MGT 239 I

Total 25 + 3 pts; 9:40AM - 11:30AM, May 6, 2009.

1. [14pts] Consider *daily* operation of a supply chain consisting of one supplier and 10 manufacturers $i = 1, \dots, 10$. In each day, each manufacturer i has probability $p_i = 0.7$ to place an order (to the supplier) with order quantity D_i following a *discrete uniform distribution* $U[1, 10]$. Manufacturers' ordering decisions are independent of each other. Suppose the supplier's production cost is negligible. In each day, the supplier first produces 50 units of the product, then receives all the orders from manufacturers and satisfies them as much as possible. Each unit of unsatisfied demand costs the supplier \$10K, and each unit of leftover costs \$2K to dispose. Assume sample size $N = 365$.
(a) [8pts] What is the distribution (histogram) of the daily total order quantities received by the supplier?
(b) [3pts] What is the 90% confidence interval of mean daily total cost for the supplier?
(c) [3pts] What is the supplier's probability of shortage? And what is the probability that the supplier has leftover?

2. [14pts] Consider *weekly* operation of a component manufacturing system, who has daily demand D_t following discrete uniform distribution $U[0, 10]$ during each weekday, but discrete uniform $U[3, 7]$ during weekend (Saturday and Sunday, respectively). The system is controlled by two-parameter (s, S) policy where $0 \leq s \leq S$. It works as follows. In each day, the system first observes its initial inventory x_t carried over from day $(t-1)$. If $x_t \leq s$, it initiates production and brings the inventory level (after the production) up to S, and then shuts down the production line; if $x_t > s$, it does not initiate production at all. The production line, if used in a day, incurs a *fixed* cost \$20K, *regardless of production quantity.* If production line is not used in that day, no production cost incurs. After that, the demand materializes and the system satisfies it as much as possible. The leftover inventory, if any, incurs per unit holding cost \$ 4K and carries over to the next day $(t+1)$; each unit unmet demand costs \$ 12K. The initial inventory of Monday x_1 is always 0 and the leftover inventory on Sunday after incurring holding cost is disposed at zero cost. Currently S is set to $S = 8$ in (s, S) policy. We need to set s to control the system. Assume sample size $N = 100$ (weeks).
(a) [8pts] If $s = 0$, what is the distribution (histogram) of the weekly total cost?
(b) [3pts] What is the 90% confidence interval of the expected weekly total cost under $s = 0, 1, \dots, 8$, respectively? Graph these intervals as a function of s and find the optimal s.
(c) [3pts] Assume $(s = 0, S = 8)$. The weekly *fill rate* is defined by the total demand fulfilled of a week divided by the total demand of that week. What is the 90% confidence interval of mean weekly fill rate? And what is the probability that the system does not use production line during weekend?