

# Arrogance can be a virtue: Overconfidence, information acquisition, and market efficiency<sup>☆</sup>

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## Abstract

In behavioral finance, overconfidence has been established as a prevalent psychological bias, which can make markets less efficient by creating mispricing in the form of excess volatility and return predictability. In this paper, we develop a model in which overconfidence causes investors to overinvest in information acquisition when this information could improve market efficiency by driving prices closer to true values. We study the impact of overconfidence on mispricing and information acquisition, comparing their net effect on prices. We derive several novel implications. First, overconfidence generally improves market pricing provided the level of overconfidence is not too high. Pricing can also improve even when overconfidence is arbitrarily high, depending on the amount of private information acquired relative to publicly available information.

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## 1. Introduction

In behavioral finance, psychological biases are conjectured to make markets less efficient by generating asset-pricing anomalies such as momentum, reversals, post-announcement drift, and closed-end fund discounts, just to name a few (See Barberis and Thaler, 2003; DeBondt and Thaler, 1995). Behavioral finance has emerged primarily to explain these anomalies that appear inconsistent with rational, efficient markets. Among various known psychological biases, overconfidence has come to be viewed by behavioralists as an important factor in financial markets because it has been shown by experimental psychologists to exist in many aspects of human behavior. DeBondt and Thaler (1995) state that “perhaps the most robust finding in the psychology of judgment is that people are overconfident.” (For overviews of the relevant psychology literature on overconfidence, see Odean, 1998; Daniel, Hirshleifer, and Subrahmanyam, 1998). In addition, overconfidence seems to explain patterns of trading and prices such as excess trading volume (Odean, 1999), long-term reversals (Daniel, Hirshleifer, and Subrahmanyam, 1998), and excess volatility (Odean, 1998).

The motivating idea of this paper is that psychological biases and overconfidence, in particular, might actually make markets more efficient. Specifically, overconfident investors believe that they can earn extraordinary returns and will consequently invest resources in acquiring information pertaining to financial assets. Anecdotally, it seems that professional investors spend a great deal of time and other resources acquiring information about individual companies, industries, and the macroeconomy to make their investment decisions. They invest these resources in spite of it being unclear that they can even achieve returns that recoup these costs (See Elton, Gruber, Das, and Hlavka, 1993; Malkiel, 1995; Gruber, 1996). In the classic paradigm of Grossman (1976), rational investors have no incentive to acquire information in the absence of noise because they can free-ride by observing prices, which perfectly aggregate all available information. Hence, overconfident investors could introduce information into the market that drives security prices closer to their true values.

In the prior literature on overconfident investment, overconfidence generates mispricing, thereby making prices less efficient as measured by “price quality” or the mean-squared error (MSE) between prices and discounted payoffs (e.g., Odean, 1998). Thus, prior models have shown that overconfidence makes markets less efficient unless there are rational arbitrageurs to bring prices to their correct values (See model B from Odean, 1998; Kyle and Wang, 1997). With endogenous information, however, the incentive of overconfident investors to acquire information is a possible countervailing effect that makes prices more informative and efficient even in the absence of rational traders. This possibility was mentioned by Rubinstein (2001) in an argument for efficient markets.

While overconfidence can express itself in other ways, surely it causes many investors to spend too much on research. ...As a result, there is a sense in which asset prices become hyper-rational; that is, they reflect not only the information that was cost-effective to impound into prices but also information that was not worthwhile to gather and impound. Overspending on research is not in one’s self-interest, but it does create a positive externality for passive investors who now find that prices embed more information and markets are deeper than they should be.

In this paper, we combine both the rational and the behavioral perspective to study which of these two effects is larger, the incentive to acquire information or the mispricing caused by overconfidence.

We develop a model based on Grossman (1976) of investors with differential information who face the decision of how much to invest in information acquisition. Investment in information increases the precision of the private signal observed regarding the payoff of the risky asset as in Verrecchia (1982) and Litvinova and Ou-Yang (2003). In addition, investors are overconfident, overestimating the precision of their private signals as well as the productivity of their investment in information. Investors trade assets and invest in information in our model because they overestimate their ability to earn excess returns from investment in contrast to the rational case.

We use this model to address the question of whether overconfidence with endogenous information acquisition can improve market efficiency as measured by price quality. We find first that a market with overconfident investors generally has better price quality than a rational market as long as the level of overconfidence is not too high. When this cognitive bias is not large, the effect of overconfidence on introducing new information into prices dominates its effect on driving prices away from rational values. In addition, we find that price quality can even improve when the level of overconfidence is arbitrarily high, depending on the amount of private information acquired relative to publicly available information. Price quality improves in this case when there is an abundance of private information produced, i.e., when the number of active investors and per capita expenditure in information acquisition are sufficiently high. These results run contrary to the behavioral presumption that cognitive biases make markets function less efficiently. Our analysis shows that overconfident investors can indeed improve the quality of prices through their information acquisition activities even in the absence of rational traders.

The model also gives us novel implications regarding the empirical properties of price quality and, in particular, measures of overreaction in prices such as excess volatility and return predictability. First, the degree of overreaction in prices is decreasing in the number of active investors and the precision of public information and increasing in overconfidence. Second, more precise private information can either decrease or increase mispricing. More accurate private information can increase the informativeness and rationality of prices, but it can also make prices less rational given that investors are overconfident and can overreact to additional private information. The fact that additional private information can either decrease or increase overreaction is a unique prediction of our model, and there appears to be existing evidence in both of these directions. These results also provide empirical tests of our model based on the cross-sectional properties of price overreaction across assets, which have yet to be extensively explored.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 describes the setup of our differential information model with overconfidence and endogenous information acquisition. Section 4 solves the equilibrium for trade and information acquisition. Section 5 analyzes the properties of price quality in our model. Section 6 concludes.

## 2. Literature review

Our paper is principally related to two literatures on financial investment: one on information acquisition and the other on overconfidence. The first literature is based

mainly on the rational expectations paradigms of Grossman and Stiglitz (1980) and Grossman (1976). These seminal papers first articulated the idea that investors have no incentive to collect information in a rational market without noise because information becomes revealed in prices. Many papers have since analyzed the information acquisition of rational investors in the presence of noise including Grossman and Stiglitz (1980), Verrecchia (1982), and other subsequent papers. Our model is similar to those of Verrecchia (1982) and Litvinova and Ou-Yang (2003) in that agents face a variable, not a fixed, cost of information when a higher investment in this cost leads to more precise information. Our paper differs from this prior literature in that we study the information acquisition of overconfident instead of rational investors in markets without noise. These investors acquire costly information even in the absence of noise because they overestimate the value of their private information relative to the aggregated information in prices.

One paper that studies overconfidence in information acquisition is that of Hirshleifer, Subrahmanyam, and Titman (1994), which also finds that such overconfidence can lead to inefficient outcomes. Their model differs from ours significantly, however, in that their investors are overconfident about their ability to obtain information before others and not about the precision of this information. As a result, these investors process information rationally when it arrives. Inefficiency in their model stems from the misallocation of information across assets whereas inefficiency in ours stems from the biased processing of information for a specific asset and the resulting distortion on prices.

