Designing and Pricing Certificates

Nima Haghpanah

joint with Nageeb Ali, Xiao Lin, Ron Siegel

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Certification

Labor markets, Financial markets, Products

What certificates would an agent acquire and disclose?

How would a profit-maximizing certifier design and price certificates?

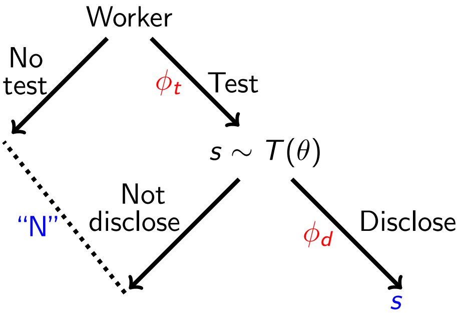
A worker, a certifier, a competitive labor market

Ability $\theta \sim U\{0,1\}$

- <u>unknown to all</u>

A test-fee structure (T, ϕ):

1. Test $T : \{0,1\} \to \Delta(S)$
   WLOG $E[\theta|s] = s$
2. Testing fee ϕ_t
   Disclosure fee ϕ_d



Market observes s or "N"
Market offers wage $= E[\theta]$

Profit-maximizing test-fee structures?

$$\sup_{\text{test-fee structure}} \ \sup_{\text{equilibria}} \ \text{Profit} = \text{Full surplus } E[\theta] = 0.5$$

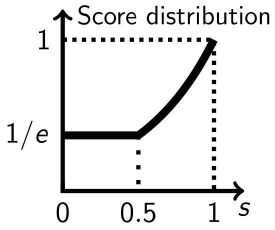
Fully reveal, $\phi_t = 0.5, \phi_d = 0$

- Another equilibrium: worker doesn't take test. Profit $= 0$

$$\sup_{\text{test-fee structure}} \ \inf_{\text{equilibria}} \ \text{Profit} = 0.5 \cdot (1 - 1/e) \approx 0.31$$

"Robustly optimal" test-fee structure:

1. Is unique
2. Zero testing fee
3. Not fully revealing: continuum of scores



Related Work

Profit-maximizing certification:

- Lizzeri (1999). Informed worker, mandatory disclosure:
  - Signaling vs. voluntary disclosure
- DeMarzo, Kremer, Skrzypacz (2019). "favorable" selection

Adversarial equilibrium selection in information/mechanism design:

- Dworczak and Pavan (2020), Halac, Kremer, Winter (2020), Halac, Lipnowski, Rappoport (2020), ...

Information design and unit-elastic distributions:

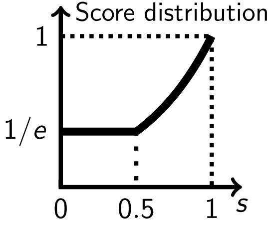
- Roesler and Szentes (2017), Ortner and Chassang (2018), Condorelli and Szentes (2020), ...
  - Indifference condition vs. worst-equilibrium condition

Next

Identify optimal test with $\phi_t = 0$ and $\phi_d = 0.5$

$$\sup_{\text{test}} \; \inf_{\text{equilibria}} \; \text{Probability of disclosure}$$

Exponential distribution maximizes $\inf_{\text{equilibria}}$ Probability of disclosure



Disclosure stage: threshold structure

Equilibrium threshold τ:

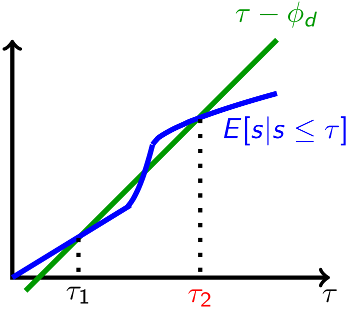
$$\tau - \phi_d = w_N = E[s|s \leq \tau]$$

