Balancing Covariates via Propensity Score Weighting: The Overlap Weights

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April 6th, 2017

Joint work with Fan Li (Duke) and Alan Zaslavsky (Harvard)
Outline

1. Causal inference in observational studies - a brief overview
2. Introduce a general class of balancing weights
3. Propose overlap weights
4. Illustrate with examples
Causal Inference in Observational Studies

• Ideal goal: estimate the causal effect of a treatment using observational data

• Problem: Without randomization to treatment groups, severe covariate imbalance is likely

• Realistic goal: Balance observed covariates between treatment groups
Example: Right Heart Catheterization

- Right heart catheterization (RHC) is an invasive diagnostic procedure to assess cardiac function.
- What is the causal effect of right heart catheterization on survival?
- 2184 treatment (RHC), 3551 control (no RHC)
- Observational data (Murphy and Cluff, 1990)
- Covariate imbalance
Example: Right Heart Catheterization

**Figure:** Imbalance in APACHE, Acute Physiology and Chronic Health Evaluation Score, measured before procedure.
Overview

- **Notation:**
  - Population density of the covariates $X$ is $f(x)$
  - Density for group $Z \in \{0, 1\}$ is $f_z(x) = P(X = x \mid Z = z)$

- **GOAL:** make $f_1(x) \propto f_0(x)$

- One solution: Use weights, $w_z(x)$, such that $f_1(x)w_1(x) \propto f_0(x)w_0(x)$
The Propensity Score

- Propensity score is $e_i(x) \equiv \Pr(Z_i = 1|X_i = x)$
- Covariate density for group $Z = 1$:

$$f_1(x) = P(X = x | Z = 1)$$
$$= \frac{P(Z = 1|X = x)P(X = x)}{P(Z = 1)}$$
$$\propto e(x)f(x)$$

- Likewise, $f_0(x) \propto (1 - e(x))f(x)$
- The propensity score is key for balancing covariates!
Balancing weights

- We propose the following class of balancing weights, satisfying $f_1(x)w_1(x) \propto f_0(x)w_0(x)$:

\[
\begin{align*}
  w_1(x) & \propto \frac{h(x)}{e(x)}, \\
  w_0(x) & \propto \frac{h(x)}{1-e(x)},
\end{align*}
\]

where $h(\cdot)$ is a pre-specified function.

- The weighted covariate distributions in the two groups have the same target density $\propto f(x)h(x)$:

\[
\begin{align*}
  f_1(x)w_1(x) & \propto f(x)e(x)\frac{h(x)}{e(x)} = f(x)h(x), \\
  f_0(x)w_0(x) & \propto f(x)(1-e(x))\frac{h(x)}{1-e(x)} = f(x)h(x).
\end{align*}
\]
Estimand

- Conditional average controlled difference (ACD):
  \[ \tau(x) \equiv \mathbb{E}(Y|Z = 1, X = x) - \mathbb{E}(Y|Z = 0, X = x) \]

- In a causal context, under SUTVA and unconfoundedness, with potential outcomes \( Y_i(1), Y_i(0) \), \( \tau(x) \) is the average treatment effect (ATE) conditional on \( x \):
  \[ \tau(x) = \mathbb{E}(Y(1) - Y(0)|X = x) \]

- Estimand is average over a target population with density \( \propto f(x)h(x) \):
  \[ \tau_h \equiv \frac{\int \tau(dx)f(x)h(x)\mu(dx)}{\int f(x)h(x)\mu(dx)} \].
## Examples of target population, estimands, and balancing weights