Our paper is part of the literature on overconfident investment and is related particularly to models in which overconfidence increases the perceived precision of information, including those of Odean (1998) and Daniel, Hirshleifer, and Subrahmanyam (1998). Like us, Odean (1998) finds that overconfidence can increase price quality although his mechanism is different from ours. Namely, more overconfident investors trade more aggressively in a market with noise and thereby reveal more information, which is accurately reflected in prices through trade with rational market-makers. The model of Kyle and Wang (1997) also has this feature although their focus is on the profits and survival of overconfident traders when they compete strategically with rational traders.<sup>1</sup> In contrast, we find that overconfidence can increase price quality through greater information acquisition even in the absence of rational traders who bring prices closer to rational values.

Odean (1998) also addresses the information acquisition of overconfident investors in a separate model, studying fixed costs of information in a model based on Grossman and Stiglitz (1980). In contrast, our paper studies a more general variable cost model in a differential information market in which investors choose not simply whether to invest in information but precisely how much to invest. As such, we can derive implications related to both the amount invested in information acquisition and the number of these acquiring investors. The distinction between these variables is critical because they can have different effects on price quality and excess volatility as is discussed in this paper.

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<sup>1</sup>In the models of Odean (1998) and Kyle and Wang (1997), higher overconfidence unambiguously leads to higher price quality because prices are set by rational market-makers in a framework based on Kyle (1985). However, in a setting where both overconfident and rational investors determine prices as in Daniel, Hirshleifer, and Subrahmanyam (2001), higher overconfidence can lead to greater mispricing and can, therefore, lead to lower price quality.

Our paper is perhaps most related to that of [Garcia, Sangiorgi, and Urošević \(2005\)](#), who study the effect of overconfidence on information acquisition and prices in a model with both overconfident and rational traders. They obtain a very different result from ours that overconfidence has no effect on market efficiency and prices. Their result, however, relies on some highly specialized assumptions, one of which is that all overconfident investors as well as many rational investors in the economy find it profitable to pay fixed costs to obtain fundamental information. This assumption is at odds with the advice of finance academics and practitioners that rational investors ought to avoid incurring information and other costs associated with active management based on fundamental analysis. Our paper, in contrast, does not make any such restrictions and studies the effect of overconfidence on pricing over the entire range of parameter values in the economy.<sup>2</sup> The results of our model can also be extended to a model with rational investors as discussed in section 3.

Our paper is fundamentally related to that of [Berg and Lein \(2005\)](#), who also study the social benefits of overconfidence in financial markets. Their model pertains to overconfidence on the part of uninformed traders in the information of informed speculators, which enhances confidence in prices and leads to increased trading and liquidity. Our paper, in contrast, deals with a more standard notion of overconfidence grounded in psychology in which individuals overestimate their own abilities and not the abilities of others. Finally, [Bernardo and Welch \(2001\)](#) also articulate the idea that overconfident individuals can produce information that is socially but not privately valuable in the vastly different setting of overconfident entrepreneurship. Whereas our model studies the effect of overconfidence and information production on prices in a market context, their model studies private and social welfare in a nonmarket context.

### 3. Model setup

In this section, we characterize a market with  $N$  identical agents, each with a constant absolute risk aversion (CARA) utility, normal distribution on both unconditional and conditional asset returns, costly differential information acquisition, and overconfidence about the precision of their private signal. This model is based on [Grossman \(1976\)](#) except that it adds information acquisition and overconfident investors.<sup>3</sup>

The model has three events. At time 0, agents decide how much costly information to acquire. At time 1, agents observe their private signals as well as a public signal and trade with each other in a competitive asset market. Finally, at time 2, the assets pay off, and all agents consume.

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<sup>2</sup>Relaxing the assumption of full participation by overconfident investors in the model of [Garcia, Sangiorgi, and Urošević \(2005\)](#) would cause prices to be affected by investors' level of overconfidence. Namely, the informativeness of prices would be strictly increasing in overconfidence, precisely as in our model. Higher overconfidence would increase both information acquisition and trading aggressiveness among overconfident investors, which would increase price informativeness.

<sup>3</sup>In our model, we study an economy with homogeneous agents for simplicity. Our model can be extended to include heterogeneous agents with all of our conclusions pertaining to market efficiency remaining the same. For example, we have analyzed a model with a subset of rational agents as in [Garcia, Sangiorgi, and Urošević \(2005\)](#), and details of this analysis are available upon request.

- **Assets:** There is one risk-free asset with zero net return serving as a numeraire and one risky asset that pays off  $\tilde{x} \sim N(0, \tau^{-1})$  at time 2. Investors have zero endowment of both the risky asset and the risk-free numeraire at time 0.
- **Agents:** There are  $N$  identical investors with CARA utility function:

$$U(w) = -\exp(-\lambda w), \quad (1)$$

where  $\lambda$  is the investor's coefficient of absolute risk-aversion and  $w$  is the investor's wealth at time 2.

- **Information:** Prior to trade at time 1, all investors costlessly observe a public signal about the payoff of the risky asset:  $\tilde{\pi} = \tilde{x} + \tilde{\eta}$ , where  $\tilde{\eta} \sim N(0, \delta^{-1})$  and is independent of  $\tilde{x}$ . In addition, investor  $i$  observes a private signal:  $\tilde{s}_i = \tilde{x} + \tilde{\varepsilon}_i$ , where the  $\tilde{\varepsilon}_i$ 's are independent and identically-distributed (i.i.d.) for  $i = 1, 2, \dots, N$  and are independent of both  $\tilde{x}$  and  $\tilde{\eta}$ . The private signal has a precision that depends on the investor's investment in information at time 0,  $c$ .

The actual precision of  $\tilde{\varepsilon}_i$  is given by  $\alpha(c, \xi) = \xi \cdot \phi(c)$ , where  $\xi$  parameterizes the level of this precision. We assume that the precision function is increasing and concave in  $c$  and that zero investment generates no information, i.e.,

$$\frac{\partial \alpha}{\partial c} > 0, \quad \frac{\partial^2 \alpha}{\partial c^2} < 0 \quad \text{and} \quad \alpha(0, \xi) = 0. \quad (2)$$

The investor is overconfident and overestimates the precision of her private signal where her level of overconfidence is given by the parameter  $\gamma$ . The case  $\gamma = 1$  represents the benchmark of rationality, and overconfident investors have  $\gamma > 1$ . Investor  $i$  believes the precision of  $\tilde{\varepsilon}_i$  to be  $\beta(c, \gamma, \xi) > \beta(c, 1, \xi) = \alpha(c, \xi)$ , where  $\alpha(c, \xi)$  is the actual precision. Hence, the actual distribution is  $\tilde{\varepsilon}_i \sim N(0, \alpha(c, \xi)^{-1})$  whereas investor  $i$  believes it to be  $\tilde{\varepsilon}_i \sim N(0, \beta(c, \gamma, \xi)^{-1})$ . We use the terms *objective* and *subjective* to refer to the actual and overconfident distributions, respectively.