Worst equilibrium τ is largest intersection:

$$\tau' - \phi_d \neq E[s|s \leq \tau'], \forall \tau' > \tau$$

Claim: Robustly optimal test-fee structure,

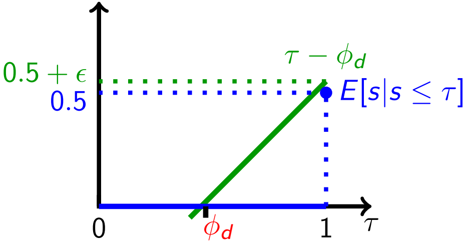
- Worker participates with probability 1 in all equilibria



Fully revealing test

Worst equilibrium threshold $= \phi_d$

- Probability of disclosure $= 0.5$



Ability		Score s	Prob
0	→	0	$\frac{1}{2}$
1	→	1	$\frac{1}{2}$

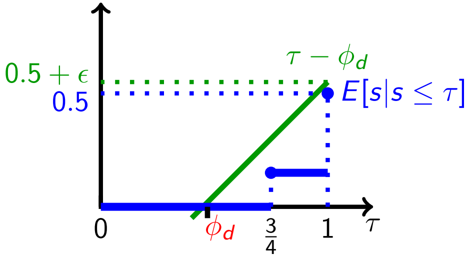
testing fee $= 0$

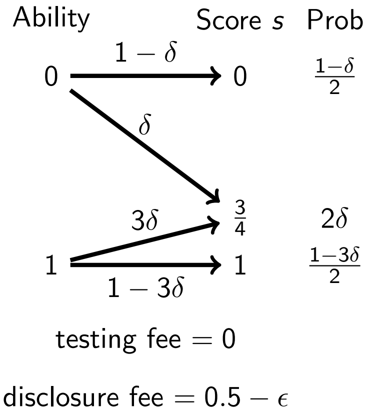
disclosure fee $= 0.5 - \epsilon$

Improvement by a noisy test

Worst equilibrium threshold $= \phi_d$

- Probability of disclosure > 0.5





“Robustly optimal” test subject to $\phi_t = 0$, $\phi_d \simeq 0.5$

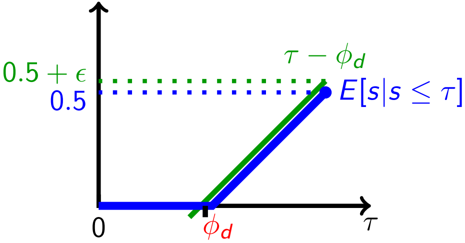
Worst equilibrium threshold $= \phi_d$

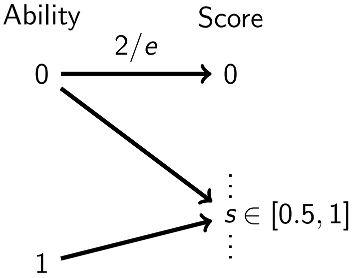
- Probability of disclosure $1 - 1/e \approx 0.63$

$$\phi_d = \frac{\int_0^\tau G(s)ds}{G(\tau)}$$

$$= \left(\frac{d}{d\tau} \left(\ln(\int_0^\tau G(s)ds) \right) \right)^{-1}$$

$$\Rightarrow G(\tau) = \frac{c}{\phi_d} e^{\tau/\phi_d}$$





testing fee $= 0$

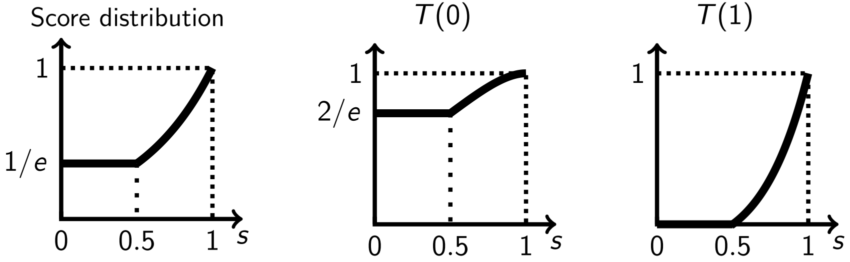
disclosure fee $= 0.5 - \epsilon$

Robustly optimal test-fee structure

Proposition

There is a unique robustly optimal test-fee structure. It consists of testing fee $\phi_t^* = 0$, disclosure fee $\phi_d^* = 0.5$, and test T below.

