MAD 03/00
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Experiment 7 – Rotational Dynamics

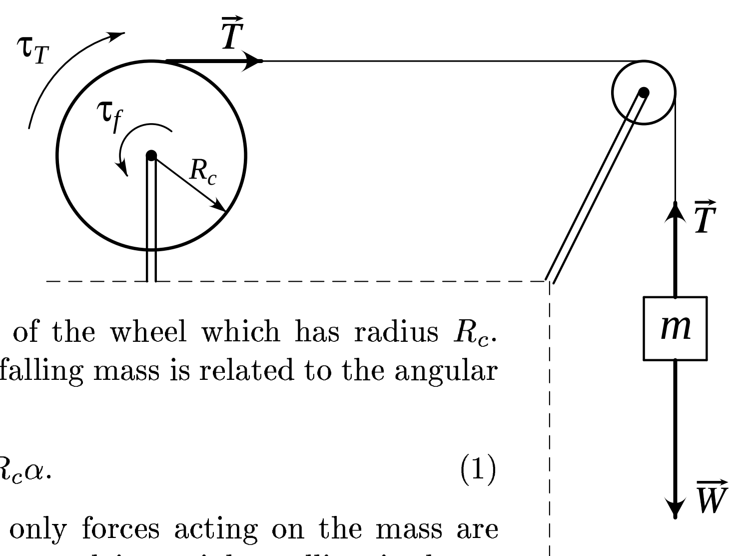
Objective

To study angular motion and the concept of moment of inertia. In particular, to determine the effect of a constant torque upon a disk free to rotate, and to measure the resulting angular motion and determine the moment of inertia of the disk.

Equipment

- Science Workshop 750 Computer Interface.
- Science Workshop Version 2.3.2 Data Acquisition Software for Windows.
- Pasco Scientific Smart Pulley ME-9387.
- String.
- Moment of inertia apparatus.
- Vernier caliper.
- Balance.
- Weights and hanger.
- Miscellaneous clamps and supports.

Theory



The apparatus for this experiment is sketched at the right. A mass is attached to the end of a string which is threaded through a "smart pulley" and wound around the heavy metal wheel which comprises the moment of inertia apparatus. Suppose that the string is wrapped around a part of the wheel which has radius R_c. Then, the (linear) acceleration of the falling mass is related to the angular acceleration of the wheel by

$$a = R_c\alpha. \tag{1}$$

Now focus on the falling mass. The only forces acting on the mass are the tension in the string pulling it up and its weight pulling it down. According to Newton's 2nd Law, the acceleration of the mass is related to these forces as follows:

$$\sum F_y = W - T = ma \tag{2}$$

For convenience, we have chosen the *downward* direction to be positive. Inserting the weight of the mass ($W = mg$) and using the relationship (1) between the acceleration of the mass and the angular acceleration of the disk produces

$$mg - T = mR_c\alpha, \qquad \text{or} \qquad T = mg - m\alpha R_c. \tag{3}$$

Now let's turn to the wheel. The string tension produces a torque on the wheel whose magnitude is given by

$$\tau_T = TR_c. \tag{4}$$

Since the tension acts tangential to the edge of the disk, the lever arm for the torque is simply the distance R_c from the center of rotation. The magnitude of the tension is as was found in Eq. (3). This torque makes the disk rotate in the clockwise direction, as indicated on the figure. There is also a measurable amount of friction between the disk and its support: it acts in the form of a frictional torque, and is approximately constant. The frictional torque is counterclockwise: it opposes the motion of the disk. Therefore, the net torque acting on the disk is

$$\begin{aligned}\tau_{net} &= \tau_T - \tau_f \\ &= TR_c - \tau_f \\ &= (mg - m\alpha R_c)R_c - \tau_f \\ &= mgR_c - m\alpha R_c^2 - \tau_f,\end{aligned} \tag{5}$$

where we have inserted the expressions for the torque due to the tension [Eq. (4)] and the magnitude of the tension [Eq. (3)]. Now the rotational form of Newton's 2nd Law relates the net torque to the angular acceleration of the disk:

$$\tau_{net} = \mathcal{I}\alpha, \tag{6}$$

Inserting the net torque determined in Eq. (5) gives us

$$mgR_c - mR_c^2\alpha - \tau_f = \mathcal{I}\alpha, \tag{7}$$

or, solving for α:

$$\alpha = \frac{gR_c}{\mathcal{I} + mR_c^2}\, m - \frac{\tau_f}{\mathcal{I} + mR_c^2}. \tag{8}$$

Eq. (8) gives the exact dependence of the angular acceleration of the disk as a function of the size of the falling mass. In the setup we will be using, the mass m is small: that is we have $mR_c^2 \ll \mathcal{I}$ (you will be asked to verify this later). Therefore, we may simplify Eq. (7) to read

$$\alpha = \frac{gR_c}{\mathcal{I}}\, m - \frac{\tau_f}{\mathcal{I}}. \tag{9}$$

This is a much nicer relationship to deal with, since a plot of α versus m allows us to determine the moment of inertia $\mathcal{I}$ from the slope (which will be $gR_c/\mathcal{I}$), and the frictional torque from the intercept (which will be $-\tau_f/\mathcal{I}$).