As in Odean (1998), we assume that the investor's subjective precision has the following functional form:  $\beta(c, \gamma, \xi) = \gamma \alpha(c, \xi)$ .<sup>4</sup> With this function, we are making the standard assumption that subjective precision is increasing in overconfidence. In addition, we are making a critical assumption that the marginal precision per unit cost is increasing in overconfidence, i.e.,

$$\frac{\partial \beta}{\partial \gamma} > 0, \quad \frac{\partial^2 \beta}{\partial c \partial \gamma} > 0. \quad (3)$$

We are assuming, therefore, that the investor not only overestimates the precision of her information but also the productivity in her investment in information. This property gives a more overconfident investor the propensity to invest more in information acquisition. Our motivation for this assumption comes from several observations. First, professional investors who invest more in information acquisition appear to be more overconfident about their ability to earn excess returns. In a rational model of information investment, higher information investment stems from more productive information investment, which

<sup>4</sup>We work with this multiplicatively separable subjective precision function to make the analysis simpler, but our results hold under weaker mathematical conditions. For example, our asymptotic results hold whenever the overconfidence function  $(\beta(c, \gamma, \xi)/\alpha(c, \xi))$  is weakly concave in cost.

leads to higher risk-adjusted returns, both gross and net of costs. The empirical studies of Carhart (1997) and Elton, Gruber, Das, and Hlavka (1993), however, show that higher expense ratios in mutual funds lead to lower net returns adjusted for risk. Hence, fund managers who invest more in information acquisition overestimate their ability to earn higher gross returns that recoup these costs.<sup>5</sup> In addition, higher overconfidence appears to lead to higher investment of resources in other economic activities as well. For example, entrepreneurs appear to be self-selected for extremely high overconfidence in their own abilities as well as a high level of work ethic in their new business. Cooper, Woo, and Dunkelberg (1988) show in their empirical study of entrepreneurial overconfidence that “this extreme optimism probably does contribute to the heavy personal commitments observed here, in which the median entrepreneur devoted more than 60 h per week to the business” in p. 98.

Finally, we assume that the investor overestimates neither the precision of others’ signals nor the precision of public information as in Odean (1998), Kyle and Wang (1997), and Daniel, Hirshleifer, and Subrahmanyam (1998). Specifically, investor  $i$  believes the precision of  $\tilde{\eta}$  to be  $\kappa_1 \cdot \delta$  and  $\tilde{\epsilon}_j$  to be  $\kappa_2 \cdot \alpha(c)$ , where  $i \neq j$  and  $\kappa_1, \kappa_2 \leq 1$ . Because the specific value of the  $\kappa$ ’s do not affect any of the results in this paper, we henceforth assume that  $\kappa_1 = \kappa_2 = 1$  for simplicity.

#### 4. Equilibrium

In this section, we study the equilibrium for trade and information acquisition followed by a discussion of the asymptotic properties and comparative statics of information acquisition.

##### 4.1. Overconfident expectations equilibrium at time 1

We define an equilibrium for the time 1 asset market and solve for equilibrium demands and price of the risky asset. Investor  $i$ ’s wealth at time 2 is simply her profit per share of the risky asset times her demand,  $\theta_i$ , minus her information investment,  $c$ :

$$\tilde{w}_i = \theta_i(\tilde{x} - p) - c. \quad (4)$$

To distinguish between objective and subjective statistics, we use the subscript  $i$  to indicate that the inference is from investor  $i$ ’s point of view, e.g.,  $E_i[\tilde{x}|\pi, s_i, p]$ ,  $cov_i(s_i, \tilde{x})$ , etc. Statistics without a subscript are objective statistics.

In this paper, we focus on symmetric Nash equilibria of the time 0 information acquisition game in which all investors choose the same subjective precision of their signals in equilibrium. We denote this equilibrium subjective and its corresponding objective precision by  $b$  and  $a$ , respectively.

<sup>5</sup>There is at least one alternate explanation for this empirical finding. Namely, fund expenses reflect managerial perks, not information investment. Though we do not directly address the empirical question of how to distinguish our hypothesis from this alternative, we point out first that the prior mutual fund literature on returns and expenses treats fund expenses as a proxy for information investment. In addition, we use our model to derive empirical implications in Section 5.2 regarding the relationship between these expenses and observable measures of price overreaction that can serve as a test for our hypothesis.



An overconfident expectations equilibrium (OEE) at time 1 is a demand function,  $\theta_i(\pi, s_i, p)$ , and equilibrium price,  $p(\pi, s_1, s_2, \dots, s_N)$ , defined by the following two conditions.

- (i) Agents choose demand to maximize their subjective expected utility. With time 2 wealth given by Eq. (4),  $\theta_i(\pi, s_i, p)$  solves the following maximization problem:

$$\max_{\theta_i} E_i\{U(\theta_i(\tilde{x} - p) - c) | \pi, s_i, p\}. \quad (5)$$

We assume that the investor can costlessly infer information through prices.

- (ii) The asset market clears

$$\sum_{i=1}^N \theta_i(\pi, s_i, p) = 0. \quad (6)$$

Proposition 1 summarizes the linear OEE.

**Proposition 1.** *There exists an OEE such that the equilibrium price is given by*

$$p = A\pi + B\bar{s}, \quad (7)$$

where

$$A = \frac{\delta}{(N + \gamma - 1)a + \delta + \tau}, \quad B = \frac{(N + \gamma - 1)a}{(N + \gamma - 1)a + \delta + \tau}, \quad \bar{s} = \frac{s_1 + s_2 + \dots + s_N}{N}.$$

Based on this price, trader  $i$ 's demand  $\theta_i(\pi, s_i, p)$  is

$$\theta_i(s_i, p) = \frac{(\gamma - 1)a}{\lambda} (s_i - \bar{s}). \quad (8)$$

**Proof.** See Appendix.

The demand function in Eq. (8) is intuitive. Investors buy (sell) if their private signal,  $s_i$ , is greater (less) than the average social signal,  $\bar{s}$ . In addition, the size of their trade increases with overconfidence and decreases with risk-aversion. As investors approach rationality ( $\gamma \rightarrow 1$ ), their demands converge to zero, consistent with the no-trade theorem of rational markets as discussed in Brunnermeier (2001).

Equilibrium prices are too responsive to private information because investors are overconfident, and this responsiveness,  $B$ , is increasing in overconfidence or  $\gamma$ . In other words, prices overreact to private information because individual demands overreact to this information. Hence, prices are excessively volatile relative to fundamentals, and there is overreaction at time 1 followed by reversal or correction when the asset pays off.

#### 4.2. Information acquisition equilibrium at time 0

At time 0, the investor decides how much to invest in information acquisition,  $c$ , and thereby, the subjective signal precision of her signal,  $\beta(c, \gamma, \xi)$ . In a standard backward induction, she chooses her precision so as to maximize the expected utility of her investment at time 1 given the precision of others' signals. These equilibrium subjective and



objective precisions are  $b$  and  $a$ , respectively. Proposition 2 summarizes the investor's utility maximization problem.

