Leveraged investor disclosures and concentrations of risk

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Abstract

We analyze a model where investors (e.g., hedge funds) need to borrow from lenders with heterogeneous risk-exposures and risk-management motives. Investors may obtain advantageous terms of borrowing by disclosing their investment strategy, thereby revealing its correlation to the lender’s existing risk exposure. Investors risk being “front-run” by their lender if they disclose, however. We show that in the presence of front-running, the “unraveling” result of full disclosure may not hold. In addition, disclosure regulation results in a loss of welfare since investors compelled to disclose will mitigate front-running by choosing a lender with sufficiently high correlation, thus exacerbating concentrations of risk.

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The near-collapse of Long-Term Capital Management in 1998 sparked a furious interest in the level of transparency between leveraged market participants and their lenders. LTCM conducted its activities in a highly non-transparent way, disclosing very little information to its creditors, counterparties, and investors. The Basel Committee’s policy report on the crisis, for example, documents that “counterparties did not generally receive meaningful information about leverage or the concentration of exposure in certain types of...
positions, risk factors, trading strategies... to enable them to form a more comprehensive picture of the true risk profile of LTCM.”

The conventional wisdom that emerged was that better information sharing could have mitigated the severity of the crisis by fostering more prudent risk management. In addition to providing monitoring and control over the size and leverage of LTCM’s portfolio, greater transparency could have potentially reduced the concentrations of risk that many institutions were exposed to. Specifically, many institutions were vulnerable because they held highly correlated assets in their proprietary and loan portfolios that simultaneously experienced losses. The crisis opened the question of whether regulation might be needed to foster transparency between leveraged investment institutions and their creditors. The Basel Committee’s report proposed fostering both bilateral disclosures to specific creditors and public disclosures to the entire market. Their proposal for implementing the former was through either the bank supervisory process or the formation of a credit register with centralized information on the exposures of leveraged institutions, which could be selectively released to relevant counterparties. Our paper focuses on this brand of disclosure, i.e., bilateral disclosures between financial institutions with a credit relationship.

The “unraveling” argument of Milgrom (1981) and others says that such regulation is unnecessary because there ought to be full voluntary disclosure in the absence of regulation. The argument is that better types will always want to disclose to differentiate themselves from worse types so that all types will become known in equilibrium. The argument applies when there are no costs associated with disclosure.

This paper posits that the main barrier or cost to information sharing by leveraged investors is that disclosed information can often be exploited through competitive trade in the markets. If the disclosure reveals information regarding an investor’s proprietary trading strategies, for example, the creditor (typically a bank or securities firm) can often duplicate this strategy and extract profits. We refer to this duplication as front-running. Anecdotally, LTCM and other hedge funds clearly saw the threat of front-running as a primary motive to withhold information. Lowenstein (2000) documents that “[John Meriwether] felt that investment banks were rife with leaks and couldn’t be trusted not to swipe his trades for themselves. Indeed, most of them were plying similar strategies. Thus, as a precaution, Long-Term would place orders for each leg of a trade with a different broker. Morgan would see one leg, Merrill Lynch another, and Goldman yet another, but nobody would see them all.” As its first recommendation for enhancing information sharing between counterparties, an industry-formed policy group on counterparty risk management (the Counterparty Risk Management Policy Group or CRMPG) has proposed improving “information barriers between the firm’s traders... and the credit risk managers who determine counterparty credit” because of “the intensely competitive nature of the relationship between credit providers and credit users in aspects of their respective market businesses.” Such “Chinese walls” have existed in the past but have clearly not been entirely effective at containing information leakage.

In this paper, we develop a theory of disclosure in the presence of front-running costs. We seek to understand first, the conditions under which a leveraged investor will voluntarily disclose information and second, the conditions under which a regulator ought to alter free-market disclosure practices to improve social welfare. Although we have used the LTCM crisis to motivate these questions, we do not attempt to capture the features of LTCM per se in our model. We attempt to answer these questions with a more general
model since LTCM was atypical of hedge funds in many respects in its dealings with counterparties and capital providers. ¹

We study these issues in a stylized model where an investor’s market exposure is unknown and can be disclosed to heterogeneous lenders with different preferences for this exposure. In our model, an investor (e.g., a hedge fund manager) has information about a profitable but risky investment opportunity in one of two possible markets. The investor has limited capital and needs a loan from the banking sector to make the investment. Banks have existing risky assets with heterogeneous exposures to the possible markets of the investment. These banks also have concave objectives as a result of risk-management motives and consequently want to avoid new correlated risks.

The investor may obtain favorable terms of lending by disclosing her investment strategy, thereby revealing its correlation to the lender’s existing assets. The cost of disclosure is that the lender can front-run and attempt to duplicate the strategy. We find that, in equilibrium, the investor may choose disclosure or non-disclosure depending on parameters in the economy. These parameters affect the relative cost of front-running versus the interest rate on the loan. Specifically, we find that there is non-disclosure for sufficiently high NPV investments and bank risk tolerance as well as high investor capitalization when the choice of lenders is limited. If investors disclose, they will not, in general, prefer to disclose to a lender with minimum correlation and highest tolerance to the investment’s risk. Although a bank with lower correlation will offer a superior interest rate, it will also front-run the investor more aggressively because it views the risk more favorably.

We next analyze the welfare effects of disclosure regulations. Such regulations should be designed to prevent risk concentrations and other conditions that may lead to systemic market and institutional failures. We obtain the surprising result that disclosure regulation results in a loss of welfare if the investor has access to a wide spectrum of lender risk exposures. If compelled to disclose, the investor will actually prefer a lender with higher and not lower correlation in order to suppress front-running. Hence, disclosure regulations will exacerbate concentrations of risk and increase lending costs in the economy.

The existing literature on disclosure is extensive. We should first mention the papers of Jovanovic (1982), Verrecchia (1983), and Dye (1986), which analyze disclosure with costs. In these papers, disclosures have an exogenous and “deadweight” cost whereas in ours, the cost of front-running is an endogenous transfer from one party to another. Our paper is most related to those of Campbell (1979), Bhattacharya and Ritter (1983), Ueda (2004), and Yosha (1995), which study optimal disclosure policies when there are exploitation costs associated with disclosure. These papers deal with disclosures by firms about their real investments that competitors can exploit in the product market. Our paper is specific to disclosures between financial institutions concerning financial market information. Exploitation costs are particularly relevant in financial markets because of the ease of duplication and the inefficacy of legal protections for proprietary information. In addition,

¹For instance, a principal issue regarding LTCM was that it took leveraged positions with a multitude of counterparties and that no single counterparty was aware of the magnitude of LTCM’s positions with other counterparties. In our model, we deal with a single leveraged investor and a single lender, ignoring the issue of combining information from many different lenders. We thank an anonymous referee for bringing these issues to our attention.

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the financial market setting with endogenous prices yields a set of implications distinct from the product market setting.

This paper proceeds as follows. Section 1 motivates some of our modeling assumptions with accounts of actual institutional practices. Section 2 describes the setup of the model. Section 3 describes the equilibrium in a benchmark case where there is no front-running. In this case, disclosure is costless, and there is full disclosure in accord with the Milgrom argument. In Section 4, we consider the equilibrium with front-running but where disclosure is mandatory. This case is intended to isolate the investor’s choice of lender correlation. Section 5 discusses the voluntary equilibrium and examines the investor’s choice of disclosure versus non-disclosure. In Section 6, we examine the effects of regulation on disclosure policies and welfare. In Section 7, we study a few variations and extensions of our model. Section 8 concludes.

1. Motivation

Prior to formal presentation of the model, we begin with an exposition to motivate some of our modeling assumptions. First, our model assumes that correlation between a lender’s existing assets and a leveraged investor’s trading strategy is a primary consideration in risk assessment. Institutional practice leads us to believe this assumption to be reasonable. For example, CRMPG’s policy recommendations emphasize the importance of concentration or correlation analysis, stating that risk-management “information should highlight possible concentrations of market and credit risk resulting from positive correlation among the firm’s own principal positions, counterparties’ positions with the firm and collateral received or posted.”

A related critique is that our model also assumes that correlation decreases front-running. One could imagine, however, that a bank with higher correlation would conceivably front-run more than a bank with lower correlation since it is more highly invested in similar assets and could possess differential expertise in that market. We would like to highlight the fact that our analysis can be adapted to more general conditions that cause banks to have heterogeneous preferences for the investor’s risk, not necessarily related to correlations. Various characteristics may drive these heterogeneous preferences, including differing expertise and financial technology, capitalization, number of lenders in a syndicate, etc. With heterogeneity in these characteristics, mandating disclosure is harmful because the investor will choose a lender for whom the risk is costly in order to suppress front-running. Hence, our welfare conclusions can maintain their validity outside the specific form of the model presented here.

