Physical Optics. **Diffraction.**

- **Huygens-Fresnel principle (recap)**
- **Single slit diffraction**
  - “Lazy physicist's” Method
  - Intensity distribution
- **Circular aperture diffraction**
- **Double slit diffraction**
  - “Lazy physicist's” Method II
  - Intensity distribution
- **Diffraction gratings**
**Diffraction. Limitations of Geometrical Optics (GO)**

- **Large hole** bright spot
  - Can treat beam as PW
  - GO works!

- **Smaller hole**
  - CAN’T treat beam as PW
  - if $a$ comparable with $\lambda$

- **Point-like hole**
  - Point-like slit will emit spherical wave

*Can predict based on Huygens-Fresnel principle!*
Diffraction = class of wave phenomena such as spreading and bending of waves passing through an aperture or by an object.

will estimate width of fringes, then compute intensity distribution on screen.
Lazy physicist’s method. Units analysis

All physical quantities have units.
Any equation must have same units on right- and left-hand sides.
Quite often it is possible to deduce a relation between physical quantities merely by considering their units – units analysis.

Strategy:
1. Identify all the quantities that should be related

2. Construct a dimensionless combination* of the quantities.
   Set it equal to a constant.

3. Solve for the quantity of interest

*may not be unique
Lazy physicist’s method. Units analysis

1) Pendulum

1) \(m, l, g, \omega\)
2) \(m^0 l^1 g^{-1} \omega^2 = \text{const}\)
3) \(\omega \sim \sqrt{g/l}\)

precise answer

2) Falling ball

1) \(m, h, g, v\)
2) \(m^0 h^{-1} g^{-1} v^2 = \text{const}\)
3) \(v \sim \sqrt{gh}\)

wrong by a factor of \(\sqrt{2}\)

Can find the answer only up to a numerical factor.

3) How about mean free path (\(m\)), concentration (\(m^{-3}\)) and molecule size (\(m\))?
1. Relevant quantities:
   - \( a \) = slit width (m)
   - \( \lambda \) = wavelength (m)
   - \( L \) = distance (m)
   - \( A \) = fringe width (m)

2. The relation should have the form:
   \[ \frac{XY}{ZW} = \text{const} \quad \text{or} \quad XY \sim ZW \]

3. Reciprocity principle implies that \( a \) and \( A \) should be on one side of the equation, i.e.
   \[ aA = \lambda L \quad \text{or} \quad A = \frac{\lambda L}{a} \]
   
   *compare with the Young double slit experiment!*
Single slit diffraction. Intensity.

Applying Huygens-Fresnel principle:
1. Break wavefront into small wavelets
2. Each wavelet acts as a point source with the same frequency, wave number, amplitude and different phase @P due to different pathlength
3. Interference of the secondary waves results in the diffraction pattern with the intensity:

\[ I = I_0 \left( \frac{\sin \alpha}{\alpha} \right)^2 \]

where \( \alpha = \frac{\pi}{\lambda} a \sin \theta \)

minima correspond to \( \sin \alpha = 0 \), i.e. \( \alpha = m\pi \)

width of central fringe:

\[ A = 2y_1 = \frac{L\lambda}{a} \]
A 0.10-mm-wide slit is illuminated by light with $\lambda=589\text{nm}$. Consider a point $P$ on a screen at $30^\circ$ from the central axis of the slit. Find the phase difference between the Huygens wavelets arriving at $P$ from the top and midpoint of the slit.
(for next Monday’s recitation)

1. Take all the necessary measurements (after the lecture) and estimate the width of the single slit used.

2. Try to observe a (single slit) diffraction pattern using two razors. The more fringes you can observe the better.
Circular aperture diffraction. Resolvability.

What pattern do we expect?

Light coming through a circular aperture, like a circular lens in a telescope or spy satellite camera, is not simply focused at a single point (the focal point) as geometrical ray optics led us to believe.

Location of the first minimum: $\sin \theta \sim \lambda / d$. More precisely: $\sin \theta = \frac{1.22 \lambda}{d}$.

In fact, this diffraction effect is an important limiting factor in the resolution of distant point objects located close together.

If their diffraction patterns overlap too much (fig. a) then we cannot separate them.

If their diffraction patterns are well separated (fig. c) we can

Rayleigh criterion: we can resolve (distinguish) two objects if their angular separation is greater than $\theta_R = \sin^{-1}(1.22 \lambda / d) \sim 1.22 \lambda / d$.

$(d = \text{size of telescope})$
Circular aperture. Sample problem.

Small step for the Nittany Lion...

On the Moon surface, would you be able to tell State College and Philadelphia apart with the naked eye? State College and Altoona (40 mi)?

Would you be able to see the Beaver Stadium?