**Proposition 2.** *The investor maximizes her expected utility by choosing  $c$  such that*

$$\max_c \left[ -c - \left( \frac{1}{\lambda} \right) \log f(a, b, \beta) \right] \quad \text{s.t. } c \geq 0, \quad (9)$$

where

$$\begin{aligned} [f(a, b, \beta)]^2 = & \frac{a\beta}{(\delta + \tau)^2} N^2 \left( \frac{b + a(N-1)}{\delta + \tau} + 1 \right)^2 \left( \frac{\beta + a(N-1)}{\delta + \tau} + 1 \right)^{-1} \\ & \times \left[ \left( \frac{b + a(N-1)}{\delta + \tau} \right)^2 \left( \frac{a + \beta(N-1)}{\delta + \tau} \right) + \frac{a\beta}{(\delta + \tau)^2} N^2 \right]^{-1}. \end{aligned} \quad (10)$$

**Proof.** See Appendix.

We write the objective in log utility form (normalized by risk-aversion) for the sake of economic interpretation. The term  $-(1/\lambda) \log f(a, b, \beta)$  is the dollar certainty equivalent of investment at time 1 given social objective precision,  $a$ , the social subjective precision,  $b$ , and the investor's choice of subjective precision,  $\beta$ . The investor obviously maximizes this dollar benefit of information minus the dollar cost of information,  $c$ . The first-order condition of this optimization is given by

$$-\left( \frac{1}{\lambda} \right) \frac{\partial \log f(a, b, \beta)}{\partial \beta} \cdot \frac{\partial \beta}{\partial c} = 1. \quad (11)$$

The left-hand side of this equation is the marginal benefit per dollar information investment, which is equal to the marginal cost of information, 1. This problem is highly nonlinear in terms of  $c$  (and  $\beta$ ) and has no closed-form solution. We next study the comparative statics and asymptotic behavior of the investor's information investment as a function of different parameters.

#### 4.3. Properties of information acquisition

From our assumption of a symmetric Nash equilibrium, we plug  $\beta = b = \gamma a$  into Eq. (11). We also denote equilibrium information investment by  $c^*$  where equilibrium objective precision is therefore defined as,  $a = \alpha(c^*, \xi)$ . Although this problem does not have a closed-form solution for  $c^*$ , we can study the comparative statics of equilibrium information investment as a function of model parameters. The following proposition gives their comparative statics and asymptotics as a function of the social level of risk-aversion ( $\lambda$ ) and overconfidence ( $\gamma$ ). All proofs for this section are in the Appendix.

**Proposition 3.** (i)  $c^*$  is nonincreasing in  $\lambda$ . As  $\lambda \rightarrow 0$  and as  $\lambda \rightarrow \infty$ ,  $c^* \rightarrow \infty$  and  $c^* \rightarrow 0$ , respectively.

(ii)  $c^*$  is nondecreasing in  $\gamma$ . At  $\gamma = 1$  and as  $\gamma \rightarrow \infty$ ,  $c^* = 0$  and  $c^* \rightarrow c_\infty^*$ , respectively, where  $c_\infty^*$  is the solution to

$$\frac{1}{2\lambda\alpha} \cdot \frac{\partial \alpha}{\partial c} = 1. \quad (12)$$

Part 1 of Proposition 3 is an extension of the Verrecchia (1982) result that information investment is decreasing in the agent's risk-aversion,  $\lambda$ , for a rational agent. The reason is that higher  $\lambda$  decreases the size of demand [Eq. (8)] and consequently decreases the investor's subjective expected profit. Hence, higher  $\lambda$  gives the investor less profit incentive to collect information. We extend Verrecchia's result to the case of overconfident investors. In addition, our results apply to social instead of individual parameters and choice variables. In other words, an increase in the aggregate risk-aversion of the market decreases per capita investment in information across investors.

Equilibrium investment in information is nondecreasing in the social level of overconfidence according to Part 2 of Proposition 3. This result is true because of our assumption in Eq. (3) that the marginal subjective precision of information investment is increasing in  $\gamma$ . Higher overconfidence thus leads to higher information investment because it makes investors perceive higher productivity in their information acquisition. When investors are rational and  $\gamma = 1$ , investors have no incentive to collect information, i.e.,  $c^* = 0$ . This is simply the familiar result of Grossman (1976) that price perfectly aggregates information in the absence of noise. Consequently, rational investors do not need to collect or even observe private information because they can simply observe prices. When investors are overconfident and  $\gamma$  is greater than 1, on the other hand, our model provides resolution to the Grossman–Stiglitz puzzle without exogenous noise trading. Namely, investors trade and acquire information because they overestimate their ability to profit from this activity.

As  $\gamma \rightarrow \infty$ , investors' information investment approaches the limit of  $c_\infty^*$ , which is finite, in general. Investors generally have a finite incentive to collect information even when overconfidence is infinite because of two competing effects. The first term of the product in the first-order condition of Eq. (11),  $-(1/\lambda)(\partial \log f / \partial \beta)$ , is the marginal dollar benefit per unit precision, which becomes proportional to  $1/\gamma$  as  $\gamma \rightarrow \infty$ . Intuitively, the subjective precision of the signal becomes high as  $\gamma$  gets larger, which eventually creates diminishing marginal returns to further precision. The second term,  $(\partial \beta / \partial c)$ , is the marginal precision per dollar cost. Again by assumption, this term is increasing and proportional to  $\gamma$ . Hence, these two effects cancel and information investment approaches a finite value.

In addition,  $c_\infty^*$  does not depend on  $N$ ,  $\delta$ , or  $\tau$  because these terms do not appear in Eq. (12). Intuitively, investors perceive their private information to be infinitely precise and ignore all other information when they are infinitely overconfident. Hence, their information investment is unaffected by the number of investors in the market ( $N$ ) and the precision of either their priors regarding the asset's value or that of the public signal ( $\tau$  and  $\delta$ , respectively). We denote the objective precision of information in the limit as  $\gamma \rightarrow \infty$  by  $a_\infty = \alpha(c_\infty^*, \xi)$ .

Proposition 4 gives equilibrium information investment as a function of the precisions of public information ( $\xi$ ), public information ( $\delta$ ), their priors ( $\tau$ ), and the number of investors ( $N$ ).

**Proposition 4.** (i)  $c^*$  is nondecreasing in  $\xi$ . As  $\xi \rightarrow 0$ ,  $c^* \rightarrow 0$ .

(ii)  $c^*$  is nonincreasing in  $\delta$  and  $\tau$ . As  $\delta \rightarrow \infty$  or  $\tau \rightarrow \infty$ ,  $c^* \rightarrow 0$ .

(iii)  $c^*$  is nonmonotonic in  $N$ . As  $N \rightarrow \infty$ ,  $c^* \rightarrow 0$ .

Parts 1 and 2 of this proposition are straightforward. First, the higher the precision of the investor's private information ( $\xi$ ), the higher investor's incentive to acquire information. Our model also yields the intuitive result that more precise existing

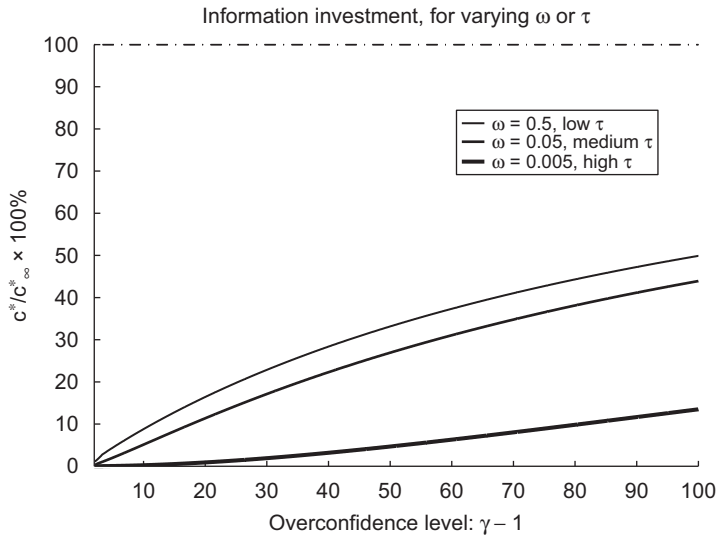


Fig. 1. Information investment as a function of  $\gamma$ .  $N = 100$ ,  $\lambda = 0.0025$ ,  $\omega = 0.5, 0.05$ , and  $0.005$ .

information in the form of either priors ( $\tau$ ) or public information ( $\delta$ ) decreases the incentive to acquire private information.