Our results simply require that front-running and good lending terms be associated with one another. There is anecdotal evidence that intermediaries can offer attractive lending terms in order to benefit from viewing the trading activities of counterparties. For example, Lowenstein (2000) documents that Merrill Lynch’s intent in financing LTCM was precisely this, stating that “Merrill salesmen . . . rationalized that financing LTCM was the key to being able to trade with the firm and that trading would enable Merrill to see Long-Term’s order flow.” Furthermore, it was well known that “investment banks that also operate

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2We present an alternative model in a previous draft of this paper (Ko, 2001) where the heterogeneity arises from differing financial technology in the market. The results of this model are identical to the ones presented here.
proprietary bond-trading desks ... publicly boasted exploiting their knowledge of 'customer flow.'” Hence, front-running and good lending terms were often conjoined.

Finally, our model assumes that the investor either fully discloses the contents of her portfolio or discloses nothing. In reality, disclosures of general asset allocations absent information about specific trading strategies could reveal information about correlations that is not overly prone to front-running. However, even disclosures that reveal only general or aggregated information is not immune to exploitation as noted by CRMPG’s policy document. Furthermore, this type of general disclosure is often insufficient to reveal the true risk profile of a leveraged financial institution. The CRMPG document states that “…the Group recognizes the particular sensitivity of information about specific portfolio positions held by a counterparty … Nevertheless, there will be cases in which credit providers will and should feel a need for regular access to that type of information—in effect moving further along the transparency continuum—in order to manage properly more credit intensive activities, larger than normal exposures or exposure to high risk counterparties.”

2. Model setup

The model has three dates. At date 0, lending contracts are established and disclosures are made. At date 1, investments are made. At date 2, assets pay off, and all agents consume.

2.1. Assets

There are three assets in the economy. The first is a riskless asset with a zero rate of interest. We refer to this asset as “cash”. The other two are risky assets with payoffs specified below.

- Asset \( x \) has payoff: \( \tilde{V}_x \cdot \tilde{X} \).
- Asset \( y \) has payoff: \( \tilde{V}_y \cdot \tilde{Y} \).
- \( \tilde{V}_x, \tilde{V}_y, \tilde{X}, \text{ and } \tilde{Y} \) are binary and independent random variables. \( \tilde{V}_x \) and \( \tilde{V}_y \) have outcomes of \( V \) or 0 with equal probability. \( \tilde{X} \) and \( \tilde{Y} \) have outcomes of 2 or 0 so that \( E[\tilde{X}] = E[\tilde{Y}] = 1 \). The factors and assets are shown in Fig. 1.

\( \tilde{V}_x \) and \( \tilde{V}_y \) are private information and represent the conditional expected value of the risky assets. \( \tilde{X} \) and \( \tilde{Y} \) are fundamental risk factors and no agents have information about them.

![Fig. 1. These four random variables are independent, and all outcomes above are equally likely.](image)
2.2. Agents

There are three types of agents.

- **Investor:** There is one investor who is of either type \( x \) or type \( y \) with equal probability. The investor’s type is private information. An investor of type \( x \) observes \( \hat{V}_x \), and an investor of type \( y \) observes \( \hat{V}_y \).
  
  The investor is endowed with a small amount of cash: \( A \). We also refer to \( A \) as the investor’s capital.

- **Banks:** There is a continuum of banks with heterogeneous exposures to the fundamental risk of the assets. We model the bank’s endowment as a large amount of cash plus a risky payoff exposed to either \( \tilde{X} \) or \( \tilde{Y} \). Specifically, the bank’s terminal payoff is \( W + \tilde{\alpha} \cdot \tilde{X} + (1 - \tilde{\alpha}) \cdot \tilde{Y} \) where \( \tilde{\alpha} \) is independent of all other random variables and equal to 1 with probability \( \rho_x \) and 0 with probability \( \rho_y = 1 - \rho_x \). A continuum of banks with each correlation parameter ensures that these banks will act competitively in offering lending terms. We assume that \( \rho_x \) is common knowledge and has a distribution in the economy with support of \([0, 1]\) although we relax this assumption in Section 7.3.

  Banks are risk-averse and have a monotone and concave utility function, \( u \). This utility function does not represent the utility of an individual investor in the bank, but is instead a reduced form for the institution’s risk-management policy. Froot et al. (1993), among others, show how frictions such as costly access to external capital markets can concavify a firm’s objective function.4 In our analysis, we will utilize the following measure of risk aversion in our binary setting, i.e., the fractional difference in marginal utility between the high and low wealth outcomes of a bank’s risky endowment, given by the parameter \( k \):

  \[
  k = \frac{u'(W) - u'(W + 2q)}{u'(W + 2q)}
  \]

  This parameter measures the difference in slope between low and high wealth outcomes for the bank and reflects the additional cost of losses in the low wealth state. In this paper, we use a power utility function for simplicity. Finally, banks do not exogenously receive the private information of investors, \( \hat{V}_x \) or \( \hat{V}_y \).

- **Counterparty:** There are two counterparties. Counterparty \( x \) is endowed with one unit of asset \( x \), and counterparty \( y \) is endowed with one unit of asset \( y \). Counterparties have a private value for assets \( x \) and \( y \). Specifically, they achieve a payoff of \( V' \) from

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3 An alternative modeling choice would be for the bank’s risky endowment to be a linear combination of \( \tilde{X} \) or \( \tilde{Y} \) with deterministic weights. The results of this model are the same except that it does not allow for the linear approximation employed in this paper and consequently does not yield analytic expressions. With these heterogeneous fixed endowments, we are assuming a market with frictions such as adverse selection that does not allow for perfect risk sharing between banks.

4 In a comprehensive model, the bank’s objective function should be endogenous because this function is determined endogenously by capital structure and investment decisions, the cost of capital, etc. We assume, however, that the investment is not large relative to the bank’s existing assets so we can simply take this objective function as given. In addition, we assume that other agents in the model are risk-neutral for simplicity since the absence or presence of their risk management does not affect our results.

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assets $x$ and $y$ in all states where $V'$ satisfies the following inequalities:

$$V > V' > \max\left\{ \frac{V}{2}, \Delta \right\}$$

(2)

The right inequality first ensures that the investor’s capital alone is insufficient to make an investment in the asset and that external finance is needed as a result. The left inequality implies that there is a gain from trade when $\hat{V}_j = V' (j \in \{x, y\})$. In this state, the counterparty should sell the asset to the other agents since they have a higher expected value for the asset than the counterparty. When $\hat{V}_j = 0$, however, there is no gain from trade since the counterparty values the asset more. There is also no gain when the agent has no information since $V'$ is greater than the unconditional expected value of the asset, i.e., $V' > V/2$. One motivation for this private valuation comes from tax considerations. The counterparty’s value can come from a higher tax rate that decreases both capital gains and capital losses, consistent with the fact that $V > V' > 0$. Anecdotally, it seems that it is not uncommon for institutions to face restrictions that prevent them from realizing the full value from financial assets in certain states of the world and that these restrictions can generate substantial profits for arbitrageurs. For example, Edwards (1999) states, “Even staunch believers in efficient markets will readily admit that price inefficiencies may exist when regulations restrict the flow of capital into particular financial sectors or into particular investment strategies. For example, the short-sale portfolio restrictions imposed on institutional money managers may create an opportunity that hedge funds can exploit.”

In this paper, we use a first-order approximation to compute the bank’s utility from asset and loan payoffs. Hence, we implicitly assume that the risk of the asset is small relative to the risk of the bank’s existing endowment, or that: $q \gg V$. This assumption is made only to obtain analytic expressions for prices and interest rates and is not critical to our results. We have verified through numerical analyses (available upon request) that our approximation is highly accurate even when the asset presents substantial risk to the bank.

We end this section by motivating our assumption that a bank’s type is common knowledge while an investor’s is not, which we make mainly for simplicity. The results of our model are consistent with one in which the bank’s type is private information and can be disclosed to the investor as well as vice versa. Banks would need to disclose their type to investors in trades where two-way variation collateral is required, and where both institutions are mutually exposed to the other’s credit risk. Such a model would be quite elaborate, unfortunately. Alternatively, as we mention in the Introduction, one can think of the bank’s risk aversion as being driven by characteristics such as differing financial technology in the market, which are more likely to be commonly known (as in Ko, 2001).