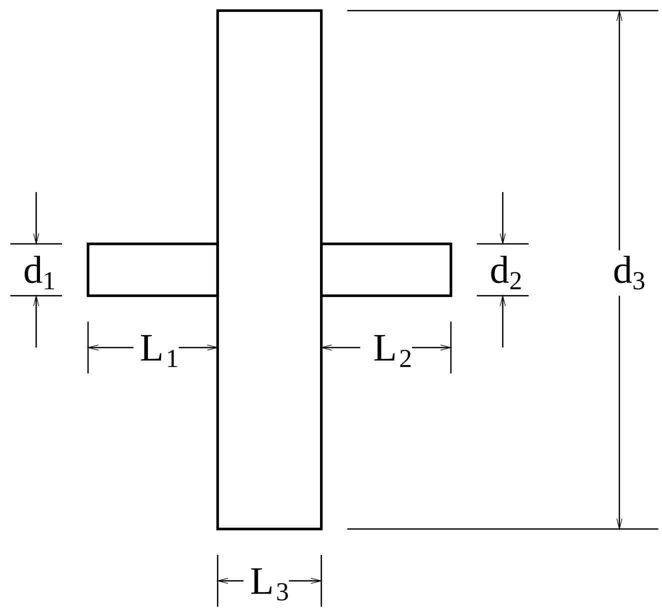
Procedure

1. Measure the mass of the wheel M by removing it from the stand and placing it on the 5-kg capacity balance on the front table. Instead of using the sliding scale on the

balance (which can only be read to ±5 g), use the set of weights to obtain the mass to an accuracy of ±1 g. Record the resulting mass in kilograms:

mass of wheel (M)	

2. Using the vernier caliper, measure and record all of the dimensions outlined in the picture below. Do not assume that the two axles have the same lengths or diameters. Since the caliper does not open up wide enough to measure d_3 directly, instead measure the distance from the outer edge to the axle and determine d_3 from this information. You should read the caliper to its full capability, that is to the nearest 0.002 cm.



d_1		L_1	
d_2		L_2	
d_3		L_3	

3. Moment of inertia calculation. In order to compute the moment of inertia of the wheel, we break it into three pieces: the two "wings" on the sides plus the large central disk. Since each piece is effectively a cylinder rotating about the cylinder axis, the total moment of inertia is

$$\mathcal{I} = \tfrac{1}{2}M_1R_1^2 + \tfrac{1}{2}M_2R_2^2 + \tfrac{1}{2}M_3R_3^2, \tag{10}$$

or, equivalently, since we've measured the diameters instead of the radii:

$$\mathcal{I} = \tfrac{1}{8}M_1d_1^2 + \tfrac{1}{8}M_2d_2^2 + \tfrac{1}{8}M_3d_3^2, \tag{11}$$

To determine the masses M_1, M_2, M_3 of the different parts of the wheel, we assume that it has a uniform density. Then, the fraction of the total volume occupied by a given part multiplied by the total mass will give the mass of just that part. Since the volume of a cylinder is $V = \pi L R^2 = \frac{1}{4}\pi L d^2$, the mass of the jth piece is

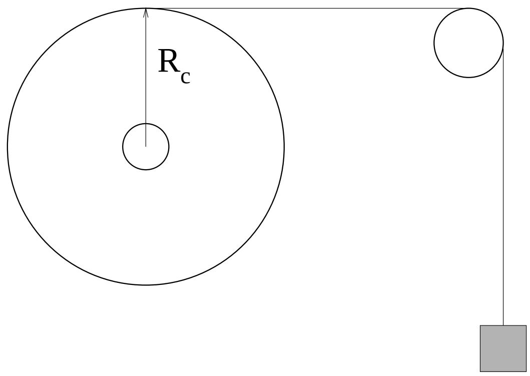
$$\begin{aligned} M_j &= M \, \frac{\frac{1}{4}\pi L_j d_j^2}{\frac{1}{4}\pi L_1 d_1^2 + \frac{1}{4}\pi L_2 d_2^2 + \frac{1}{4}\pi L_3 d_3^2} \\ &= M \, \frac{L_j d_j^2}{L_1 d_1^2 + L_2 d_2^2 + L_3 d_3^2}. \end{aligned} \tag{12}$$