The number of investors ( $N$ ), on the other hand, can either decrease or increase information acquisition. A greater number of investors can decrease the incentive to acquire information because of an increasing free-rider problem. Namely, the aggregate signal becomes more precise as  $N$  gets larger and consequently increases the investor's incentive to free-ride on others' information by simply observing prices. As a result, the investor's incentive to collect her own private information goes to zero as  $N$  approaches infinity and the aggregate signal becomes infinitely precise. Higher  $N$  can also increase information acquisition because it increases price volatility, which can increase the opportunity for disagreement between the investor's inference and the prevailing market price. Information investment is increasing in  $N$  when price volatility is low, i.e., when  $\gamma$ ,  $\xi$ , or  $N$  are sufficiently low. Under these conditions, the marginal impact of  $N$  on price volatility and the chance for disagreement is high.

Figs. 1 and 2 show numerical solutions of equilibrium information investment (normalized by asymptotic information investment,  $c_{\infty}^*$ ) as a function of economy-wide overconfidence,  $\gamma$ . We use a power function for subjective precision:  $\beta(c, \gamma, \xi) = 2\gamma\xi c^{1/2}$ .<sup>6</sup>

We would first like to graph the relationship between information investment and public and private information precisions. As can be seen from the objective function in Eq. (10), information investment in our model depends only on the ratio between private information precision ( $a$ ,  $b$ , and  $\beta$ ) and the posterior precision from public information ( $\delta + \tau$ ). For this reason, we define a variable,  $\omega$ , which is a function of the ratio between these private and public information precisions, and graph information investment for various  $\omega$ . Specifically,

<sup>6</sup>In all of our figures,  $\lambda$  simply scales the absolute amount of information investment up or down and does not affect any of the comparative statics of our solutions. Therefore, we simply set  $\lambda$  to an arbitrary quantity ( $\lambda = 0.0025$ ) because we normalize our information investment (by maximum information investment,  $c_{\infty}^*$ ) and focus on comparative statics only, not absolute quantities.

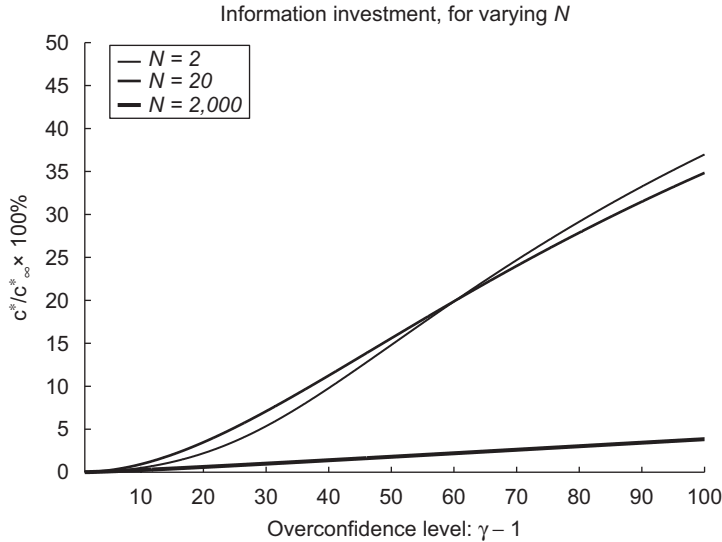


Fig. 2. Information investment as a function of  $\gamma$ .  $\omega = 0.01$ ,  $\lambda = 0.0025$ ,  $N = 2, 20$ , and  $2,000$ .

$\omega$  represents the reduction in the objective variance of  $\tilde{x}$  from observing a private signal of precision,  $a_\infty$ , relative to observing only public information and is defined by

$$\omega = 1 - \frac{\delta + \tau}{a_\infty + \delta + \tau} = \left(1 + \frac{\delta + \tau}{a_\infty}\right)^{-1}. \quad (13)$$

This relationship comes from the well-known additive property of precision for normal random variables, where  $\delta + \tau$  is the posterior precision after observing the public signal only and  $a_\infty + \delta + \tau$  is the posterior precision conditional on observing both the private and public signals.

In Fig. 1, we hold the number of investors fixed at  $N = 100$  and graph information investment for  $\omega$  of 50%, 5%, and 0.5% (i.e., a highly overconfident investor's private information reduces objective variance by 50%, 5%, and 0.5%, respectively). Fig. 1 shows simply that equilibrium information investment is increasing in aggregate overconfidence and increasing in the ratio between private and public information precisions.

Fig. 2 shows that information acquisition is nonmonotonic in  $N$ , in general. Fig. 2 graphs information precision for  $N$  of 2, 20, and 2,000 for a fixed low  $\omega = 1\%$ . Information investment for  $N = 20$  is higher than for  $N = 2$  when overconfidence is low and lower when overconfidence is high whereas investment is uniformly lower than both for  $N = 2,000$ . This figure shows that information acquisition is increasing in  $N$ ,  $\gamma$ , or  $\omega$  are sufficiently low as described previously.

## 5. Price quality

In our model, the objective expectation of overconfident investors' profit and utility is lower than for rational investors similarly to Odean (1998). The reason is that investors in our model make zero expected profit gross of information costs because they are

homogeneous agents in a zero-sum market. Because overconfident investors, in general, invest in costly information acquisition and incur utility costs by trading in risky assets, their objective profit and utility is lower than when they are rational and neither invest in information nor trade. The fact that overconfident investors lose money in our model naturally raises the question of whether overconfidence can survive in the long-run, which we defer discussing until the end of this section.

Though information acquisition does not generate positive profits and, therefore, has no private value, it can have social value in the sense that it potentially drives prices closer to the true value of the asset. We use “price quality” as a metric of this social value as well as the overall efficiency of the market. As in Odean (1998), price quality is measured using mean-squared error between the asset’s payoff and its price; the lower this mean-squared error, the higher is price quality. Higher price quality increases the accuracy of asset pricing and can thereby improve the functioning of the market. More accurate prices can, for instance, drive real investment policies of firms closer to first-best as in the model of Stein (1996).

The mean-squared error between the asset’s payoff and its price can be rewritten in the following form:

$$\begin{aligned}
 \text{MSE} &= E[(\tilde{x} - \tilde{p})^2] = E[(\tilde{x} - E[\tilde{x}|\tilde{\pi}, \tilde{p}] + E[\tilde{x}|\tilde{\pi}, \tilde{p}] - \tilde{p})^2] \\
 &= E[(\tilde{x} - \tilde{p}_r + \tilde{p}_r - \tilde{p})^2] \\
 &= E[(\tilde{x} - \tilde{p}_r)^2] + E[(\tilde{p}_r - \tilde{p})^2] + 2E[(\tilde{x} - \tilde{p}_r)(\tilde{p}_r - \tilde{p})] \\
 &= \underbrace{E[(\tilde{x} - \tilde{p}_r)^2]}_{\text{Rational MSE}} + \underbrace{E[(\tilde{p}_r - \tilde{p})^2]}_{\text{Pricing error}}. \tag{14}
 \end{aligned}$$

Above,  $\tilde{p}_r$  is “the rational price” or the price of the risky asset that would prevail if all investors were rational. Because this asset is in zero net supply, there is no risk premium in its price. Hence, the rational price would simply be the objective expectation of asset’s payoff conditional on public information and private information aggregated in price:  $\tilde{p}_r = E[\tilde{x}|\tilde{\pi}, \tilde{p}]$ . The expectation of the cross-term in Eq. (14) is equal to zero because the rational error term is orthogonal to all available information at time 1.