2.3. Timing and Actions

**Loan commitments**: The investor in this model requires financing to invest in either asset $x$ or $y$. We assume that this financing is provided by banks only through loan

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commitments. The contract is a one-sided loan commitment since the borrower has the option to take the loan or not at date 1. We exogenously take the size of the loan to be \( 1 - \Delta \), which, when added to the investor’s capital of \( \Delta \), gives the investor total assets of \$1. Giving borrowers and lenders choice over the size of the loan changes no results of the model.

The contract specifies the face value of the loan, i.e., the maximum claim that the lender has on the borrower’s assets at date 2: \( R \) (we abuse terminology slightly and also refer to \( R \) as the loan’s interest rate). All claims are limited liability. It can be shown that parties will endogenously choose debt without having to make an exogenous restriction since debt claims are optimal from risk-sharing considerations. It is likely that in other reasonable models, banks will have an incentive to offer equity contracts to curtail their own incentive to front-run. However, it may still be optimal for the investor to retain a substantial portion of the equity in order to provide ongoing incentives to monitor and maintain profitable positions.

The terms of the loan are negotiated prior to the arrival of the investor’s information: \( \tilde{V}_j \). Thus, this model captures the common practice of negotiating a credit facility prior to the identification of a specific trade. The results of the model are nearly identical if the loan is negotiated after the information arrives (except that this model adds a minor technical complication).

Date 0: Prior to date 0, all agents observe their type. At date 0, the following game is played to determine the terms of the loan.

1. The investor commits to disclosure or non-disclosure and solicits potential interest rates from lenders.
2. If the investor commits to disclose, each bank offers interest rates of \( R_x \) and \( R_y \) if the investor is of type \( x \) and type \( y \), respectively. Otherwise, each bank offers an interest rate of \( R_N \) if the investor has committed to not disclosing her type.
3. The investor chooses an offered interest rate from at most one bank for the loan.

The investor (of type \( j \)) then observes \( \tilde{V}_j \). If the investor has agreed to disclose, the lending bank observes the investor’s type and signal.

The timeline is shown in Fig. 2.

Date 1: Trade at date 1 occurs in first-price sealed-bid auctions for assets \( x \) and \( y \).

1. The investor and banks may simultaneously submit bids for both auctions: \( p_x \) and \( p_y \).
2. Counterparty \( x \) can choose to sell asset \( x \) to the highest bidder for this asset at the highest price and similarly for counterparty \( y \). For simplicity, we assume that ties go to the investor.

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6We can endogenize banks as the sole lenders in the economy by assuming the existence of investors with high-risk, negative-NPV investments that only banks can distinguish. In addition, the results of the model are the same if the contract is a loan (that is always taken) and not a credit line, but the analysis is more complicated.

7If the asset were divisible, a third option would be for the investor to not take a loan and make the investment from existing capital, \( \Delta \). It can be shown, however, that the investor will always prefer taking a loan to make the investment. This proof is available upon request.

8Even if the disclosure is made after the trade is executed, front-running is still an issue for several reasons, one of which is that many types of positions often take weeks or months to accumulate according to the author’s informal discussions with hedge fund traders.
The counterparty will, of course, accept the bid only if it is no less than his private value, \( V_0 \).

Date 2: All assets pay off, and agents consume.

This model admits the possibility that the investor can sell her information to the bank and commit not to front-run by giving its capital to the bank at date 1 in exchange for a fee paid at date 2. This possibility is uneconomic since hedge funds generally do not sell their information for reasons such as the fact that they possess some intellectual capital essential to the management of their portfolio. Consequently, we simply rule out this possibility by assuming that banks are the only lenders in the economy.

2.4. Claims and objective

Suppose the investor takes a loan with face value \( R \), and buys asset \( j \in \{x, y\} \) for an amount \( p \), when \( \tilde{V}_j = V \) (the investor will not buy the asset if \( \tilde{V}_j = 0 \) since it pays off zero with certainty). The investor’s total assets consist of cash of \( 1 - p \) plus the risky asset. These provide a gross payoff at date 2 of: \( 1 + V \cdot \tilde{J} - p \) \((J \in \{X, Y\})\). The investor takes an equity claim on this payoff, and the bank takes a debt claim.

The bank’s net payoff is as follows:

\[
\min\{1 + V \cdot \tilde{J} - p, R\} - 1 + \Delta
\]

since the bank lends \( 1 - \Delta \) at date 1 and takes a debt claim on the date 2 payoff. This debt is risky and exposed to factor \( \tilde{J} \) since the investor may default in the low state when \( \tilde{J} = 0 \).

The investor’s net payoff on her equity claim is given as follows:

\[
\max\{1 + V \cdot \tilde{J} - p - R, 0\} - \Delta
\]

The investor will, therefore, attempt to minimize the objective: \( p + R \). This objective is the price paid in the auction plus the face value of debt or, roughly speaking, the investor’s front-running plus interest rate costs. In our discussion of equilibrium, our focus will be on the investor’s choice of lender type and disclosure policy, i.e., the investor can choose the correlation of the lender and whether to disclose or not. In these choices, the investor trades off interest rate versus front-running costs.

We now analyze the equilibria of different versions of this game.
3. Benchmark: equilibrium with no front-running

In this section, we consider the simplest benchmark case, one in which banks cannot participate in the auction at date 1 and consequently, cannot front-run the investor. Disclosures are costless, therefore, and investors ought to voluntarily disclose their information in accord with the Milgrom unraveling result. In addition, investors should match with uncorrelated lenders since these lenders are most tolerant to the investor’s risk and can offer minimum interest rates.

We start by considering the investor’s trade at date 1. The investor has already procured terms for a loan at date 0. The investor will attempt to trade as long as she can earn positive profit from the loan. If $E\{1 + V \cdot f_1 + V / C_1 \cdot e^J / C_0 \cdot V_0 / C_0 \cdot R_0 \cdot e^v \}$, the investor will buy the asset by bidding the minimum price in the auction, $V_0$. If $e^v = 0$, the investor will not buy the asset because it pays off zero with certainty.

We now analyze the interest rates offered by banks at date 0. In this paper, we consider equilibria in which banks offer their reservation interest rates for $R_x$, $R_y$, and $R_N$. Hence, lenders will offer disclosure interest rates in equilibrium that depend only on their correlation ($\rho$) with that type of investor. We denote this equilibrium disclosure rate by $R^*_\rho$ (i.e., banks with $\rho = \rho$ will offer $R_x = R^*_\rho$ and $R_y = R^*_{1-\rho}$, while banks with $\rho = \rho$ will offer $R_x = R^*_{1-\rho}$ and $R_y = R^*_\rho$). We assume throughout this paper for simplicity that the correct inference for non-disclosure is that the investor has equal probability of being either type. This assumption can be justified by introducing an infinitesimal cost that the investor must pay to learn the type of the bank. Investors will not pay this cost if they choose non-disclosure because there is no reason to differentiate between different types of lenders (since there is no difference in their front-running). Hence, both types $x$ and $y$ investors will mix equally among different banks offering the minimum interest rate, and lenders must infer that the investor has a correlation 0 or 1 with equal probability.

The following are the equilibrium interest rates for disclosure by an investor of correlation $\rho$ ($R^*_\rho$) and for non-disclosure ($R^*_N$):

$$R^*_\rho = 1 - V' + 2(1 + \kappa_\rho)(V' - \Delta)$$

$$R^*_N = 1 - V' + 2(1 + \kappa_{1/2})(V' - \Delta)$$

(5)

where

$$\kappa_\rho = \frac{k\rho}{2 + k(1 - \rho)}$$

(6)

These competitive interest rates can be understood from a decomposition of the investment assets. The total assets can be decomposed into two parts: $1 - V'$ of the assets are kept as riskless cash and the remainder is invested in the risky asset. The investor has borrowed an amount, $1 - \Delta$, which needs to be repaid. $R^*_\rho$, therefore, consists of two parts. The first is the riskless repayment of $1 - V'$, and the second is the risky repayment of $V' - \Delta$. This second term is multiplied by 2 because it is received with probability $\frac{1}{2}$. This term is also discounted at a rate of $\kappa_\rho$, which is increasing in the bank’s risk aversion, $k$, and its correlation, $\rho$, both of which make the risk more costly to the bank. For non-disclosure, the bank’s discount rate is $\kappa_{1/2}$ because the banks infer a correlation of $\frac{1}{2}$, i.e., the investor has a correlation of 0 or 1 with equal probability.
At date 0, the investor will obviously disclose and select a bank with a correlation of zero in order to obtain the minimum interest rate (since there are no front-running costs to consider) as stated in the following proposition:

**Proposition 1.** With no front-running, the investor in equilibrium chooses disclosure to a bank with correlation of zero at an interest rate of \( R^*_0 \).

