How to Sell Hard Information
(Old title: Designing and Selling Hard Information)

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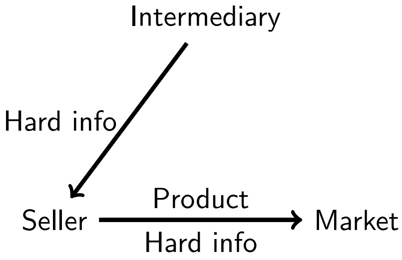
A market for hard information

Why do these intermediaries exist?

- They often increase efficiency
- This paper: intermediary makes considerable profit
  - even though there is no efficiency gain
  - even in the worst equilibrium
  - Intermediary reduces payoff of both seller and market

Study intermediary's profit-maximization problem



First a model with no intermediary

Value (to seller) $= 0$
Value (to market) $\theta \in \{0, 1\}$, prob. 0.5 each

- Unknown to all

Competitive market plays Bertrand game

- price $= E[\theta] = 0.5$
- Efficient allocation

	Seller	Market
Payoff	0.5	0

Seller $\xrightarrow{\text{Product}}$ Market

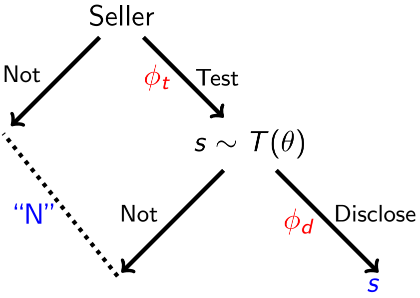
Now a model with an intermediary

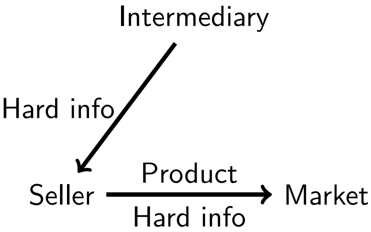
Test-fee structure (T, ϕ):

1. Test $T : \{0, 1\} \to \Delta(S)$
2. Testing fee ϕ_t
   Disclosure fee ϕ_d

Market observes s or "N"

- price $= E[\theta|\text{observation}]$





Revenue-maximizing test-fee structures?

$$\max_{\text{test-fee structure}} \quad \max_{\text{equilibria}} \quad \text{Revenue} = 0.5 = E[\theta] \text{ “full surplus”}$$

Fully reveal, $\phi_t = 0.5, \phi_d = 0$

- Equilibrium with revenue $= 0.5$
  - Seller: take test, reveal if and only if score $= 1$
  - Market: price(0) $= 0$, price(1) $= 1$, price(N) $= 0$
- Another equilibrium with revenue $= 0$
  - Seller: don’t take test
  - Market: price(0) $= 0$, price(1) $= 1$, price(N) $= 0.5$

Revenue-maximizing test-fee structures?

$$\max_{\text{test-fee structure}} \min_{\text{equilibria}} \text{Revenue} = 0.5 \cdot (1 - 1/e) \approx 0.31$$

With intermediary, in worst equilibrium

	Seller	Market	Intermediary
Payoff	0.29	0	0.31

Without intermediary: allocation efficient and payoffs are

	Seller	Market
Payoff	0.5	0

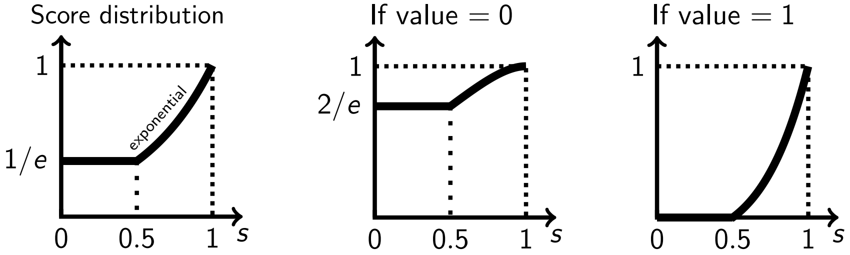
Intermediary makes considerable profit

- Even though there is no efficiency gain
- Even in the worst equilibrium

Optimal test-fee structure

Proposition

There is a unique robustly optimal test-fee structure. It consists of testing fee $\phi_t^* = 0$, *disclosure fee* $\phi_d^* = 0.5$, *and test* T *below.*



Why is testing free?

