Physics 211: Lab

Oscillations. Simple Harmonic Motion.

Reading Assignment:
Chapter 15

Introduction:
As we learned in class, physical systems will undergo an oscillatory motion, when displaced from a stable equilibrium. Moreover, if the initial displacement is small, the oscillations will be harmonic, i.e. a physical quantity \( x(t) \) describing the state of the system will be governed by the following differential equation:

\[
\frac{d^2 x(t)}{dt^2} + \omega^2 x(t) = 0,
\]

where the angular frequency \( \omega \) is a constant determined by the parameters of the system but does not depend on the initial displacement. The most generic solution to this equation is given by a cos-function

\[
x(t) = A \cos(\omega t + \theta_0),
\]

where the amplitude \( A \) and the initial phase \( \theta \) are determined by the initial conditions. Note that the quantity \( x \) is not necessarily displacement, it may be any physical quantity associated with the system, for instance, the electric current, temperature, concentration of a chemical substance or even a stock price on the market. In this lab we shall restrict ourselves to two particular mechanical oscillatory systems: the mathematical pendulum and a spring oscillator depicted in the figures below.

The frequency of the spring oscillator was derived in class and is given by:

\[
\omega = \sqrt{\frac{k}{m}}.
\]
In Activity II you will experimentally check this formula. We now turn to deriving
the frequency (and hence the period) of the mathematical pendulum. There are, of course,
more than one way of deriving it. Let us, for a change, consider the angular method. That
is we characterize the oscillations by the deflection angle $\phi$. The oscillations occur around
the stable equilibrium which corresponds to $\phi=0$, i.e. the bottom point. Viewing the
oscillations as rotational motion around the pivot point, where the string is attached to
ceiling, we can write down Newton’s second law in its angular form:

$$\tau = I \alpha .$$  \hfill (4)

There are two forces acting on the bob: tension and force of gravity, but the former one
has a zero arm length. Therefore the net torque is produced by gravity and equals
(CHECK!)

$$\tau = -mg\ell \sin \phi .$$ \hfill (5)

(What does the minus sign mean here?). The rotational inertia of the bob, which can be
treated as a point particle, is simply $I=ml^2$. Writing the angular acceleration as the second
time derivative of the angle and using Eq.(5), Newton’s second law (4) becomes:

$$\frac{d^2 \phi}{dt^2} \equiv \alpha = \frac{\tau}{l} = -\frac{mg\ell \sin \phi}{ml^2} \equiv -\frac{g \sin \phi}{l} .$$ \hfill (6)

Comparing the very beginning of the equation with its very end, and bringing both terms
on the lefthand side of the equation results in

$$\frac{d^2 \phi}{dt^2} + \frac{g}{\ell} \sin \phi = 0 .$$ \hfill (7)

This equation is the exact equation of motion which is valid for any deflection angle. The
solution is complicated and associated with elliptic functions. However in the case small
oscillations: $\phi << \pi/2$, the sin-function can be very well approximated by the angle
itself, which yields the following equation

$$\frac{d^2 \phi(t)}{dt^2} + \frac{g}{\ell} \phi(t) = 0 ,$$ \hfill (8)

which clearly has the form of the simple harmonic motion, Eq. (1). Here the oscillating
quantity is the angle $\phi$, and the coefficient in front of $\phi(t)$ should be identified with $\omega^2$.
Therefore the angular frequency of the mathematical pendulum is given by
\[ \omega = \sqrt{\frac{g}{l}}, \quad (9) \]

as we had hand-wavingly obtained in class based on units analysis. Note that the frequency does not depend on the mass of the bob. In Activity I you shall use this formula to measure the acceleration of gravity.

**Simple Harmonic Motion.**

**Equipment List:**
- Stand with vertical rod
- Long string and weight
- Stopwatch
- Low-friction track with clamp
- Two springs with different \( k \)
- Cart and additional bar masses
- Motion Sensor

**Lab Activity I: Mathematical Pendulum**

**Goals:**
- Determine the period and angular frequency of a mathematical pendulum.
- Check how the period depends on the amplitude of the oscillations.
- Using the relation between the frequency and the acceleration of gravity find \( g \) with sufficient precision.

