Mimetic Discrete Models with Weak Material Laws, or Least Squares Principles Revisited.

P. Bochev[1,2]

[b]Sandia National Laboratories P.O. Box 5800, MS 1110, Albuquerque, NM 87185-1110
pbboche@sandia.gov

To a casual observer, compatible (or mimetic) methods and least squares principles for PDEs are polar opposites. Mimetic methods inherit key conservation properties of the PDE, can be related to a naturally occurring optimization problem, and require specially selected, dispersed degrees of freedom. The conventional wisdom about least squares is that they rely on artificial energy principles, are only approximately conservative, but can work with standard C^0 nodal (or collocated) degrees of freedom. The latter is considered to be among the chief reasons to use least squares methods.

In this talk we demonstrate that exactly the opposite is true about least-squares methods. First, we will argue that nodal elements, while admissible in least squares, do not allow them to realize their full potential, should be avoided and are, perhaps, the least important reason to use least squares! Second, we will show that for an important class of problems least squares and compatible methods are close relatives that share a common ancestor. In fact, we will prove that in some circumstances, least squares and compatible methods compute *identical* answers. The price paid for gaining favorable conservation properties is that one has to give up what is arguably the least important advantage attributed to least squares methods: one can no longer use C^0 nodal elements for all variables.

To carry out this agenda we use algebraic topology to guide our analysis and develop a common framework for compatible discretizations. Using a reduction and a reconstruction maps between differential forms and cochains we define mutually consistent sets of *natural* and *derived* discrete operations that preserve the invariants of the DeRham homology groups and obey a discrete Stokes theorem. By choosing a specific reconstruction operator we obtain well-known mixed FE, mimetic FD and covolume methods and explain when they are equivalent.

The key concept in our approach is the natural inner product on cochains. This inner product is sufficient to generate a combinatorial Hodge theory on cochains but avoids complications attendant in the construction of robust discrete Hodge-star operators. For problems that require approximations of material laws we employ equivalent constrained optimization problems that enforce the laws weakly, instead of using their explicit discretization. We then consider three possible mimetic discretizations of the optimization problem. Two of them give familiar Galerkin and/or mixed type methods. The third one reduces the optimization problem to a *mimetic* least squares principle whose minimizers are, under certain conditions, identical with the solutions of the other two mimetic discretizations. We conclude by a series of numerical examples that assert our findings. This talk is based on joint work with M. Gunzburger (CSIT, Florida State University) and M. Hyman (Theoretical Division, Los Alamos National Laboratory).

---

[1] Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed-Martin Company, for the United States Department of Energy's National Nuclear Security Administration under contract DE-AC-94AL85000.

[2] This work was partially funded by the Applied Mathematical Sciences program, U.S. Department of Energy, Office of Energy Research