Example: Fully reveal, $\phi_t = 0, \phi_d < 0.5$

- Unique equilibrium with revenue $= \frac{\phi_d}{2}$
  - Seller: take test, reveal if and only if score $= 1$
  - Market: price(0) $= 0$, price(1) $= 1$, price(N) $= 0$
- Why is not-taking-test not an equilibrium?
  - Market: price(0) $= 0$, price(1) $= 1$, price(N) $= 0.5$
    - Seller's payoff $= 0.5$
  - Deviation: take test, reveal if score $= 1$
    - Seller's payoff $= \frac{1}{2} \times 0.5 + \frac{1}{2} \times (1 - \phi_d) > 0.5$

Seller takes test because of option value

Why exponential?

A demand-theoretic justification

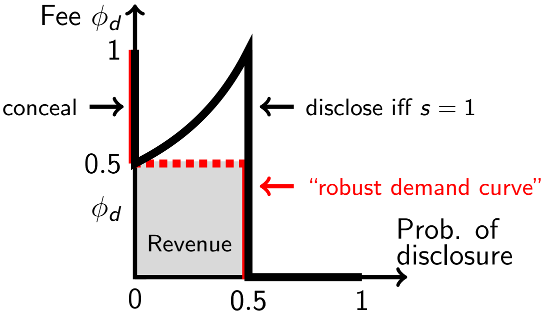
1. Start with a fully revealing test
2. See how adding small noise increases revenue
3. Taken to extreme: exponential distribution

Fully reveal, free testing ($\phi_t = 0$)

(Robustly) optimal disclosure fee ϕ_d?

- $\phi_d \lesssim 0.5$. Revenue $\lesssim 0.25$
- $\phi_d \geq 0.5$: conceal any s
- $\phi_d \in [0, 1]$: disclose iff $s = 1$

$Pr[s\|\theta]$	$s = 0$	$s = 1$
$\theta = 0$	1	0
$\theta = 1$	0	1



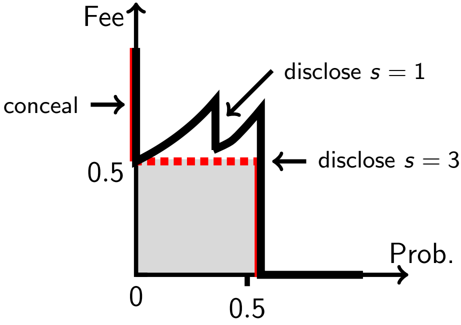
Benefit of noise (free testing $\phi_t = 0$)

- $\phi_d \lesssim 0.5$: Revenue > 0.25

Increase p subject to

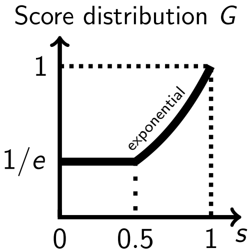
$$\underbrace{\frac{3}{4} - \phi_d}_{\text{Disclose medium score}} > \underbrace{\frac{3p}{1+3p}}_{\text{Nondisclosure price}}$$

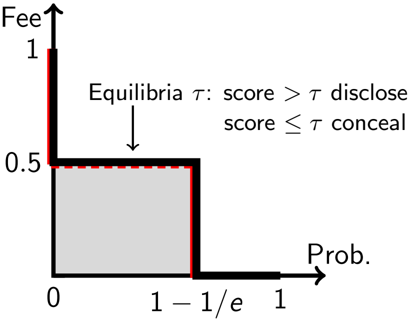
$Pr[s\|\theta]$	$s=0$	$s=3/4$	$s=1$
$\theta=0$	$1-p$	p	0
$\theta=1$	0	$3p$	$1-3p$



Robustly Optimal Test ($\phi_d \lesssim 0.5$ & free testing $\phi_t = 0$)

$$\phi_d = \tau - E[\text{score} \mid \text{score} \le \tau]$$
$$= \frac{\int_0^\tau G(s)ds}{G(\tau)}$$
$$= \left(\frac{d}{d\tau} \left(\ln(\int_0^\tau G(s)ds) \right) \right)^{-1}$$
$$\Rightarrow G(\tau) = \alpha e^{\tau/\phi_d}$$





Conclusions

Arbitrary prior over $\theta \in [0, 1]$

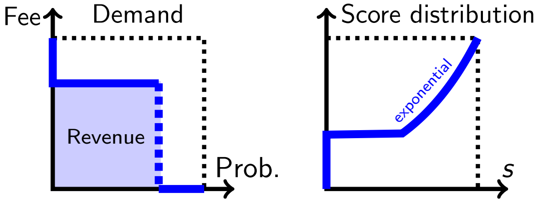
1. $0 <$ intermediary's revenue $<$ full surplus
2. "generalized exponential" distribution, disclosure fee > 0

Intermediary makes considerable profit

- even though there is no efficiency gain
- even in the worst equilibrium

How?

- High option value: low testing fee
- Exponential test to maximize disclosure probability



Thanks!