If not, a telescope of what size would you need?

(Neglect dispersion in the atmosphere.)
Diffraction & Uncertainty Principle.

**Plane wave**
- Fixed direction
- Infinitely large in space

**Point-like hole**
- Fixed location
- Infinitely many directions

Irrespective of the shape of the hole in the screen:
- More localized position implies larger spread in direction (and vice versa)
- Object size and image size inversely proportional to each other
  
  (will revisit when studying Quantum Mechanics)
Double slit diffraction. **Pattern?**

**Figures:**
- a) $a<\lambda$, double slit
- b) $a>\lambda$, single slit
- c) $a>\lambda$, double slit

Multiply $\theta$ by $L$ to get geometrical scales on the screen:
- $A=$envelope width, $D=$fringe width
Double slit diffraction. Lazy physicist’s method II.

Scales.

Both the object and the image may have several characteristic scales.

The Uncertainty Principle implies:

- smallest object scale is responsible for largest image scale: \( A = \lambda L / a \)
- largest object scale is responsible for smallest image scale: \( D = \lambda L / d \)

Intensity.

\[
I(\theta) = I_m (\cos \beta)^2 (\sin \alpha / \alpha)^2
\]

\( \alpha = (\pi a / \lambda) \sin \theta = \text{single slit diffraction factor} \)

\( \beta = (\pi d / \lambda) \sin \theta = \text{double slit interference factor} \)

Limiting cases:

\( a \rightarrow 0 \) (Young’s double slit) \( \alpha \rightarrow 0 \), \( \Rightarrow \) \( I \rightarrow I_m (\cos \beta)^2 \)

\( d \rightarrow 0 \) (Single slit diffraction) \( \beta \rightarrow 0 \), \( \Rightarrow \) \( I \rightarrow I_m (\sin \alpha / \alpha)^2 \)
Double slit diffraction. Sample problem.

Suppose that the central diffraction envelope of a double slit diffraction pattern contains 11 bright fringes and the first diffraction minima eliminate (are coincident with) bright fringes. How many bright fringes lie between the first and second minima of the diffraction envelope?

- 1 fringe in center, 5 on either side, 6\textsuperscript{th} on either side coincides with 1\textsuperscript{st} diffraction minimum
- Bright fringes occur at angles given by \( d \sin \theta = m_2 \lambda \); diffraction minima by \( a \sin \theta = m_1 \lambda \)
  - At first diffraction min, \( a \sin \theta = \lambda \) and \( d \sin \theta = 6\lambda \), so \( d/a = 6 \)
  - At second diffraction min, \( a \sin \theta = 2\lambda \) and \( d \sin \theta = m_2 \lambda \), so \( d/a = m_2/2 \)
  - Thus \( m_2 = 12 \)
- Meaning: the 12\textsuperscript{th} fringe is dark, and from the problem so is the 6\textsuperscript{th}, so in between are fringes number 7, 8, 9, 10, 11: 5 fringes
If we increase the number of slits dramatically, we get a device known as a diffraction grating: \( N_{\text{slits}} \sim \text{thousands per millimeter} \)

A diffraction grating can be used to determine which wavelengths of light—and thereby which molecules—are present in a source: from a burning chemical sample, from a lamp, from a star

If \( N \) is infinitely large, no new scale is introduced. Hence do not expect any new structure. The bright lines become extremely narrow.

If \( N \) is finite, there is new scale associated with the size if the grating. The bright lines become more pronounced.
Diffraction gratings.

Here’s what excited hydrogen molecules look like when they emit light and that light is passed through a grating.

Hydrogen has four colors, seen at orders $m=1$ and $m=2$ by this grating. At $m=0$, they are all superimposed to make white. Can be used as a “fingerprint”.

**Properties:**

**Dispersion:** how far apart a grating spreads lines (the farther, the better)

$$D = m/(d\cos\theta)$$

**Resolving power:** Narrowness of each line (the narrower, the better)

$$R = Nm \quad \Delta \theta_{hw} = \lambda/(Ndcos\theta)$$
A diffraction grating 20mm wide has 6000 rulings. At what angles will intensity maxima appear on a screen if the radiation incident on the grating has wavelength 589nm?
Recap.

• **Diffraction**
  - Quantifying single slit diffraction \( a \sin \theta = m \lambda \)
  - Circular aperture diffraction \( \sin \theta = 1.22 \lambda / d \)
  - Uncertainty principle \( a \sim 1/A, \ d \sim 1/D \)
  - Double slit diffraction

• **Diffraction gratings**
  \[ D = m / (d \cos \theta) \]
  \[ \Delta \theta_{hw} = \lambda / (Nd \cos \theta) \]

Prepare for Midterm!