<table>
<thead>
<tr>
<th>target population</th>
<th>$h(x)$</th>
<th>estimand</th>
<th>weight $(w_1, w_0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>combined</td>
<td>1</td>
<td>ATE</td>
<td>$(\frac{1}{e(x)}, \frac{1}{1-e(x)})$ [HT]</td>
</tr>
<tr>
<td>treated</td>
<td>$e(x)$</td>
<td>ATT</td>
<td>$(1, \frac{e(x)}{1-e(x)})$</td>
</tr>
<tr>
<td>control</td>
<td>$1 - e(x)$</td>
<td>ATC</td>
<td>$(\frac{1-e(x)}{e(x)}, 1)$</td>
</tr>
<tr>
<td>truncated</td>
<td>$1(\alpha &lt; e(x) &lt; 1 - \alpha)$</td>
<td>ATTrunc</td>
<td>$(\frac{1(e(x)&lt;1-\alpha)}{e(x)}, \frac{1(\alpha&lt;e(x)&lt;1-\alpha)}{1-e(x)})$</td>
</tr>
<tr>
<td>combined</td>
<td>$e(x)(1 - e(x))$</td>
<td>ATO</td>
<td>$(1 - e(x), e(x))$</td>
</tr>
</tbody>
</table>
The weighted estimator, \( \hat{\tau}_h \), is the weighted difference in means between the treatment and control group:

\[
\hat{\tau}_h = \frac{\sum_{i=1}^{n} Y_i Z_i w_1(x_i)}{\sum_{i=1}^{n} Z_i w_1(x_i)} - \frac{\sum_{i=1}^{n} Y_i (1 - Z_i) w_0(x_i)}{\sum_{i=1}^{n} (1 - Z_i) w_0(x_i)}
\]

**Theorem**

\( \hat{\tau}_h \) is a consistent estimator of \( \tau_h \).
Asymptotic Variance of $\hat{\tau}_h$

**Theorem**

As $n \to \infty$, the expectation of the conditional variance of the estimator $\hat{\tau}_h$ given the sample $X$ converges:

$$n \mathbb{E}_X \mathbb{V}[\hat{\tau}_h|X] \to \int f(x)h(x)^2 \left[ \frac{v_1(x)}{e(x)} + \frac{v_0(x)}{1-e(x)} \right] \mu(dx)/C_h^2,$$

where $v_z(x) = \mathbb{V}(Y(x)|X=x)$ and $C_h = \int h(x)f(x)d\mu(x)$.

**Corollary**

Assuming $v_0(x) = v_1(x) = v$, $h(x) \propto e(x)(1-e(x))$ gives the smallest asymptotic variance for $\hat{\tau}_h$, and as $n \to \infty$,

$$n \min_h \mathbb{E}_X \mathbb{V}_X[\hat{\tau}_h|X] \to v/C_h^2 \int f(x)e(x)(1-e(x))\mu(dx).$$
Overlap weights

We propose the overlap weights with $h(x) = e(x)(1 - e(x))$:

$$w_1(x) \propto \frac{h(x)}{e(x)} = \frac{e(x)(1 - e(x))}{e(x)} = 1 - e(x)$$

$$w_0(x) \propto \frac{h(x)}{1 - e(x)} = \frac{e(x)(1 - e(x))}{1 - e(x)} = e(x).$$

Target density $f(x)e(x)(1 - e(x))$ defined by covariate overlap:
Overlap population, $f(x)e(x)(1 - e(x))$, gives more weight to
- units with $e(x) = 1/2$
- units who, based on their covariates, could be in either treatment group
- “marginal” units who may get either treatment
- units in “clinical equipoise"
- the region of $x$ with the most overlap between groups
Exact Balance

**Theorem**

When the propensity scores are estimated from a logistic regression model with main effects, \( \text{logit}\{e(x_i)\} = \beta_0 + \beta' x_i \), the overlap weights lead to exact balance in the means of any included covariate between treatment and control groups:

\[
\frac{\sum_i x_{i,k} Z_i (1 - \hat{e}_i)}{\sum_i Z_i (1 - \hat{e}_i)} = \frac{\sum_i x_{i,k} (1 - Z_i) \hat{e}_i}{\sum_i (1 - Z_i) \hat{e}_i}.
\]
Right heart catheterization (RHC)

RHC vs. non RHC

Table: Estimated treatment effect (in %) with different weights

<table>
<thead>
<tr>
<th></th>
<th>unweighted</th>
<th>overlap</th>
<th>HT</th>
<th>ATT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\tau}_h$</td>
<td>7.36</td>
<td>6.54</td>
<td>5.93</td>
<td>5.81</td>
</tr>
<tr>
<td>SE($\hat{\tau}_h$)</td>
<td>1.27</td>
<td>1.32</td>
<td>2.46</td>
<td>2.67</td>
</tr>
</tbody>
</table>
Racial Disparity in Medical Expenditure