**Proof.** See Appendix A.

Hence, there is full disclosure in a benchmark where disclosure is costless, in accord with the Milgrom argument, as well as optimal risk matching with uncorrelated lenders.

4. Mandatory disclosure equilibrium

We now add the feature that banks can front-run and start by considering the situation in which the investor’s disclosure is mandatory because of regulation, for example. In this case, the investor can choose the correlation of the lender to whom she will disclose her information. As before, a bank with lower correlation will offer a lower interest rate than a correlated bank because it views the risk more favorably. This bank will also front-run the investor more aggressively for the same reason, however. This tradeoff makes the choice of lender correlation a priori unclear.

We denote the equilibrium interest rate offered by a bank with correlation of \( \rho \) by \( R^*_\rho \). We denote by \( p^*_\rho \) the price paid in the auction if the investor discloses to a bank with correlation \( \rho \).

We again work backwards and analyze the market at date 1. Banks can now bid against the investor in the auction, and they will do so only if they observe the investor’s information because of inequality 2. As before, parties will only bid in the auction if \( \bar{V}_j = V \).

We first consider what happens when there is disclosure but the investor has sufficient capital to make the investment and does not borrow from a bank. The investor can no longer necessarily win the auction by bidding \( V' \) since the lender will compete by bidding its reserve price, i.e., the price that makes the lender’s NPV for the investment equal to zero. To win the auction, the investor must bid the lender’s reserve price or \( V' \) (the counterparty’s reserve price), whichever is higher. A bank with correlation of \( \rho \) will discount the expected value of the asset, \( V \), by a rate of \( \kappa_\rho \) as defined in Eq. (6). Hence, its reserve price is

\[
\bar{p}_\rho = (1 + \kappa_\rho)^{-1} V.
\]

If the investor discloses to a bank with correlation of \( \rho \), the equilibrium no-loan auction price is given by

\[
p^*_\rho = \max\{V', (1 + \kappa_\rho)^{-1} V\} \tag{7}
\]

If there is a loan commitment, the bank has a reserve price in the auction that makes it indifferent between buying the asset itself at that price and making the loan to the investor. At date 0, the bank will offer competitive interest rates, which makes the equilibrium

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9The lender will bid in the auction in spite of the fact that it cannot win and doing so extracts value from the loan because the auction is closed-bid. Such expropriation does not, however, rely on a fragile set of assumptions, but happens generally whenever a partial claimant has the potential to exploit the information of the firm. For similar models of expropriation, see Tirole (1998).
auction price at date 1 equal to the no-loan price given above. The reason is that the bank will compare buying the asset to a loan of zero NPV just as in the absence of a loan.

At date 0, the competitive interest rates are given by

$$R^*_p = 1 - p^*_p + 2(1 + \kappa_p)(p^*_p - \Delta)$$

These interest rates can be understood using the same decomposition as in the previous section. The $1 - p^*_p$ term represents a riskless repayment and the $p^*_p - \Delta$ term represents a risky repayment, which the correlated bank discounts by a factor that depends on $k$. Since the investor chooses a bank to minimize $p^*_p + R^*_p$, the following proposition holds:

**Proposition 2.** If disclosure is mandatory, the investor will choose a bank with correlation $\rho^* = \min\{1, \frac{1}{1 + \frac{2}{k}(1 - V'/V)}\}$.

**Proof.** See Appendix A.

A bank with correlation of $\rho^* = \frac{1}{1 + \frac{2}{k}(1 - V'/V)}$ has a reservation value for the asset exactly equal to $V'$, i.e., $\hat{\rho}_{\mu^*} = V'$. The investor, therefore, wants to borrow from a lender with correlation just high enough that front-running goes to zero. If the lender’s correlation is any lower, so that there is positive front-running (i.e., $\hat{p}_{\rho} > V'$), the investor gains by increasing the correlation of its lender. Though this additional correlation results in a loss from increased interest rates, it results in an even greater gain from decreased front-running. The gain is greater because it comes from a discount on the total investment whereas the loss comes from a discount on the loan, which is only a partial investment in the asset. Once $\hat{p}_{\rho} = V'$, the investor will not seek a bank with higher correlation since increasing correlation beyond this point will only increase interest rates without decreasing front-running. If $(1 + \frac{2}{k})(1 - V'/V) > 1$, the investor cannot find a lender with correlation high enough to fully suppress front-running and will simply choose a lender with the highest possible correlation of 1. The fact that the investor wants a lender that does zero front-running simplifies the analysis that follows.

5. Voluntary equilibrium

We now consider the equilibrium in a free market where investors have the choice of disclosure versus non-disclosure. We denote by $p^*_N$ and $R^*_N$, the price paid in the auction at date 1 in the case of non-disclosure and the interest rate for non-disclosure, respectively.

If the investor does not disclose her information regarding the asset’s value, the lender will not bid in the auction at date 1 (as a result of inequality 2). The investor can, therefore, buy the asset at an equilibrium price of

$$p^*_N = V'$$

Since there is no front-running, the competitive interest rate for non-disclosure is consequently the same as in the case of no front-running as given in

$$R^*_N = 1 - V' + 2(1 + \kappa_{1/2})(V' - \Delta)$$

If the investor commits to disclose at date 0, the equilibrium offered interest rates and auction prices are as in Section 4. As before, she wants to choose a lender with correlation, $\rho^*$, to suppress front-running (i.e., $\rho^*_{\mu^*} = V'$). In this case, front-running costs will be equal for both disclosure and non-disclosure, and only their respective interest rate costs need be
compared. Hence, the investor only needs to compare the discount rates associated with disclosure and non-disclosure or alternatively, the respective correlations inferred by the lender (ρ* and \( \frac{1}{2} \)) which determine this rate. In the case where ρ* = 1, the investor will choose non-disclosure since both the front-running and interest rate costs of disclosure are higher when the investor discloses to bank of correlation 1. We, therefore, have the following proposition:

**Proposition 3.** The investor will choose non-disclosure if \( \rho^* = (1 + 2/k)(1 - V'/V) \geq \frac{1}{2} \) and otherwise disclose to bank with correlation of ρ*.

**Proof.** See Appendix A.

The investor’s decision can also be characterized in terms of model parameters as follows:

**Corollary 1.** There exist \( (V/V')_d \) and \( k_d \) such that the investor will choose non-disclosure if and only if \( V/V' \geq (V/V')_d \) and \( k \leq k_d \).

Corollary 1 says that the investor will not disclose if the NPV of the investment (as captured by \( V/V' \)) is sufficiently high. The investor will avoid disclosure because front-running is especially costly for a high NPV investment. To be accurate, an investor with such an investment must disclose to a bank with high correlation to suppress front-running, which results in a high interest rate for disclosure.

The investor will also opt to disclose if banks’ risk-aversion parameter, \( k \), is sufficiently high. The reasoning here is that higher \( k \) increases the interest rate for non-disclosure by more than the relevant interest rates for disclosure because banks are more averse to non-transparent than transparent risks. To be precise, the investor will never voluntarily disclose to a lender with correlation greater than \( \frac{1}{2} \) because doing so would lead to both higher front-running and interest rate costs than non-disclosure. Non-disclosure, therefore, leads to higher correlation and greater sensitivity to risk aversion than the marginal choices of disclosure. Hence, an increase in \( k \) has a greater impact on increasing the interest rate for non-disclosure than the marginal interest rates for disclosure.

6. Regulations and welfare

In this section, we analyze the effect of regulation on welfare in our economy. We assume that the regulator can set overall guidelines pertaining to disclosure policy, capitalization, and risk management but cannot control the specific lending activities and assets of institutions in the economy. The assumption here is that monitoring of specific activities is infeasible either because of high direct costs or indirect costs such as information leakage.