Suspend the weight from the string provided. Note that a longer string results in a greater period of oscillations, hence allows for a better accuracy when measuring time. Deflect the bob from the vertical by a small angle (no greater than 15-20°) and let it oscillate.

1. Explain how to get a better accuracy when measuring the period.

2. Record the value of the period: \( T_1 = \)

3. Repeat the experiment with a different initial deflection angle, say half of the one you used in Q2. Record the value of the period \( T_2 = \)

4. How does \( T_2 \) compare to \( T_1 \)? Explain.

5. Compute the acceleration of gravity

\[ g = \]

Clearly explain how you obtained it and provide your derivations.
6. Compare the result above with the theoretical value and compute the percent difference below.

7. Decrease the length of the pendulum twice and measure the new period of oscillations. Is the value consistent with the theory? Explain.

**Lab Activity II: Spring Oscillator**

**Goals:**
- Determine the period and angular frequency of a spring oscillator and compare their values with theoretical prediction.
- Check how the period depends on the amplitude of the oscillations.
- Check how the period depends on the mass of the cart and the spring constant.
- Investigate how kinetic and potential energies depend on time and check conservation of total mechanical energy.

**Setting Up the Graphs:**

Set up Data Studio™ to read the data collected by the motion sensor located at the base of the track. Check to make sure that the motion sensor is oriented towards the cart and is sending out a narrow beam signal.

Create a graphing window that contains the three following graphs: **Position vs. Time**, **Velocity vs. Time**, and **Acceleration vs. Time**.

Using the Experiment Calculator, create new calculation called “Position from Equilibrium”. Based on that, create calculations “Potential Energy of the Spring”, “Kinetic Energy” and “Total Mechanical Energy”.

1. Measure the mass of the cart and record the value. \( m = \)

2. Determine the spring constant of each spring. Explain how you did it and record the values:
   \[ k_1 = \quad k_2 = \]

3. Using the ruler located along the length of the track, estimate the distance from the motion detector to the closest end of the cart when the cart is at rest in the equilibrium position. Record this value in the table below.

<table>
<thead>
<tr>
<th>Distance of Cart from Motion Detector at Equilibrium (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

4. Stretch the spring some initial length (but do not over-stretch the spring!) and then release the cart from rest. Press Record to collect data for approximately five cycles of simple harmonic motion, and then press Stop. Insert the graphs of \( x(t) \), \( v(t) \), and \( a(t) \).
5. How are these graphs related to one another?
   • What is the location and direction of the cart when the velocity is at a maximum or minimum value?
   • What is the location of the cart when the velocity is equal to zero?
   • When the acceleration of the cart is at a maximum or minimum value, what is the velocity and location of the cart?

**Determine the Angular Frequency of the System:**

6. Using a method of your choice, measure the amount of time that it takes the cart to complete 1 cycle.

7. From your measurement, record and/or calculate the period, frequency, and angular frequency of the resulting simple harmonic motion of the cart/spring system in the table below.

<table>
<thead>
<tr>
<th>Period – T (s)</th>
<th>Frequency – f (Hz)</th>
<th>Angular Frequency – ω (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
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</table>

8. Determine the theoretical value for the angular frequency, ω, of the cart and spring system.

<table>
<thead>
<tr>
<th>(Theoretical) Angular Frequency - ω (rad/s)</th>
</tr>
</thead>
</table>

9. What is the value of the % error between the experimental value of ω and the theoretical value of ω (assumed to be “true”)?

**Energy:**

10. Insert the three graphs with energy below.

11. What is the period of the Kinetic Energy? Potential Energy?

12. Is total Energy constant over time? Explain.

**Checking the dependence of the period on the parameters of the system:**

13. Change the mass of the cart and determine the new period of oscillations. Is it consistent with the theory?

14. Change the spring and determine the new period of oscillations. Is it consistent with the theory?

15. Assemble an inclined spring oscillator with the same spring constant and mass of the cart. How does the new period compare to the horizontal oscillator?

16. Assemble a vertical spring oscillator with the same spring constant and mass of the cart. How does the new period compare to the horizontal oscillator?