As indicated from Eq. (14), MSE is composed of two parts. The rational MSE is the residual uncertainty after making rational inferences from available information and decreases when there is more information acquisition and therefore, higher objective information precision. The other part is the departure from rational pricing or pricing error caused by the cognitive bias of overconfidence.

We end this section with a brief discussion on the long-run survival of overconfidence in financial markets. Although overconfident investors lose money as a result of information acquisition in our model, they could actually make money in a more complete model in which they can exploit noise traders, for example. This issue has been analyzed in several papers including Benos (1998) and Kyle and Wang (1997), but the results of Hirshleifer and Luo (2001) are most applicable here. These authors find that price-taking overconfident traders can better exploit noise traders and earn higher returns than their rational counterparts through their aggressive trade. Although this model does not consider information acquisition, Hirshleifer (2001) alludes to the fact that overconfident investors could earn even higher returns as a result of their investment in information.

Although the theoretical literature finds that it is possible for overconfidence to survive in markets under certain conditions, empirical studies provide a more mixed picture of

overconfident investors and their long term prospects. [Biais, Hilton, Mazurier, and Pouget \(2005\)](#), for example, show that higher overconfidence causes traders to lose money in a zero-sum experimental market. In addition, [Odean \(1999\)](#) finds that individual investors lose money, on average, as a result of excessive trading in actual brokerage account data. Other experimental evidence, on the other hand, indicates that overconfidence is a pervasive bias among market participants including professional investors (e.g., [Staël von Holstein, 1972](#); [Glaser, Langer, and Weber, 2003](#)). A full exploration of this important issue lies outside the limited scope of our model, however, and we therefore leave its study for future research.

### 5.1. Price quality and overconfidence

In this section, we study the effect of overconfidence on price quality and, specifically, whether overconfidence has a greater effect on information acquisition or pricing error. We find that overconfidence can indeed improve price quality, contrary to the idea that behavioral biases necessarily make markets function less efficiently.

To this end, we start by studying price quality for rationality,  $\gamma = 1$ , and then consider how it changes with overconfidence.

**Proposition 5.** *Equilibrium MSE at  $\gamma = 1$  is given by*

$$\text{MSE}|_{\gamma=1} = \frac{1}{\delta + \tau}. \quad (15)$$

When investors are rational, there is zero acquisition of private information as in Proposition 3. For this reason, price is simply the rational expectation of the asset's payoff conditional on public information. Hence, MSE for a rational market is equal to the asset's posterior variance conditional on public information:  $E[(\tilde{x} - E[\tilde{x}|\tilde{\pi}])^2] = (\delta + \tau)^{-1}$ . We now analyze equilibrium MSE as overconfidence approaches infinity and find that price quality can improve relative to a rational market even when overconfidence is extremely high.

**Proposition 6.** *The limit of equilibrium MSE as  $\gamma \rightarrow \infty$  is given by*

$$\text{MSE}|_{\gamma \rightarrow \infty} = \frac{1}{a_{\infty} N}. \quad (16)$$

In other words, mean-squared error is decreasing in the precision of aggregate private information:  $a_{\infty} N$ . More precise information obviously reduces rational MSE as we mentioned in the previous section. On the other hand, information precision has two competing effects on pricing error. Higher information precision increases the informativeness of prices, which makes prices more “rational” and can decrease the gap between rational and non-rational prices. Higher information precision also increases the volatility of both rational and nonrational prices, however, which can increase the average deviation between the two. When overconfidence is very high, the first effect dominates. The reason is that price volatility asymptotes to a finite maximum as  $\gamma \rightarrow \infty$  so that information precision has a diminishingly small marginal effect on this component.<sup>7</sup> In addition, the

<sup>7</sup>Price volatility does not become arbitrarily large as overconfidence goes to infinity. Infinite overconfidence simply means that investors ignore all information except their private signal, not that there will be willing buyers (sellers) at arbitrarily high (low) prices.

precision of public information and priors have no effect on mean-squared error when investors are extremely overconfident because they ignore everything but private information. Hence, public information and priors do not affect demands or prices. Aggregate private information precision, therefore, decreases both rational MSE and pricing error, which decreases total equilibrium MSE.

Simply stated, price quality for a market in which investors are extremely overconfident can be greater or less than that for a rational market, depending only on the amount of private information acquired relative to publicly available information. Asymptotic price quality will be greater than rational price quality when  $(a_{\infty}N)/(\delta + \tau) > 1$ , i.e., when the number of active traders and investment in information acquisition are sufficiently high and public information is sufficiently poor.

We next consider how mean-squared error changes as a function of  $\gamma$ , focusing first on the local behavior of MSE near rationality. We find the surprising result that a small amount of overconfidence generally improves price quality over a rational market.

**Proposition 7.** *Equilibrium MSE is a nonincreasing function of  $\gamma$  at  $\gamma = 1$ .*

The effect of overconfidence on mean-squared error near rationality is twofold. The marginal effect of  $\gamma$  is negative on rational MSE while it is zero on pricing error at  $\gamma = 1$ . The first effect is true because an increase in overconfidence results in additional information acquisition by Proposition 3, which decreases rational MSE. The marginal effect of  $\gamma$  on pricing error is zero at  $\gamma = 1$  because equilibrium pricing error attains a minimum of zero at  $\gamma = 1$  and consequently has zero derivative at this point. Total equilibrium MSE, therefore, is nonincreasing at  $\gamma = 1$  by these two effects.

According to this proposition, a market with a small amount of overconfidence generally has better price quality than a rational market. A small amount of cognitive bias has a small effect on creating pricing errors, while having a comparatively large effect on introducing information into prices.

To compare MSE relative to that of a rational market, we normalize mean-squared error by  $(\delta + \tau)^{-1}$ , the posterior variance of the asset's value conditional on public information. Hence, we define the variable of  $\widehat{\text{MSE}}$  as the following normalized quantity:

$$\widehat{\text{MSE}} = \frac{E[(\tilde{x} - \tilde{p})^2]}{E[(\tilde{x} - E[\tilde{x}|\tilde{\pi}])^2]} = \frac{1 + (N + \gamma - 1)^2[a/(N(\delta + \tau))]}{((N + \gamma - 1)[a/(\delta + \tau)] + 1)^2}. \quad (17)$$

From Eq. (17),  $\widehat{\text{MSE}}$  depends only on the ratio of private to public information precision ( $a/(\delta + \tau)$ ). In Fig. 3, therefore, we graph equilibrium  $\widehat{\text{MSE}}$  as a function of  $\gamma$  for fixed  $N$  and various ratios of private to public information precision ( $\omega$ ) as before. From Proposition 6, asymptotic MSE in our economy is higher than that of a rational market for private-to-public precision ( $\omega$ ) sufficiently low. Hence, asymptotic price quality is inferior for  $\omega = 0.5\%$  and  $5\%$  and superior for  $\omega = 50\%$ .