We should mention that our welfare analysis omits various relevant factors including asymmetric information between the investor and external capital providers or between banks (e.g., that generates heterogeneous endowments) and tax motives for trade which might affect external claimants such as the government. Our analysis is, therefore, limited in scope in that we only consider front-running and its effect on risk matching between
borrower and lender. We view our results as indicative, therefore, and a starting point for a more comprehensive analysis, which would incorporate the effect of other factors as well as study the effect of regulation on parties outside of our model.11

In this section, we focus on Pareto analysis as well as an analysis of social surplus in the economy as given by the sum of investor and counterparty expected profits: \( \Pi_i + \Pi_{cp} \). The lender’s utility does not enter into this division of surplus since banks act competitively and realize only their reservation utility in equilibrium. The investor and counterparty’s profits are given as follows when \( \bar{V}_j = V \) (profits are zero when \( \bar{V}_j = 0 \):

\[
\Pi_i = E[\max\{1 + V \cdot \bar{J} - p^* - R^*\} - \Delta] = \frac{1}{2}[1 + 2V - p^* - R^*] - \Delta = V - p^* - \kappa(p^* - \Delta) \tag{11}
\]

\[
\Pi_{cp} = p^* - V'
\]

where we have substituted the equilibrium costs: \( p^* + R^* = 1 + 2(1 + \kappa)(p^* - \Delta) \). Hence, social surplus is given by

\[
\Pi_i + \Pi_{cp} = V - V' - \kappa(p^* - \Delta) \tag{12}
\]

where \( V - V' \) reflects the risk-neutral gain from trade, \( \kappa \) the bank’s discount rate for the risk in equilibrium, and \( p^* - \Delta \) the bank’s net investment in the risky asset. Total surplus, therefore, captures the gains from the transaction net of risk-related costs borne by the lender. It is clear from this expression that social losses stem from increased correlations and risk exposures of the lender.

The following proposition summarizes the effect of disclosure regulation on welfare:

**Proposition 4.** If \( \rho^* < 1 \), disclosure regulation is Pareto-suboptimal.

If \( \rho^* = 1 \), disclosure regulation results in a loss of social surplus.

If \( \rho^* < 1 \), our welfare analysis is simple since the investor discloses to a bank that does zero front-running. In this case, the equilibrium auction price is equal to the counterparty’s reservation value for both disclosure and non-disclosure, i.e., \( p^*_\rho = p^*_N = V' \). The transaction, therefore, always results in zero gain for the counterparty as well as zero gain for the lender because of competition. As a result, social welfare is determined entirely by the investor’s profit. Changing the disclosure regime results in a loss for the investor because she chooses this regime to maximize her expected profit. This loss is a Pareto loss since the utility of the counterparty and the lender remain the same in any regime.

If \( \rho^* = 1 \), on the other hand, disclosure results in non-negative front-running since \( p^*_\rho \geq V' \). Hence, mandating disclosure is no longer necessarily Pareto-suboptimal since it will result in higher auction prices and a gain for the counterparty. Mandating disclosure will, however, result in a loss of social surplus since it increases correlations and the risk of the loan. Specifically, the investor will disclose to a bank with correlation of 1 so that borrowing costs increase from \( \kappa_{1/2}(V' - \Delta) \) to \( \kappa_1(p^*_\rho - \Delta) \).

In either case, disclosure regulation results in a loss of welfare and an associated increase in asset correlations or concentrations of risk. For example, if \( \rho^* < 1 \), the investor chooses

11Our model also omits the investor’s incentive to gather information from our analysis. If we incorporate this incentive into our model, regulations have the dual effect of decreasing the investor’s profit and the investor’s incentive to gather information. For this reason, regulation simply becomes less attractive than before, and our arguments for suboptimality become even stronger.
the disclosure regime to minimize correlation with the lender, thereby minimizing her interest rate costs. Any external change in the regime increases risk concentrations and consequently decreases welfare.

We should clarify that these results are derived under the assumption that the investor has access to the entire spectrum of lender correlations from 0 to 1. In the next section, we relax this assumption and find that mandating disclosure can actually increase surplus and decrease concentrations of risk under certain conditions. Our results generally indicate that regulators should consider the incentive of leveraged investors to curtail front-running by matching with correlated and other suboptimal lenders. Regulators should also be aware of the conditions under which these considerations matter, one of which is a wide dispersion in risk attitudes across lenders. From an observational standpoint, a regulator could observe the range of lending terms offered as a way of gauging the range in risk attitudes. We discuss further conditions in the next section.

7. Extensions

In this section, we develop two extensions of our model. The first extension examines the case in which the investor has a limited choice of lender correlations. We then analyze a natural extension in which there is variation not just in lenders’ correlations but also in their risk aversion.

7.1. Limited choice of lender correlation

In this section, we analyze the case where the investor has limited choice of lender type and find that mandating disclosure now has the potential to increase surplus and decrease concentrations of risk. Our model remains identical in all respects except that the distribution of lender correlations no longer necessarily has support of [0, 1].

Suppose that the investor chooses a bank with optimal correlation of \( \rho^* \) (no longer necessarily equal to \( \min(1,(1 + (2/k)(1 - V''/V)) \)) if compelled to disclose. It is still true that if \( \rho^* > \frac{1}{2} \), the investor will prefer non-disclosure because it results in lower front-running and interest rate costs than disclosure. The converse, however, is no longer true. The investor may not disclose even if \( \rho^* < \frac{1}{2} \) since there may now be non-zero front-running with disclosure. Specifically, she may not be able to find a lender such that \( \bar{p} = V \) if there is a scarcity of lender correlations. The investor will choose disclosure versus non-disclosure based on parameter values according to the following proposition:

**Proposition 5.** There exist \( (V/V')_d, (\Delta/V')_d, \) and \( k_d \) such that the investor will choose non-disclosure if and only if \( V/V' \geq (V/V')_d, \Delta/V' \geq (\Delta/V')_d \) and \( k \leq k_d \).

**Proof.** See Appendix A.

This proposition is identical to Corollary 1 except that the disclosure decision now depends on the investor’s capitalization ratio, \( \Delta V' \). Decreasing the investor’s capital, \( \Delta \), increases interest rate costs because it increases the credit risk borne by the lender. It increases the cost of non-disclosure by more than that of disclosure for the same reason as before: banks are more averse to non-transparent risks than the relevant transparent risks. Previously, the front-running costs for disclosure and non-disclosure were equal so that the
investor simply needed to consider her respective discount rates and inferred correlations. Since these front-running costs can now differ, not only does the discount rate matter but also the size of the loan (as determined by $D$) in the comparison of dollar interest rate and dollar front-running costs.

The following proposition summarizes the effect of disclosure regulation on welfare:

**Proposition 6.** If $\rho^* > \frac{1}{2}$, mandating disclosure results in a loss of social surplus. If $\rho^* < \frac{1}{2}$, mandating disclosure can increase social surplus.

If $\rho^* > \frac{1}{2}$, mandating disclosure will increase correlations as well as lenders’ risk exposures so that borrowing costs will increase in Eq. (12). If $\rho^* < \frac{1}{2}$, the investor may no longer voluntarily disclose as just discussed. Hence, mandating disclosure has the potential to decrease correlations and risk exposures in the economy and increase surplus. Regulation, however, can also decrease surplus if $\rho^* < \frac{1}{2}$ because it can increase the bank’s net investment in the risky asset from $V' - \Delta$ to $p^* - \Delta$ in Eq. (12).

Mandating disclosure can be beneficial when the investor prefers disclosure to a low correlation lender. Since a lower correlation lender front-runs more and offers lower interest rates than a higher correlation lender, one can roughly deduce the conditions under which this preference arises. That is, front-running is less costly for low NPV investments and borrowing more costly for low investor capitalization and high bank risk aversion. A regulator, therefore, might rely on a “rule-of-thumb” which says that enhancing information sharing through regulations ought to be considered only when institutions are poorly capitalized, financially weak, and earn low rates of return on their investments.

Another interesting aspect of this version of the model is the impact of capital regulations. The Basel committee has proposed and discussed the possibility of imposing capital requirements on leveraged institutions as a means of averting financial crises. To this end, we consider a variation of our model where the investor has a convex cost of capital, $\Delta + C(\Delta)$, and $C(\Delta)$ represents any net costs of raising capital $\Delta$. For example, these costs can represent the net costs of external finance. The investor optimally chooses an amount of capital, $\Delta^*$, prior to date 0. In addition, we assume that the investor’s capital is observable to the banks (e.g., through a disclosure of NAV). With this added feature, the following statement can be made concerning capital requirements:

**Proposition 7.** Imposing a minimum capital requirement on the investor is Pareto-suboptimal.

When capital requirements compel the investor to hold additional capital above $\Delta^*$, there is a loss of welfare that comes from two sources. First, according to Corollary 1, additional capital can cause the investor to switch from disclosure to non-disclosure. Hence, the counterparty can lose rents from reduced front-running as a result of non-disclosure. Second, the investor also loses profits because the voluntary amount of capital, $\Delta^*$, results in optimal profits whereas any regulation pushes profits away from this optimum.
7.2. Variation in risk aversion

In Section 1, we mentioned the fact that our results apply even when lenders have general heterogeneous preferences for the risk of the investment. We now develop an extension of our model in which banks may have differing risk-aversion parameters, \( k \).