Continuum of scores even though abilities are binary.



Arbitrary prior over $\theta \in [0, 1]$ with mean μ

Proposition

Robustly optimal profit $\leq (1-\mu)(1-e^{\frac{-\mu}{1-\mu}}) < \mu$.

Proposition

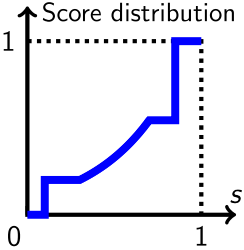
There exists a robustly optimal test-fee structure with a "step-exponential-step" score distribution.

Disclosure fee > 0

- Contrast with "maximize value and extract via testing fee" intuition.

Testing fee?

- Positive for log-concave priors
- May be zero (e.g., for binary prior)



Precluding no-testing equilibria

$$\mu < \underbrace{\int_0^1 \max\{\mu, s - \phi_d\}\, dG}_{\text{Option Value}} - \phi_t,$$

Rearranging:

$$\phi_t < \int_{\mu+\phi_d}^1 [s - (\mu + \phi_d)] dG, \tag{P}$$

Lemma

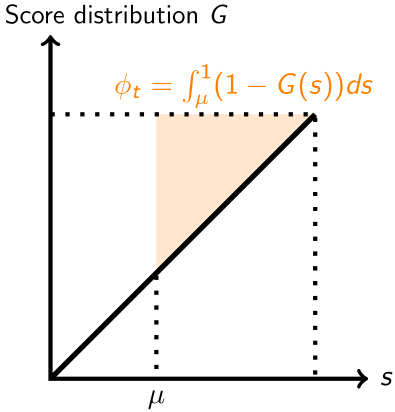
1. *If (P), $\forall$ equilibria: worker takes test with probability 1*
2. *If !(P), $\exists$ equilibrium: worker takes test with probability 0*

Proves earlier claim: Robustly optimal test-fee structure,

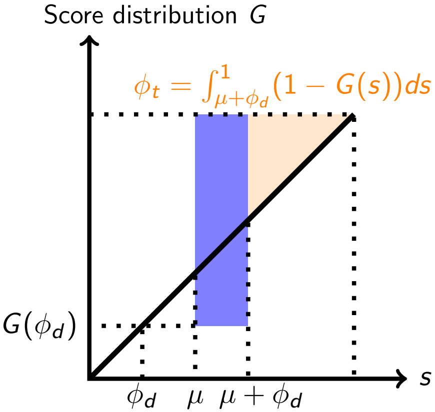
- Worker participates with probability 1 in all equilibria

Optimality of positive disclosure fee

profit $= \phi_t$

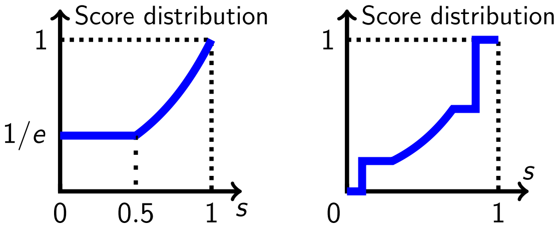


profit $= \phi_t + \phi_d(1 - G(\phi_d))$



Extensions

1. Small amount of private information
   - Full surplus extraction remains impossible
   - Step-exponential-step distributions are approximately optimal
2. Technological constraints: Certifier has a set of feasible tests
   - Assumption: feasible to garble a feasible test
   - Step-exponential-step is optimal
3. Score-dependent disclosure fees
   - Allows for slightly higher profit, still not full surplus



Thanks!