. Pareto Optimality

**Theorem**

A mechanism is Pareto optimal iff \( \exists \gamma \in [0, 1], \lambda, w^* \in \Delta(\{1, \ldots, n\}) \) s.t.

1. **Allocation** \( x^* \) maximizes weighted sum of values & virtual values

2. **Weights** \( w^* \) minimize weighted virtual surplus

3. **Transfer rule** is “defined appropriately”
1. Pareto Optimality

**Theorem**

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   \[
   \text{Allocate} \iff (1 - \gamma) \left( \sum_i w_i^* \phi_i(v_i) \right) + \gamma \left( \sum_i \lambda_i v_i \right) \geq c
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   where \( \phi_i(v_i) = v_i - \frac{1-F_i(v_i)}{f_i(v_i)} \)

2. **Weights** \( w^* \) **minimize weighted virtual surplus**

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1. Pareto Optimality

**Theorem**

A mechanism is Pareto optimal iff $\exists \gamma \in [0, 1], \lambda, w^* \in \Delta(\{1, \ldots, n\})$ s.t.

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   Allocate $\Leftrightarrow (1 - \gamma) \left( \sum_i w_i^* \phi_i(v_i) \right) + \gamma \left( \sum_i \lambda_i v_i \right) \geq c$

   where $\phi_i(v_i) = v_i - \frac{1 - F_i(v_i)}{f_i(v_i)}$

2. **Weights** $w^*$ minimize weighted virtual surplus

   $w^* \in \arg \min_{w \in \Delta(\{1, \ldots, n\})} \mathbb{E} \left[ \left( \sum_i w_i \phi_i(v_i) - c \right) x^*(v) \right]$

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1. Pareto Optimality

Theorem

A mechanism is Pareto optimal iff $\exists \gamma \in [0, 1], \lambda, w^* \in \Delta(\{1, \ldots, n\})$ s.t.

1. Allocation $x^*$ maximizes weighted sum of values & virtual values

$$\text{Allocate} \iff (1 - \gamma) \left( \sum_i w_i^* \phi_i(v_i) \right) + \gamma \left( \sum_i \lambda_i v_i \right) \geq c$$

where $\phi_i(v_i) = v_i - \frac{1 - F_i(v_i)}{f_i(v_i)}$

2. Weights $w^*$ minimize weighted virtual surplus

$$w^* \in \arg \min_{w \in \Delta(\{1, \ldots, n\})} \mathbb{E} \left[ \left( \sum_i w_i \phi_i(v_i) - c \right) x^*(v) \right]$$

3. Transfer rule is “defined appropriately”

so that payment identity is satisfied
2. Fixed-share payment rules

What if agents pay out of pocket, but in fixed shares?
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What if agents pay out of pocket, but in fixed shares?

- Agent $i$ pays $\sigma_i m$, for fixed $\sigma_1, \ldots, \sigma_n$
- Payoff $v_i x - \sigma_i m$
- Relabel $\theta_i = v_i / \sigma_i$
- Payoff $\sigma_i (\theta_i x - m)$
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E.g., if $v_i$ i.i.d and $\sigma_1 \leq \ldots \leq \sigma_n$
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E.g., if $v_i$ i.i.d and $\sigma_1 \leq \ldots \leq \sigma_n$

- Then $w_1^* \leq \ldots \leq w_n^*$
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- Payoff $v_i x - \sigma_i m$
- Relabel $\theta_i = v_i / \sigma_i$
- Payoff $\sigma_i (\theta_i x - m)$

E.g., if $v_i$ i.i.d and $\sigma_1 \leq \ldots \leq \sigma_n$

- Then $w_1^* \leq \ldots \leq w_n^*$
- Pay more attention to agents who are “more on the hook”
3. Beyond Veto Bargaining

What if mechanism needs to be approved by $k < n$ agents?
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What if mechanism needs to be approved by $k < n$ agents?

1. A set of $k < n$ agents fixed a priori
3. Beyond Veto Bargaining

What if mechanism needs to be approved by $k < n$ agents?

1. A set of $k < n$ agents fixed a priori
   - Then solve problem for those $k$

Mechanism announced. Agents vote. Mechanism approved if $k < n$ agents vote "yes"

Mechanism: $x = 0, m > 0$. Silly equilibrium: all vote yes

Pay $m + 1$ if not unanimously approved. Then "yes" undominated.

Worse-case equilibria?

Mechanism announced. Agents vote to "mediator". Mechanism approved if $k < n$ agents vote "yes". Seller only learns approve/reject.

When approved, agents update belief. No longer independent.
3. Beyond Veto Bargaining

What if mechanism needs to be approved by \( k < n \) agents?

1. A set of \( k < n \) agents fixed a priori
   - Then solve problem for those \( k \)

2. Mechanism announced. Agents vote. Mechanism approved if \( k < n \) agents vote “yes”
3. Beyond Veto Bargaining

What if mechanism needs to be approved by $k < n$ agents?

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   - Mechanism: $x = 0, m >> 0$. Silly equilibrium: all vote yes
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   - When approved, agents update belief. No longer independent.
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Posted pricing is not even \textit{approximately optimal}.
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What does posted pricing mean?

- A mechanism is posted pricing if \( m = px \) for some fixed \( p \in \mathbb{R} \)
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Then, exists an instance with i.i.d values where

- As number of agents $\to \infty$, $\frac{\text{posted pricing revenue}}{\text{optimal revenue}} \to 0$
Related Literature

Mechanisms for public goods
▶ e.g., d’Aspremont Gérard-Varet 1979, Güth Hellwig 1986

Voting mechanisms without transfers
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Mechanisms for public goods
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Voting mechanisms without transfers
  e.g., Rae 1969, Azrieli Kim 2014

Reduced-form implementation
  e.g., Matthews 1984, Border 1991

BIC-DIC equivalence
  e.g., Manelli Vincent 2010, Gershkov et al 2013
Single seller, single product, “single” buyer

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- Pay more attention to “weaker” agents
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The End!