1. If
\[ A = \int_0^\infty \frac{\sin x}{x} \, dx \]
then what is
\[ \int_0^\infty \frac{\sin^2 x}{x^2} \, dx \]
in terms of \( A \).

2. Evaluate
\[ \sum_{n=1}^{2008} \frac{1}{\sqrt{n^2 + \sqrt{n(n+1)}} + \sqrt{(n+1)^2}} \]

3. Let \( f : \mathbb{R} \to \mathbb{R} \) be a function such that \( f(x, y) + f(y, z) + f(z, x) = 0 \) for all real numbers \( x, y, \) and \( z \). Prove that there exists a function \( g : \mathbb{R} \to \mathbb{R} \) such that \( f(x, y) = g(x) - g(y) \) for all real numbers \( x \) and \( y \).

4. Find the limit
\[ \lim_{n \to \infty} \int_0^\pi (\sin x)^n \, dx \]

5. Let \( x \) be a real number. Define the sequence \((x_n)_{n \geq 1}\) recursively by \( x_1 = 1 \) and \( x_{n+1} = x_n + nx_n \) for \( n \geq 1 \). Prove that
\[ \prod_{n=1}^{\infty} \left(1 - \frac{x^n}{x_{n+1}}\right) = e^{-x} \]