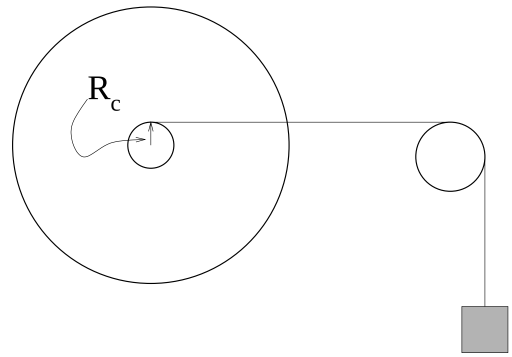
At this stage, it is easiest to leave the lengths and diameters in centimeters. We may do this since both the numerator and denominator of the fractions will be in cm^3, cancelling out.

Using Eq. (12), determine the masses M_1, M_2, M_3 of the different parts of the wheel in the space below. Your final value should contain 4 significant digits (you did record the lengths and mass to the accuracy instructed, right?), although you should keep 5 (or more) digits at all intermediate stages. After computing M_1, M_2, M_3, cross check your work by adding them up: you should recover the total mass of the wheel measured in step **1**.

Using Eq. (11) and your results for the masses, compute the moment of inertia for the wheel. At this point you must convert your lengths to meters, so that the result will be in kg·m^2.

4. Wrap the string around either the central part of the disk (left-hand diagram) or around one of the wings (right-hand diagram), run the string through the smart pulley, and adjust the height of the clamp holding the smart pulley so that the string between the disk and the pulley is horizontal. Record your choice for R_c.





5. Tell Science Workshop about the smart pulley. Click on the plug icon and drag it to input in the set-up window. Select "Smart Pulley [Linear]" when asked to choose a sensor. Next, click on the calculator icon near the lower-left of the set-up window. Arrange for the direct display of the angular position of the wheel in radians by dividing the x-position output by the smart pulley by your measured value for R_c. You must supply all of the requested information in the window. Set the calculation name to "Angular Displacement" and the short name to "theta" and the units to "radians." Finally, request a graph of the angular displacement of the wheel versus time by clicking on the graph icon. Select "position x" from the initial range of choices you are given. After the graph appears on the screen, enlarge it, and change your selection to the angular displacement calculation you have just defined. (For some reason, the software won't let you choose the calculation from the start). You can avoid that annoying auto-rescaling of the plots by clicking on the icon at the far lower-left of the graphing window and deselecting the "Auto-Scale During Sampling" option. Then, once you set the scales on the graph to something sensible, they will stay that way!

6. For 5 different values of the falling mass, record the angular displacement as a function of time. If you are wrapping the string around the central part of the disk, you should use masses of 50, 60, 70, 80, and 90 grams. If you are wrapping the string around one of the wings, you should use masses of 50, 100, 150, 200, and 250 grams. In either case, for a given run, the angular acceleration should be a constant: the angular displacement as a function of time should obey

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2. \tag{13}$$

Therefore, you may determine α by selecting the polynomial fit option. Don't forget that the coefficient reported as a_3 is actually $\frac{1}{2}\alpha$!

RUN	mass	α
1		
2		
3		
4		
5		

Print out a copy of the graph display for one of your runs, showing the region used in your fit, and include it in your write-up.

7. On 10-square-to-the-centimeter graph paper, prepare a plot of α versus m. That is, put the angular acceleration along the y axis and the size of the falling mass along the x axis. Since you want to determine the value of the intercept, do not suppress the point $(0, 0)$ on your plot! (Tip: since the intercept is at a negative value of α, you should place the x axis at a point halfway down the page.) Using two widely-spaced points on your best-fit straight line drawn through the data, compute the slope. You may place the details of this calculation in an empty place on your plot.

Measured slope: ____________________

Measured intercept: ____________________

8. Using your measured slope, the measured value of R_c, and the accepted value for g, compute the moment of inertia of the disk. Compare your result to what you expected based upon your calculation in step **3**. Can you think of any construction techniques which would invalidate that calculation?

9. Using your measured intercept and *measured* moment of inertia (*i.e.* the number obtained in step **8** above), determine the magnitude of the frictional torque.