Practice exercises for homework #1

1. (similar to 2.6)

Following Alfred Nobel’s will, there are five Nobel Prizes awarded each year. These are for outstanding achievements in Chemistry, Physics, Physiology or Medicine, Literature, and Peace. In 1968, the Bank of Sweden added a prize in Economic Sciences in memory of Alfred Nobel. You think of the data as describing a population, rather than a sample from which you want to infer behavior of a larger population. The accompanying table lists the joint probability distribution between recipients in economics and the other five prizes, and the citizenship of the recipients, based on the 1969-2001 period.

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>U.S. Citizen ((Y = 0))</td>
</tr>
<tr>
<td>-------------------------------</td>
</tr>
<tr>
<td>Economics Nobel Prize ((X = 0))</td>
</tr>
<tr>
<td>Physics, Chemistry, Medicine, Literature, and Peace Nobel Prize ((X = 1))</td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>

(a) Compute \(E(Y)\) and interpret the resulting number.

**Answer:** \(E(Y) = 0.537\). 53.7 percent of Nobel Prize winners were non-U.S. citizens.

(b) Calculate \(E(Y \mid X = 1)\).

**Answer:** \(E(Y \mid X = 1) = 0.586\).

(c) A randomly selected Nobel Prize winner reports that he is a non-U.S. citizen. What is the probability that this genius has won the Economics Nobel Prize? A Nobel Prize in the other five disciplines?
**Answer:** There is a 9.1 percent chance that he has won the Economics Nobel Prize, and a 90.9 percent chance that he has won a Nobel Prize in one of the other five disciplines.

(d) Show what the joint distribution would look like if the two categories were independent.

**Answer:**


<table>
<thead>
<tr>
<th>Economics Nobel Prize ( (X = 0) )</th>
<th>U.S. Citizen ( (Y = 0) )</th>
<th>Non-U.S. Citizen ( (Y = 1) )</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physics, Chemistry, Medicine, Literature, and Peace Nobel Prize ( (X = 1) )</td>
<td>0.386</td>
<td>0.447</td>
<td>0.833</td>
</tr>
<tr>
<td>Total</td>
<td>0.463</td>
<td>0.537</td>
<td>1.00</td>
</tr>
</tbody>
</table>

2. (Similar to 2.14)

Calculate the following probabilities using the standard normal distribution.

(a) \( \Pr(Z \leq 1.0) \)
(b) \( \Pr(Z > 1.96) \)
(c) \( \Pr(-1.96 < Z < 1.96) \)

**Answer:**

(a) 0.8413
(b) 0.0250
(c) 0.9500

3. (Similar to 2.14)

The central limit theorem states that
a. the sampling distribution of \( \frac{\bar{Y} - \mu_Y}{\sigma_T} \) is approximately normal.

b. \( \bar{Y} \rightarrow \mu_Y \).

c. the probability that \( \bar{Y} \) is in the range \( \mu_Y \pm c \) becomes arbitrarily close to one as \( n \) increases for any constant \( c > 0 \).

d. the \( t \) distribution converges to the \( F \) distribution for approximately \( n > 30 \).

Answer: a

4. (similar to 2.22) \( \text{var}(aX + bY) = \) ____________

\( E(aX + bY) = \) ____________

a. \( a^2 \sigma_X^2 + b^2 \sigma_Y^2 \)

b. \( a^2 \sigma_X^2 + 2ab \sigma_{XY} + b^2 \sigma_Y^2 \)

c. \( EX + EY \)

d. \( aEX + bEY \)

Answer: b and d

5. (similar to E3.1) 95% confidence interval for \( \mu_Y \) is: ____________, and

95% confidence interval for \( d = \mu_m - \mu_w \) is: ____________

Answer: \{ \( \bar{Y} \pm 1.96SE(\bar{Y}) \) \} and \( (\bar{Y}_m - \bar{Y}_w) \pm 1.96SE(\bar{Y}_m - \bar{Y}_w) \), where

\[
SE(\bar{Y}_m - \bar{Y}_w) = \sqrt{\frac{s_m^2}{n_m} + \frac{s_w^2}{n_w}}
\]