The following proposition summarizes the investor’s preference for lender risk aversion, \( k \), in terms of \( k_r = k\rho/(2 + k(1 - \rho)) \):

**Proposition 8.** The investor’s profit from disclosure is maximized for a lender with \( k^*_\rho = V/V' - 1 \), and this maximum profit is \( \Delta \cdot k^*_\rho \).

If the investor chooses non-disclosure, she will select a bank with minimum risk aversion, \( k_{\text{min}} \).

**Proof.** See Appendix A.

The first part of this proposition is simply a restatement of Proposition 2 for the case where there is variation in \( k \). Specifically, the above value of \( k^*_\rho \) makes this lender’s reserve price in the auction equal to the counterparty’s reserve value, \( V' \). In other words, the investor wants to disclose to a lender with risk aversion just high enough to suppress front-running. Reducing front-running to zero is again optimal because the gain from decreased front-running exceeds the loss from increased interest rates.

If the investor chooses not to disclose, she will borrow from a bank with minimum risk aversion, \( k_{\text{min}} \), to obtain the lowest interest rate. The investor, therefore, will choose disclosure over non-disclosure with respect to parameter values according to Proposition 5 except that the risk aversion that applies to non-disclosure is \( k_{\text{min}} \). Hence, the investor does not disclose if and only if \( k_{\text{min}} \not\leq k_d \).

Proposition 8 tells us that if there is disclosure, investors with higher NPV investments will tend to match with higher risk aversion and higher correlation lenders since \( k_r \) is increasing in \( k \) and \( \rho \). In addition, these investors will earn higher profits and rates of return on average. In a model without front-running, the investor’s profit would be strictly decreasing in \( k \) or the stringency of its lender’s risk management. This property is, therefore, a unique implication of a model with front-running since risk aversion can increase profits by decreasing front-running.

One can potentially develop empirical implications based on this property. The returns of leveraged investment institutions such as hedge funds can be positively related to the severity of their lenders’ risk-management policies. This relationship ought to be observed when investors disclose, i.e., when they maintain a high level of transparency with their financiers. We found that investors disclose when the NPV of their investments, their capitalization, or their lender’s risk tolerance is sufficiently low. Full development of the empirical issues related to these implications is outside the scope of this paper, and we reserve it for future research.

8. Conclusion

In this paper, we have studied disclosure by leveraged investors to their creditors when there are front-running costs. We have analyzed both voluntary and regulated disclosure in the context of a model of risk matching between institutions with heterogeneous risk exposures.
In our model, voluntary disclosure occurs for sufficiently low NPV investments, investor capitalization, and lender risk tolerance. Mandating disclosure results in a loss of welfare when it causes the investor to increase the inferred correlation with her lender in order to reduce front-running. One condition under which these concentrations of risk would arise occurs when there is a wide dispersion in risk attitudes and lending terms across lenders. Mandating disclosure can also be potentially beneficial when it results in the investor matching with a low correlation lender in order to avoid high interest rate costs. We argue that enhancing information sharing through regulation should be considered only when institutions are poorly capitalized and financially weak. We also study the effects of imposing minimum capital requirements on the risk matching and disclosure properties of our economy. We find that the effects were unambiguously negative for welfare and decreased the likelihood of voluntary disclosure.

One important manner in which to extend this research is to consider the impact of disclosure and regulation on agents outside the model. For example, information sharing can lead to front-running, which deteriorates the investor’s profit but can also improve the welfare of external parties by enhancing market liquidity and functioning. In addition, welfare losses in our model are the result of one lender’s utility loss from suboptimal risk matching. Our model does not capture welfare losses from the “systemic” risks that were the prime concern of regulators in the LTCM crisis. In particular, we do not capture the possibility that one institution’s distress could spill over into other institutions and markets, even ones not directly related to the original institution. In this respect, concentrations of risk were problematic in the LTCM situation because of the potential for “knock-on” effects whereby a liquidation of LTCM’s assets could have led to substantial losses for other institutions, possibly resulting in further defaults and liquidations. It would be interesting to extend the model to capture such effects. For example, one could incorporate these issues by introducing intermediate shocks in asset prices in an economy with multiple investors and lenders. To study the interaction of disclosure policy and lender choice with properties of asset prices including liquidity and volatility in crisis situations would be an interesting agenda for future research.

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Appendix A

We adopt the following notation:

\[ u_+(x) = u(W + 2q + x) \]
\[ u_-(x) = u(W + x) \]
\[ u_\pm(x) = \frac{1}{2}(u_+(x) + u_-(x)) \]
Suppose the price in the auction is given by \( p \). We define \( R^* \) as the reservation interest rates for a bank with correlation \( \rho \):

\[
\rho \cdot [\frac{1}{2}u_+(min(1 + 2V - p, R^*) - 1 + \Delta) + \frac{1}{2}u_-(min(1 - p, R^*) - 1 + \Delta)] + (1 - \rho) \\
\cdot [\frac{1}{2}u_+(min(1 + 2V - p, R^*) - 1 + \Delta) + \frac{1}{2}u_-(min(1 - p, R^*) - 1 + \Delta)] - u_+(0) \\
= \rho \cdot [\frac{1}{2}u_+(R^* - 1 + \Delta) + \frac{1}{2}u_-(p - R^*)] + (1 - \rho) \cdot [\frac{1}{2}u_+(R^* - 1 + \Delta) \\
+ \frac{1}{2}u_+(p + R^*) - 1 - u_+(0) = 0
\]

(14)

Applying our first-order approximation to the above equation yields

\[
\rho \cdot [u_+ + \frac{1}{2}u_+ \cdot (R^* - 1 + \Delta) + \frac{1}{2}u_- \cdot (-p + R^*)] \\
+ (1 - \rho) \cdot [u_+ + \frac{1}{2}u_+ \cdot (R^* - 1 + \Delta) + \frac{1}{2}u_- \cdot (-p + R^*)] - u_+ \\
= \frac{1}{2} [\rho u_+ + (1 - \rho)u_+] \cdot (R^* - 1 + \Delta) + \frac{1}{2} [\rho u_- + (1 - \rho)u_-] \cdot (-p + R^*) \\
= \frac{1}{2} u_+ \cdot \left[ 1 + \frac{k}{2}(1 - \rho) \right] \cdot (R^* - 1 + \Delta) + \frac{1}{2} u_- \cdot \left[ 1 + \frac{k}{2}(1 + \rho) \right] \cdot (-p + R^*) \\
\Rightarrow R^* = 1 - \Delta + \frac{1}{1 + (k/2)(1 - \rho)} (p - R^*) = 1 - R^* + 2(1 + \kappa_1)(p - R^*)
\]

(15)

where the variables \( u = u(0) \) and \( u' = u'(0) \) (also \( u' = u'(0) \)). Since banks infer \( \rho = \frac{1}{2} \) for non-disclosure, the reservation interest rate for non-disclosure is given as follows if the auction price is \( p_N \):

\[
R^*_N = 1 - p_N + 2(1 + \kappa_1/2)(p_N - \Delta)
\]

(16)

A.1. Benchmark equilibrium: no front-running

Proof of Proposition 1. Substituting \( p = p_N = V' \) (since there is no front-running) into Eqs. (15) and (16) confirms the competitive interest rates in Eq. (5). Banks will offer competitive rates since the investor’s utility is decreasing in their selected interest rate. Specifically, if the investor chooses non-disclosure at date 0, the competitive equilibrium is that banks will offer their reservation rate for non-disclosure of \( R^*_N \). If the investor commits to disclose at date 0, the competitive equilibrium is that banks with correlation of zero will offer their reservation rate of \( R^*_0 \). Banks with higher correlation cannot beat this rate because their reservation rate is higher. The investor will disclose to obtain the minimum interest rate, \( R^*_0 \) (since there are no front-running costs to consider), and, therefore, match an uncorrelated lender in equilibrium. □

A.2. Mandatory disclosure equilibrium

We first discuss the auction equilibrium in a hypothetical case where the investor has sufficient capital to buy the asset and does not take a loan. Suppose that a competing bank of correlation \( \rho \) observes that \( \tilde{V}_j = V \) (otherwise, the bank does not bid). This bank’s “no-loan” reserve price is denoted by \( \tilde{\rho} \) defined by

\[
\rho \cdot [\frac{1}{2}u_+(2V - \tilde{\rho}) + \frac{1}{2}u_-(\tilde{\rho})] + (1 - \rho) \cdot [\frac{1}{2}u_+(2V - \tilde{\rho}) + \frac{1}{2}u_-(\tilde{\rho})] - u_+(0) = 0
\]