- Goal: estimate racial disparity in medical expenditures after balancing covariates (Le Cook et al., 2010)
- Race is not manipulable so this is not a causal question
- Data: 2009 Medical Expenditure Panel Survey
  - 10,130 non-Hispanic Whites (Z = 1)
  - 4224 Blacks
  - 1522 Asians
  - 5558 Hispanics
- Three comparisons: comparing Whites to each minority
- 29 covariates (4 continuous, 25 binary)
- Logistic regression to estimate propensity scores
- Ignore survey weights here, but weighting allows easy incorporation of survey weights
Racial Disparity in Medical Expenditure

Figure: Propensity score distributions.

White– Black
Estimated Propensity Score
Z=1
Z=0

White– Asian
Estimated Propensity Score

White– Hispanic
Estimated Propensity Score
Figure: Covariate balance (absolute standardized bias) with no weights, overlap weights, and HT weights.
HT weighting gives one Asian over 30% of the weight!

78 year old Asian lady with a BMI of 55.4: $e(x) = 0.9998$
Racial Disparity in Medical Expenditure

Weighted differences in mean yearly health expenditure (SE):

<table>
<thead>
<tr>
<th></th>
<th>Unweighted</th>
<th>Overlap</th>
<th>HT</th>
<th>ATT</th>
</tr>
</thead>
<tbody>
<tr>
<td>White - Black</td>
<td>$849 (226)</td>
<td>$789 (208)</td>
<td>$774 (261)</td>
<td>$775 (302)</td>
</tr>
<tr>
<td>White - Asian</td>
<td>$2772 (225)</td>
<td>$1302 (220)</td>
<td>$2458 (576)</td>
<td>$2624 (634)</td>
</tr>
<tr>
<td>White - Hispanic</td>
<td>$2563 (177)</td>
<td>$1292 (161)</td>
<td>$512 (359)</td>
<td>$51 (508)</td>
</tr>
</tbody>
</table>
Racial Disparity in Medical Expenditure

- Common: truncate/discard units with $e(x)$ close to 0 or 1
- White - Hispanic HT weighted estimates after truncation:

<table>
<thead>
<tr>
<th></th>
<th>Estimated Propensity Score Range Kept</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[0,1]</td>
</tr>
<tr>
<td>Truncate</td>
<td>$512</td>
</tr>
<tr>
<td>Discard</td>
<td>$512</td>
</tr>
</tbody>
</table>

Results can be very sensitive to the truncation point!

*The overlap weights avoid extreme weights and avoid needing an abrupt threshold for elimination or truncation.*
Advantages of the overlap weights

**Statistical advantages**
- Minimizes asymptotic variance among balancing weights
- Perfect (exact small-sample) balance for means
- Weights are bounded, avoiding explosive weights or the need for arbitrary truncation

**Scientific advantages**
- Upweights “marginal" units who, based on their covariates, could be in either treatment group
- Rather than focusing on atypical individuals, focuses on the naturally comparable “overlap" population
Thoughts on the Target Population

- The choice of $h(x)$, and hence the target population, deserves thought.
- There is precedent for prioritizing covariate balance over the “ideal" target population:
  - randomized experiments on volunteers
  - removing non-overlapping units
  - truncating inverse probability weights
Matching is often preferred to weighting, in part because it is less sensitive to the propensity score model specification. However, the ease and automatic “perfect balance” property of the overlap weights is appealing.

The “Tudor” Solution:
- Create a matched sample, using your method of choice.
- Estimate propensity scores by logistic regression *within* the matched sample.
- Eliminate residual imbalance by applying overlap weights to the matched sample to estimate the treatment effect.
Summary

- Unified framework for using weighting to balance covariates for any target population.
- The general class of balancing weights balance covariates and include many of the existing weights.
- A new weighting method, the overlap weights, has desirable statistical and substantive properties.


Thanks for listening!

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