**Problem 1.** Compute the Laplace Transform of \( f(t) \). You may use the provided formulas.

(a) \( f(t) = t^n e^{4t} \)

**Answer:** We can use the formula given for \( t^n \) and the shift property, and change \( s \) into \( s - 4 \):

\[
F(s) = \frac{n!}{(s - 4)^{n+1}}
\]

(b) \( f(t) = te^t \sin 2t \)

**Answer:** (alternate from class) First, we use the shift Theorem of \( \sin 2t \), and get

\[
\mathcal{L}\{e^t \sin 2t\} = \frac{2}{(s-1)^2 + 2^2} = \frac{2}{s^2 - 2s + 5}.
\]

Then, we use the property of the derivative of \( F'(s) \), we get

\[
F(s) = -\frac{d}{ds} \left(\frac{2}{s^2 - 2s + 5}\right) = \frac{2(2s - 2)}{(s^2 - 2s + 5)^2}
\]

**Problem 2.** Solve the following IVP for \( y(t) \)

\[ y''' + 4y' = 8t, \quad y(0) = 0, \quad y'(0) = 0, \quad y''(0) = 1. \]

**Answer:** We first find \( y_H \):

\[
r^3 + 4r = 0, \quad r_1 = 0, \quad r_{2,3} = \pm 2i,
\]

This means the homogeneous solution would have \( \{\text{constant term, cos} 2t, \sin 2t\} \) which corresponds to Laplace transformation: \( \left\{ \frac{1}{s}, \frac{2}{s^2 + 4}, \frac{2}{s^2 + 4} \right\} \). Observe the source term, which would make our particular solution as a polynomial, meaning it would have Laplace transform in the form of \( \frac{1}{n!}t^n + 1 \), we choose not to care about the order of polynomials right now.

Assume

\[
\mathcal{L}\{y\} = Y
\]

then

\[
y''' = s^3 Y - s^2 y(0) - sy'(0) - y''(0) = s^3 Y - 1
\]

\[
y' = sY - y(0) = sY
\]

therefore we have

\[
s^3 Y - 1 + 4sY = \frac{8s}{s^2}
\]

reorganizing terms and we have:

\[
(s^3 + 4s)Y = \frac{8s}{s^2} + 1
\]

\[
Y = \frac{8 + s^2}{s^3(s^2 + 4)}
\]
we want to use partial fractions here (observe how we choose coefficient and think why)

\[
\frac{8 + s^2}{s^3(s^2 + 4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{2C}{s^3} + \frac{Es + 2D}{s^2 + 4}
\]

multiply both sides by denominator,

\[
As^2(s^2 + 4) + Bs(s^2 + 4) + 2Cs^3 + (Es + 2D)(s^2 + 4) = 8 + s^2
\]

let \( s = 0 \), gives \( C = 1 \),
let \( s = 2i \), \((2Ei + 2D)(-8i) = 4\), gives \( E = \frac{1}{4}, D = 0 \),
now it seems we don’t have much good choice of simple value here, but observe the equation
now becomes:

\[
As^2(s^2 + 4) + Bs(s^2 + 4) + 2(s^2 + 4) + \frac{1}{4}s^4 = 8 + s^2
\]

match coefficients for \( s^4, s^3 \) and get \( A = -\frac{1}{4}, B = 0 \) directly. Now take inverse Laplace, and
the solution is:

\[
y = -\frac{1}{4} + t^2 + \frac{1}{4}\cos(2t)
\]