(17)
In this auction, the investor wins by bidding \( p^*_\rho = \max(\tilde{p}_\rho, V') \) (since the investor is risk-neutral and has the highest reserve price). By our linear approximation

\[
\frac{1}{2} \left( p u^*_+ + (1 - \rho) u^-_+ \right) \cdot (2V - \tilde{p}_\rho) + \frac{1}{2} \left( p u^-_- + (1 - \rho) u^+_+ \right) \cdot (-\tilde{p}_\rho) \\
= \frac{1}{2} u^*_+ \cdot \left[ 1 + \frac{k}{2} (1 - \rho) \right] \cdot (2V - \tilde{p}_\rho) + \frac{1}{2} u^-_- \cdot \left[ 1 + \frac{k}{2} (1 + \rho) \right] \cdot (-\tilde{p}_\rho) = 0
\]

\[
\implies \tilde{p}_\rho = \left( 1 - \frac{k \rho}{2 + k} \right) V = (1 + \kappa_\rho)^{-1} V
\]

which confirms Eq. (7).

**Proof of Proposition 2.** One might guess that \( p^*_\rho \) is the equilibrium auction price at date 1 and that the bank offers its reservation interest rate given this price at date 0. To confirm this rigorously, however, we need first to analyze the auction outcome when there is a loan between the bidders. The interest rate affects how the different parties value their claim on the asset so that the equilibrium auction price depends on \( R; p^*_\rho(R) \). We also need to show that banks have the incentive to offer their reservation interest rate. Specifically, we will show that \( p^*_\rho(R) + R \) is increasing in \( R \) so that the investor prefers a lower interest rate.

First consider the auction with a loan of face value, \( R \), between the bank and the investor. The investor’s reserve price, \( \tilde{p}_\rho(R) \) is given by

\[
E[\max\{1 + 2V - \tilde{X}, \tilde{p}_\rho(R) - R, 0\}] = \Delta
\]

(19)

The bank’s reserve price, \( \tilde{p}_\rho(R) \), is given by

\[
\rho \cdot \left[ \frac{1}{2} u^*_+(\min\{1 + 2V - \tilde{p}_\rho(R), R\} - 1 + \Delta) + \frac{1}{2} u^-_-(\min\{1 - \tilde{p}_\rho(R), R\} - 1 + \Delta) \right] \\
+ (1 - \rho) \cdot \left[ \frac{1}{2} u^*_+(\min\{1 + 2V - \tilde{p}_\rho(R), R\} - 1 + \Delta) + \frac{1}{2} u^-_-(\min\{1 - \tilde{p}_\rho(R), R\} - 1 + \Delta) \right] \\
- \rho \cdot \left[ \frac{1}{2} u^*_+(2V - \tilde{p}_\rho) + \frac{1}{2} u^-_-(\tilde{p}_\rho) \right] \cdot (1 - \rho) \cdot \left[ \frac{1}{2} u^*_+(2V - \tilde{p}_\rho) + \frac{1}{2} u^-_-(\tilde{p}_\rho) \right] = 0
\]

(20)

since the bank compares winning the auction and owning the asset directly to losing and allowing the investor to take the loan. It is easy to show that \( \tilde{p}_\rho(R) \) is non-increasing in \( R \) since increasing \( R \) increases the value of the loan, which increases the needed return on the asset.

If \( \tilde{p}_\rho(R) \geq \max\{p^*_\rho(R), V'\} \), the investor will win by bidding: \( p^*_\rho(R) = \max\{p^*_\rho(R), V'\} \).

We will prove at the end of this section that \( -p^*_\rho(R) < 1 \) for \( V \) small enough. Therefore, the investor’s cost, \( p^*_\rho(R) + R \), is increasing in \( R \). In addition, the bank’s utility from a loan with rate \( R \) is increasing in \( R \) since \( p^*_\rho(R) \) is non-increasing in \( R \). Hence, the bank prefers loans with higher \( R \). Because of competition, banks will again offer their reservation interest rate, \( R^*_\rho \), in equilibrium, which is given by

\[
\rho \cdot \left[ \frac{1}{2} u^*_+(\min\{1 + 2V - p^*_\rho(R), R\} - 1 + \Delta) + \frac{1}{2} u^-_-(\min\{1 - p^*_\rho(R), R\} - 1 + \Delta) \right] \\
+ (1 - \rho) \cdot \left[ \frac{1}{2} u^*_+(\min\{1 + 2V - p^*_\rho(R), R\} - 1 + \Delta) + \frac{1}{2} u^-_-(\min\{1 - p^*_\rho(R), R\} - 1 + \Delta) \right] \\
- u^*_+(0) = 0
\]

(21)

If \( \tilde{p}_\rho \geq V' \), Eqs. (17), (20), and (21) are solved by \( \tilde{p}_\rho(R^*_\rho) = \tilde{p}_\rho \) (since \( p^*_\rho(R^*_\rho) = \tilde{p}_\rho \) in this case). If \( \tilde{p}_\rho < V' \), these equations imply that \( \tilde{p}_\rho(R^*_\rho) < \tilde{p}_\rho \). Hence, \( p^*_\rho(R^*_\rho) = \max\{\tilde{p}_\rho(R^*_\rho), V'\} = \max\{p^*_\rho, V'\} = p^*_\rho \), the equilibrium price in the no-loan auction.\(^\text{12}\)

\(^{12}\text{Proof that the investor’s reserve price is higher than that of the bank in equilibrium (or equivalently, that the investor makes positive profits) is available upon request.} \)
Banks will again offer their reservation interest rates because the investor prefers a lower rate as discussed. The investor will select a bank with correlation $\rho$ to minimize $p^*_\rho + R^*_\rho = 1 + 2(1 + \kappa_\rho)(p^*_\rho - \Delta)$ from Eq. (15). We need to consider two scenarios: (1) $\bar{p}_\rho = (1 + \kappa_\rho)^{-1} V > V'$ ($\kappa_\rho < VV' - 1$) and (2) $\bar{p}_\rho < V'$ ($\kappa_\rho > V / V' - 1$). In the first scenario, $p^*_\rho = \bar{p}_\rho$ so that

$$p^*_\rho + R^*_\rho = 1 + 2(1 + \kappa_\rho)((1 + \kappa_\rho)^{-1} V - \Delta) = 1 + 2(V - (1 + \kappa_\rho)\Delta)$$

(22)

In the second scenario, $p^*_\rho = V'$ so that

$$p^*_\rho + R^*_\rho = 1 + 2(1 + \kappa_\rho)(V' - \Delta)$$

(23)

Hence, $p^*_\rho + R^*_\rho$ is decreasing in $\rho$ for $\kappa_\rho < V / V' - 1$ and increasing in $\rho$ for $\kappa_\rho > V / V' - 1$ (since $\kappa_\rho = k\rho / 2 + k(1 - \rho)$ is increasing in $\rho$). The investor’s cost, $p^*_\rho + R^*_\rho$, is minimized by selecting a bank with correlation $\rho^*$, such that

$$\kappa^*_\rho = \frac{k\rho^*}{2 + k(1 - \rho^*)} = \frac{V}{V'} - 1 \Rightarrow \rho^* = \left(1 + \frac{2}{k}\right) \left(1 - \frac{V}{V'}\right)$$

(24)

if $\rho^* \leq 1$. Otherwise, the investor will select a bank with correlation of 1 to minimize $p^*_\rho + R^*_\rho$. □

Proof that $-\bar{p}'_\rho(R) < 1$. In this proof, we assume that $1 + 2V - \bar{p}_\rho(R) \geq 1 - \bar{p}_\rho(R)$. It is not individually rational for an investor to take an offer of $R > 1 + 2V - \bar{p}_\rho(R) \geq 1 + 2V - p^*_\rho(R)$ so the first condition must hold. Also, $R < 1 - \bar{p}_\rho(R)$ implies that $R < R^*_\rho$ since $R^*_\rho > 1 - p^*_\rho = \bar{p}_\rho(R^*_\rho)$. Since an offer of $R < R^*_\rho$ is not individually rational for a bank, the second inequality must hold. From Eq. (20), $\bar{p}_\rho(R)$ is, therefore, given by

$$\rho \cdot [u_+(R - 1 + \Delta) - u_+(2V - \bar{p}_\rho)] + u_-(\bar{p}_\rho + \Delta) - u_-(\bar{p}_\rho) + (1 - \rho) \cdot [u_+(R - 1 + \Delta) - u_+(2V - \bar{p}_\rho) + u_-(\bar{p}_\rho + \Delta) - u_-(\bar{p}_\rho)] = 0$$

