How to Sell Hard Information
(Old title: Designing and Selling Hard Information)

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A market for hard information

Why do these intermediaries exist?

▶ They often increase efficiency
▶ This paper: intermediary makes considerable profit
  ▶ even though there is no efficiency gain
  ▶ even in the worst equilibrium
  ▶ Intermediary reduces payoff of both seller and market

Study intermediary’s profit-maximization problem
First a model with no intermediary

Value (to seller) = 0
Value (to market) $\theta \in \{0, 1\}$, prob. 0.5 each
  - Unknown to all

Competitive market plays Bertrand game
  - price = $E[\theta] = 0.5$
  - Efficient allocation

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<thead>
<tr>
<th></th>
<th>Seller</th>
<th>Market</th>
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<tbody>
<tr>
<td>Payoff</td>
<td>0.5</td>
<td>0</td>
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</tbody>
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Seller \[ \rightarrow \] Product \[ \rightarrow \] Market
Now a model with an intermediary

Test-fee structure \((T, \phi)\):

1. Test \(T : \{0, 1\} \rightarrow \Delta(S)\)
2. Testing fee \(\phi_t\)
   Disclosure fee \(\phi_d\)

Market observes \(s\) or “N”

- price = \(E[\theta | \text{observation}]\)
Revenue-maximizing test-fee structures?

\[
\begin{align*}
\max_{\text{test-fee structure}} \quad \max_{\text{equilibria}} \quad \text{Revenue} = 0.5 = E[\theta] \quad \text{“full surplus”}
\end{align*}
\]

Fully reveal, \( \phi_t = 0.5, \phi_d = 0 \)

- Equilibrium with revenue = 0.5
  - **Seller**: take test, reveal if and only if score = 1
  - **Market**: price(0) = 0, price(1) = 1, price(N) = 0

- Another equilibrium with revenue = 0
  - **Seller**: don’t take test
  - **Market**: price(0) = 0, price(1) = 1, price(N) = 0.5
Revenue-maximizing test-fee structures?

\[
\begin{array}{c}
\text{max} \\
\text{test-fee structure}
\end{array}
\quad \begin{array}{c}
\text{min} \\
\text{equilibria}
\end{array}
\quad \text{Revenue} = 0.5 \cdot (1 - 1/e) \approx 0.31
\]

With intermediary, in worst equilibrium

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<th>Seller</th>
<th>Market</th>
<th>Intermediary</th>
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<tbody>
<tr>
<td>Payoff</td>
<td>0.29</td>
<td>0</td>
<td>0.31</td>
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Without intermediary: allocation **efficient** and payoffs are

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Intermediary makes considerable profit

- Even though there is **no efficiency gain**
- Even in the **worst equilibrium**
Optimal test-fee structure

Proposition

There is a unique robustly optimal test-fee structure. It consists of testing fee $\phi^*_t = 0$, disclosure fee $\phi^*_d = 0.5$, and test $T$ below.
Why is testing free?

**Example:** Fully reveal, $\phi_t = 0, \phi_d < 0.5$

- Unique equilibrium with revenue $= \frac{\phi_d}{2}$
  - **Seller:** take test, reveal if and only if score $= 1$
  - **Market:** price(0) = 0, price(1) = 1, price(N) = 0

- Why is not-taking-test not an equilibrium?
  - **Market:** price(0) = 0, price(1) = 1, price(N) = 0.5
    - Seller's payoff $= 0.5$
    - Deviation: take test, reveal if score $= 1$
      - Seller's payoff $= \frac{1}{2} \times 0.5 + \frac{1}{2} \times (1 - \phi_d) > 0.5$

Seller takes test because of **option value**
Why exponential?

A demand-theoretic justification

1. Start with a fully revealing test
2. See how adding small noise increases revenue
3. Taken to extreme: exponential distribution
Fully reveal, free testing ($\phi_t = 0$)

(Robustly) optimal disclosure fee $\phi_d$?

- $\phi_d \lesssim 0.5$. Revenue $\lesssim 0.25$
- $\phi_d \geq 0.5$: conceal any $s$
- $\phi_d \in [0, 1]$: disclose iff $s = 1$

| $Pr[s|\theta]$ | $s = 0$ | $s = 1$ |
|-----------------|--------|--------|
| $\theta = 0$    | 1      | 0      |
| $\theta = 1$    | 0      | 1      |

Revenue - Probability of disclosure

Fee $\phi_d$

$0 \quad 0.5 \quad 1$

conceal

$0.5$

“robust demand curve”

disclose iff $s = 1$
Benefit of noise (free testing $\phi_t = 0$)

- $\phi_d \ll 0.5$ : Revenue $> 0.25$

Increase $p$ subject to

$$\frac{3}{4} - \phi_d > \frac{3p}{1 + 3p}$$

Disclose medium score | Nondisclosure price
---|---
$\theta = 0$ | $1 - p$ | $p$ | $0$
$\theta = 1$ | $0$ | $3p$ | $1 - 3p$

| $Pr[s|\theta]$ | $s = 0$ | $s = 3/4$ | $s = 1$ |
|---|---|---|---|
| $\theta = 0$ | $1 - p$ | $p$ | $0$
| $\theta = 1$ | $0$ | $3p$ | $1 - 3p$
Robustly Optimal Test ($\phi_d \lesssim 0.5$ & free testing $\phi_t = 0$)

$$\phi_d = \tau - E[\text{score} \mid \text{score} \leq \tau]$$

$$= \frac{\int_0^\tau G(s)ds}{G(\tau)}$$

$$= \left(\frac{d}{d\tau} \left( \ln(\int_0^\tau G(s)ds) \right) \right)^{-1}$$

$$\Rightarrow G(\tau) = \alpha e^{\tau/\phi_d}$$

Score distribution $G$

Equilibria $\tau$: score $> \tau$ disclose, score $\leq \tau$ conceal
Conclusions

Arbitrary prior over $\theta \in [0, 1]$
1. $0 < \text{intermediary’s revenue} < \text{full surplus}$
2. “generalized exponential” distribution, disclosure fee $> 0$

Intermediary makes considerable profit
- even though there is no efficiency gain
- even in the worst equilibrium

How?
- High option value: low testing fee
- Exponential test to maximize disclosure probability

Thanks!