In accord with Proposition 7, MSE slopes downward with respect to  $\gamma$  at rationality ( $\gamma = 1$ ) although its slope is close to zero when  $\omega$  is very low. This effect occurs because overconfidence produces less private information when private-to-public precision is low as in Proposition 4.

One can also see from Fig. 3 that private information precision can have a nontrivial effect on price quality for the reasons described previously. Namely, information precision can decrease pricing error by making prices more informative but can also increase pricing



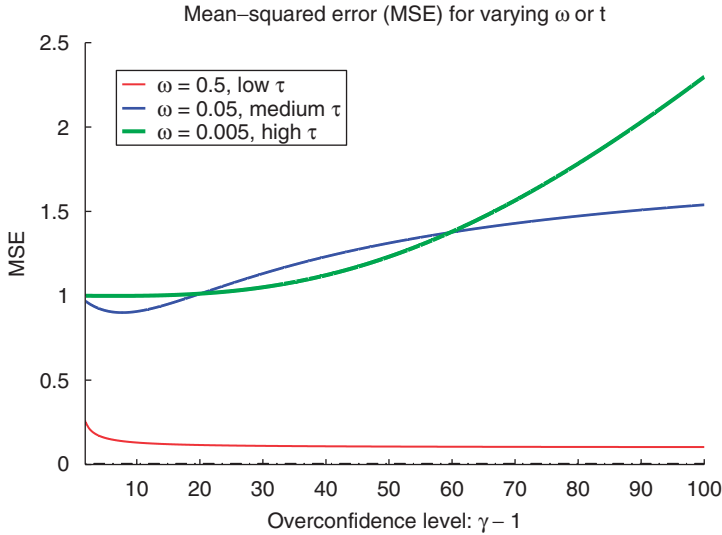


Fig. 3. Equilibrium  $\widehat{\text{MSE}}$  as a function of  $\gamma$ .  $N = 10$ ,  $\lambda = 0.0025$ ,  $\omega = 0.5, 0.05$ , and  $0.005$ .

error by increasing the volatility of prices. Higher private information precision leads to lower mean-squared error for the most part in Fig. 3 except that  $\widehat{\text{MSE}}$  for  $\omega = 5\%$  is higher than  $\widehat{\text{MSE}}$  for  $\omega = 0.5\%$  roughly between  $\gamma = 20$  and  $50$ . This feature of our model stands in contrast to the rational CARA-normal model. In the rational case, more precise information improves price quality because information is incorporated appropriately into prices. In our model, on the other hand, investors are overconfident and could overreact to additional information, which could decrease price quality.

In general, these results introduce a novel set of ideas into the tenets of behavioral finance. Namely, investors with cognitive biases can improve the quality of prices through their information acquisition activities even in the absence of rational market participants. In the case of our model of overconfidence, we obtain two interesting results. First, price quality generally improves and can never deteriorate when cognitive biases are not too large. Second, price quality can even improve when biases are arbitrarily large, depending only on the precision of privately acquired information relative to that of publicly available information.

## 5.2. Price overreaction

In this section, we attempt to map our previously studied variable of pricing error into the observable measures of mispricing and price overreaction. We can thereby generate empirical implications of our model.

Prices overreact to private information in our model because investors are overconfident about the precision of this information. One measure of this overreaction is excess volatility, i.e., price volatility in excess of fundamentals. We can measure excess volatility in our model by taking total price change variance and subtracting dividend variance,

which ought to be zero if investors are rational.<sup>8</sup> Namely, there are two periods of price changes: between time 0 and time 1, and between time 1 and time 2. We define the price of the risky asset at time 0 to be zero, the unconditional expectation of  $\tilde{x}$ , because no information has yet been observed and the price at time 2 to be the terminal value of the firm,  $\tilde{x}$ . Hence, total price change variance minus dividend variance is given by  $E[(\tilde{x} - \tilde{p})^2] + E[\tilde{p}^2] - E[\tilde{x}^2]$ . If investors are rational, the two price change increments are orthogonal so that  $E[\tilde{x}^2] = E[(\tilde{x} - \tilde{p}_r + \tilde{p}_r)^2] = E[(\tilde{x} - \tilde{p}_r)^2] + E[\tilde{p}_r^2]$  and excess volatility is equal to zero. Finally, we normalize this quantity by the dividend variance to obtain the fraction of total volatility that is excessive so that our measure is not directly dependent on the firm's number of shares, size, or volatility. We refer to this normalized quantity in a slight abuse of terminology as “excess volatility”, which we denote by  $\Gamma$ :

$$\begin{aligned}\Gamma &= \frac{1}{E[\tilde{x}^2]} \{E[(\tilde{x} - \tilde{p})^2] + E[\tilde{p}^2] - E[\tilde{x}^2]\} \\ &= \frac{1}{E[\tilde{x}^2]} \{E[(\tilde{x} - \tilde{p}_r + \tilde{p}_r - \tilde{p})^2] + E[(\tilde{p} - \tilde{p}_r + \tilde{p}_r)^2] - E[\tilde{x}^2]\} \\ &= \frac{2E[\tilde{p}(\tilde{p} - \tilde{p}_r)]}{E[\tilde{x}^2]},\end{aligned}\quad (18)$$

where we use Eq. (14) to obtain this expression. We can rewrite this expression concisely in terms of model parameters,  $\gamma$ ,  $N$ ,  $\delta/\tau$ , and  $a/\tau$ , where the last two variables represent the signal-to-noise ratios of private and public information, respectively.

$$\Gamma = \frac{2E[\tilde{p}(\tilde{p} - \tilde{p}_r)]}{E[\tilde{x}^2]} = \frac{2(a/\tau)(\gamma - 1)(N + \gamma - 1)}{N[(a/\tau)(N + \gamma - 1) + (\delta/\tau) + 1]^2}. \quad (19)$$

Excess volatility is not simply a measure of pricing error, but more specifically measures the degree of overreaction in prices, i.e., excess volatility is positive for overreaction and negative for underreaction. An alternative measure of overreaction is the degree of predictability from reversals in prices. To derive accurate quantitative results regarding predictability, it would be preferable to work with a dynamic instead of a one-period model. We can, however, do some approximate analysis in our model by taking the negative of the covariance between price increments and normalizing by the average variance of these increments. This quantity is a rough way of ascertaining the magnitude of the coefficient in a regression of past versus future prices changes. We show in the Appendix that

$$\frac{-E[\tilde{p}(\tilde{x} - \tilde{p})]}{(1/2)(E[(\tilde{x} - \tilde{p})^2] + E[\tilde{p}^2])} = (\Gamma^{-1} + 1)^{-1}. \quad (20)$$

Because predictability and excess volatility have a monotonic relationship, they are equivalent in the context of our model and we can derive properties of either one. Hence, we obtain the comparative statics of excess volatility by taking partial derivatives with

<sup>8</sup>Our measure is different from the ones put forth in Campbell, Lo, and Mackinlay (1997), Shiller (1981), LeRoy and Porter (1981), etc. These authors compare the variance of prices instead of price changes to that of discounted dividends. Our measure also captures volatility in prices in excess of cash flows and is, therefore, equivalent to these previous measures.

