Convergence Analysis of Adaptive Mixed and Nonconforming Finite Element Methods

Ronald H.W. Hoppe

Dept. of Math., Univ. of Houston, Houston, TX 77204-3008, U.S.A.

Abstract

We are concerned with a convergence analysis of adaptive mixed and nonconforming finite element methods for second order elliptic boundary value problems. We note that in case of standard conforming Lagrangian type finite element approximations, such methods have been considered in [3, 4]. The methods presented in this contribution provide a guaranteed error reduction and thus imply convergence of the adaptive loop which consists of the essential steps 'SOLVE', 'ESTIMATE', 'MARK', and 'REFINE'. Here, 'SOLVE' means the efficient solution of the finite element discretized problems with respect to a given coarse triangulation. The following step 'ESTIMATE' is devoted to the a posteriori error estimation of the global discretization error. A greedy algorithm is the basic tool of the step 'MARK' to indicate selected elements for refinement. The final step 'REFINE' deals with the technical realization of the refinement process resulting in a refined triangulation of the computational domain.
The analysis is carried out for a model problem and discretization by the lowest order Raviart-Thomas and Crouzeix-Raviart finite elements. The essential steps in the convergence proof are the reliability of the estimator, a strong discrete local efficiency, and quasi-orthogonality properties. We do not require any regularity of the solution nor do we make use of duality arguments.

References

[1] C. Carstensen and R.H.W. Hoppe. *Error reduction and convergence for an adaptive mixed finite element method.* to appear in Math. Comp.

[2] C. Carstensen and R.H.W. Hoppe. *Convergence analysis of an adaptive nonconforming finite element method.* submitted

[3] W. Dörfler. *A convergent adaptive algorithm for Poisson's equation.* SIAM J. Numer. Anal., **33**, 1106–1124, 1996.

[4] P. Morin, R.H. Nochetto, and K.G. Siebert. *Data Oscillation and convergence of adaptive FEM.* SIAM J. Numer. Anal., **38**, 2, 466–488, 2000.