(25)

We want to prove that $-\bar{p}'_\rho(R) < 1$ for power utility. We first expand Eq. (25) to first-order in $V$:

$$\rho \cdot [u'_+(R - 1 + \Delta - 2V + \bar{p}_\rho) + u'_- \cdot \Delta] + (1 - \rho) \cdot [u'_+(R - 1 + \Delta - 2V + \bar{p}_\rho) + u'_- \cdot \Delta] + O(V^2)$$

$$= A_+(R - 1 + \Delta - 2V + \bar{p}_\rho) + A_- \cdot \Delta + O(V^2) = 0$$

(26)

where $A_+ = \rho u'_+(1 - \rho)u'_+ + A_+ = \rho u'_+(1 - \rho)u'_+$. Differentiating Eq. (25) with respect to $R$ gives the following expression for $-\bar{p}'_\rho(R)$:

$$-\bar{p}'_\rho(R) = \frac{\rho u'_+(R - 1 + \Delta) + (1 - \rho)u'_+(R - 1 + \Delta)}{\rho[u'_+(2V - \bar{p}_\rho) + u'_-(\bar{p}_\rho + \Delta)] + (1 - \rho)[u'_+(2V - \bar{p}_\rho) + u'_-(\bar{p}_\rho + \Delta) - u_+'(\bar{p}_\rho)]}$$

(27)

Hence, a sufficient condition for $-\bar{p}'_\rho(R) < 1$ is

$$\rho[u'_+(R - 1 + \Delta) - u'_+(2V - \bar{p}_\rho) + u'_-(\bar{p}_\rho + \Delta) - u'_-(\bar{p}_\rho)] + (1 - \rho)[u'_+(R - 1 + \Delta) - u'_+(2V - \bar{p}_\rho) + u'_-(\bar{p}_\rho + \Delta) - u'_-(\bar{p}_\rho)] < 0$$

(28)
We take a first-order expansion of this condition:

\[
\rho [u''_+ (R - 1 + A - 2V + \bar{p}_\rho) + u''_+ \cdot A] \\
+ (1 - \rho) [u''_+ (R - 1 + A - 2V + \bar{p}_\rho) + u''_+ \cdot A] + O(V^2) < 0
\]

(29)

Substituting \(R - 1 + A - 2V + \bar{p}_\rho = -(A_-/A_+)A + O(V^2)\) from Eq. (26) and simplifying gives

\[
\rho \left[ -u''_+ \cdot \frac{A}{A_+}A + u''_+ \cdot A \right] + (1 - \rho) \left[ -u''_+ \cdot \frac{A}{A_+}A + u''_+ \cdot A \right] + O(V^2)
\]

\[
= \frac{\rho A}{A_+} [u''_+ \cdot u''_+ - u''_+ \cdot u''_+] + O(V^2) < 0
\]

(30)

It is easy to show that \(u''_+ \cdot u''_+ - u''_+ \cdot u''_+ < 0\) for power utility. Since \(A_+ > 0\), \(-\bar{p}_\rho(R < 1\)

for \(V\) small enough. □

A.3. Voluntary equilibrium

Proof of Proposition 3. It is clear that the lending bank does not bid in the auction for non-disclosure, i.e., \(p_N^*(R_N) = V'\), for any individually rational \(R_N\). Specifically, since \(V' > V/2\) (inequality 2), buying the asset always results in a loss of utility for an uninformed bank.

If the investor commits to disclosure at date 0, the equilibrium interest rates and auction prices are obviously the same as in Section 5. If the investor chooses non-disclosure at date 0, banks will offer their reservation interest rate for non-disclosure at date 1 because of competition.

First consider the case where \(\rho^* < 1\) so that \(p^*_0 = V'\). The investor will choose non-disclosure if and only if \(p^*_N + R^*_N = 1 + 2(1 + \kappa_1/2)(V' - A) \leq p^*_0 + R^*_0 = 1 + 2(1 + \kappa_\rho)(V' - A)\). Hence, the investor will not disclose if and only if \(\kappa_1/2 \leq \kappa_\rho \iff \frac{1}{2} \leq \rho^*\). If \(\rho^* = 1\), the investor will choose non-disclosure since \(p^*_1 \geq V' \iff p^*_1 + R^*_1 = 1 + 2(1 + \kappa_1)(p^*_1 - A) > p^*_N + R^*_N = 1 + 2(1 + \kappa_1/2)(V' - A)\). □

A.4. Extensions

Proof of Proposition 5. As stated previously, the investor will never voluntarily disclose to a bank with correlation greater than \(1/2\) because non-disclosure leads to lower front-running and interest rate costs. Hence, the investor will prefer disclosure if and only if there exists a \(\rho < \frac{1}{2}\) such that \(p^*_\rho + R^*_\rho - p^*_N - R^*_N < 0\). We want to show that \(p^*_\rho + R^*_\rho - p^*_N - R^*_N < 0\) if and only if \(\bar{p}_\rho + R^*_\rho - p^*_N - R^*_N < 0\) for \(\rho < \frac{1}{2}\). We first prove that \(p^*_\rho + R^*_\rho - p^*_N - R^*_N < 0 \iff \bar{p}_\rho + R^*_\rho - p^*_N - R^*_N < 0\):

\[
p^*_\rho + R^*_\rho - p^*_N - R^*_N < 0
\]

\[
\iff \bar{p}_\rho + R^*_\rho - p^*_N - R^*_N \leq \max \{\bar{p}_\rho, V'\} + R^*_\rho - p^*_N - R^*_N \equiv p^*_\rho + R^*_\rho - p^*_N - R^*_N < 0
\]

(31)
To prove the converse implication
\[
\tilde{p}_\rho + R^*_\rho - p^*_N - R^*_N < 0
\]
\[
\implies p^*_\rho + R^*_\rho - p^*_N - R^*_N = \max\{\tilde{p}_\rho - p^*_N + R^*_\rho - R^*_N, V' - p^*_N + R^*_\rho - R^*_N\}
\]
\[
= \max\{\tilde{p}_\rho - p^*_N + R^*_\rho - R^*_N, V' - p^*_N + R^*_\rho - R^*_N\} < 0
\]
(32)
since \(p^*_N = V'\) and \(R^*_\rho - R^*_N < 0\) for \(\rho < \frac{1}{2}\). In terms of parameters, the investor will prefer disclosure if and only if there exists a \(\rho < \frac{1}{2}\) such that
\[
\tilde{p}_\rho + R^*_\rho - p^*_N - R^*_N = 2(1 + \kappa_p)(\tilde{p}_\rho - \Delta) - 2(1 + \kappa_{1/2})(V' - \Delta) < 0
\]
\[
\implies \frac{V}{V'} - 1 - \kappa_{1/2} + (\kappa_{1/2} - \kappa_p)\frac{\Delta}{V'} = \frac{V}{V'} - 1 - \kappa_{1/2} \left(1 - \frac{\Delta}{V'}\right) - \kappa_p \frac{\Delta}{V'} < 0
\]
(33)
where we have substituted \(\tilde{p}_\rho = (1 + \kappa_p)^{-1}V\). This function is increasing in \(V/V'\) and \(\Delta/V'\) since \(\kappa_{1/2} - \kappa_p > 0\) for \(\rho < \frac{1}{2}\). It is also decreasing in \(k\) since \(\Delta < V'\). Hence, the investor will prefer disclosure for \(k\) sufficiently high and \(V/V'\) and \(\Delta/V'\) sufficiently low. \(\square\)

**Proof of Proposition 8.** Our proof of the optimal \(\kappa^*_p\) is identical to that of Proposition 2.

We only need show that the maximum investor profit is equal to \(\Delta \cdot \kappa^*_p\).

From Eq. (11) and using the fact that \(\kappa^*_p = V/V' - 1\):
\[
\Pi_i = V - V' - \kappa^*_p(V' - \Delta) = \left(\frac{V}{V'} - 1\right)V' - \left(\frac{V}{V'} - 1\right)(V' - \Delta)
\]
\[
= \left(\frac{V}{V'} - 1\right)\Delta = \kappa^*_p \cdot \Delta \quad \square
\]
(34)

**References**


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