respect to the parameters,  $\gamma$ ,  $N$ ,  $\delta/\tau$ , and  $a/\tau$ :

$$\frac{\partial \Gamma}{\partial \gamma} = \frac{2(a/\tau)[(1 + \delta/\tau)(N + 2(\gamma - 1)) + (aN/\tau)(N + \gamma - 1)]}{N[1 + \delta/\tau + (a/\tau)(N + \gamma - 1)]^3}, \quad (21)$$

$$\frac{\partial \Gamma}{\partial N} = -\frac{2(a/\tau)(\gamma - 1)[(a/\tau)(N + \gamma - 1)(2N + \gamma - 1) + (1 + \delta/\tau)(\gamma - 1)]}{N^2[1 + \delta/\tau + (a/\tau)(N + \gamma - 1)]^3}, \quad (22)$$

$$\frac{\partial \Gamma}{\partial (\delta/\tau)} = -\frac{4(a/\tau)(\gamma - 1)(N + \gamma - 1)}{N[1 + \delta/\tau + (a/\tau)(N + \gamma - 1)]^3}, \quad \text{and} \quad (23)$$

$$\frac{\partial \Gamma}{\partial (a/\tau)} = \frac{2(\gamma - 1)(N + \gamma - 1)}{N[1 + \delta/\tau + (a/\tau)(N + \gamma - 1)]^3} \cdot [1 + \delta/\tau - (a/\tau)(N + \gamma - 1)]. \quad (24)$$

From the signs of these derivatives, we have the following comparative statics.

**Proposition 8.**

- (i)  $\Gamma$  is increasing in  $\gamma$ , holding  $N$ ,  $\delta/\tau$ , and  $a/\tau$  fixed.
- (ii)  $\Gamma$  is decreasing in  $N$ , holding  $\gamma$ ,  $\delta/\tau$ , and  $a/\tau$  fixed.
- (iii)  $\Gamma$  is decreasing in  $\delta/\tau$ , holding  $\gamma$ ,  $N$ , and  $a/\tau$  fixed.
- (iv)  $\Gamma$  is increasing in  $a/\tau$  if  $(a/\tau)(\gamma - 1 + N) < 1 + (\delta/\tau)$  and decreasing in  $a/\tau$  if  $(a/\tau)(\gamma - 1 + N) > 1 + (\delta/\tau)$ , holding  $\gamma$ ,  $N$ , and  $\delta/\tau$  fixed.

The first implication is straightforward. Higher overconfidence ( $\gamma$ ) increases market overreaction, which increases excess volatility. A higher number of investors ( $N$ ) increases the precision of aggregate information in prices, and higher  $\delta/\tau$  increases the precision of public information. In both cases, investors weight this information more and their private information less, which decreases overreaction and excess volatility. As before, more and better private information, i.e., higher  $a/\tau$ , can decrease the degree of overreaction because it can decrease the gap between rational and irrational prices. However, additional private information can also increase overreaction by increasing the volatility of prices. Private information has high marginal impact on price volatility when  $a/\tau$ ,  $\gamma$ , and  $N$  are low and  $\delta/\tau$  is high consistent with the third comparative static.

The nonmonotonic relationship between private information and overreaction in our model distinguishes it from other models of overconfident investment. Daniel, Hirshleifer, and Subrahmanyam (1998), for example, predict unambiguous overreaction to the arrival of private information because they consider a market with no prior private information whereas our comparative statics examine the incremental effect of additional private information. Hence, our model predicts that more precise private information can increase or decrease overreaction, depending on relevant parameters. There appears to, in fact, be evidence in both of these directions. First, recent event study and time-series evidence finds that markets overreact to returns not associated with public news arrival. Chan (2003), for example, finds that stock prices overreact to abnormal returns without public news but underreact to such returns contemporaneous with public news. Daniel and Titman (2006) also report a similar finding that returns are unrelated to past accounting performance, but strongly negatively related to the component of past returns orthogonal to this publicly available fundamental information. There is also some evidence that the private information of institutional investors decreases overreaction in the returns of initial public offerings (IPOs), for example (See Shefrin, 2000, for a discussion of IPO underperformance

and overreaction.). Field and Lowry (2006) and Brav and Gompers (1997) find evidence consistent with the fact that investment by institutions decreases long-term underperformance of IPOs while investment by individuals exacerbates it.<sup>9</sup>

These comparative statics also provide empirical implications that could serve as tests of our model. One could, for example, test whether price overreaction has the proposed properties with respect to proxies for each of the model variables cross-sectionally across assets. There are several available proxies for the variables,  $N$ ,  $\delta/\tau$ ,  $a/\tau$ , and  $\gamma$ . A naive proxy for the number of investors in a stock,  $N$ , would be based on the total number of shareholders. However, because  $N$  is meant to represent the number of investors investing in information acquisition, a better measure could be based on the number of investors actively trading in the asset. For example, one could count the number of actively managed mutual funds with nonzero ownership in a given stock as a proxy for  $N$ . In the accounting literature, Barron, Kim, Lim, and Stevens (1998) develop proxies for the precisions of both public ( $\delta/\tau$ ) and private ( $a/\tau$ ) information based on the accuracy and dispersion of analyst forecasts, respectively. There are a number of alternatives as well. A company's degree of public disclosure or media coverage could also serve as measures of public information precision. In addition, an important feature of our theory is the fact that higher information investment results in higher private information precision. Consequently, per capita information investment can potentially serve as a proxy for precision of private information ( $a/\tau$ ). One method for representing this investment would be to average mutual funds' dollar investment in a stock times expense ratio across actively managed funds with nonzero investment in the stock.

For overconfidence, a standard proxy is that of trading volume as with previous models of overconfident investment (e.g., Odean, 1998) and empirical studies (e.g., Deaves, Lueders, and Luo, 2004; Glaser and Weber, 2003; Statman, Thorley, and Vorkink, 2004). Dollar trading volume in our model is increasing in overconfidence ( $\gamma$ ) as represented by the standard deviation of dollar demand times the number of investors,  $N$ :

$$N[\text{var}(\theta_i p)]^{1/2} = \frac{\gamma - 1}{\lambda} [(N - 1)a/\tau]^{1/2} \times \frac{[N((a/\tau)(N + \gamma - 1)^2 + \delta/\tau)^2 + (a/\tau)(N + \gamma - 1)^2 + N(\delta/\tau)]^{1/2}}{[(a/\tau)(N + \gamma - 1) + \delta/\tau + 1]^2}. \quad (25)$$

When using trading volume as a proxy for overconfidence, it is essential to control for information precision and the number of investors. For example, the number of investors increases trading volume while decreasing overreaction, which could potentially confound the positive relationship between overreaction and overconfidence.

The empirical literature on overreaction in the form of excess volatility and return predictability has until now focused on overreaction in the aggregate market. Our model

<sup>9</sup>There is an alternate explanation of these results that is also consistent with our model. Namely, individual investors have higher overconfidence, which increases overreaction. While we see this explanation as plausible, there are many arguments in favor of the information-based explanation including the experimental evidence of Haigh and List (2005) and Stael von Holstein, 1972 that institutional investors have more acute cognitive biases than individual investors and that they self-select to become investment professionals based on their higher overconfidence.

yields results regarding the cross-sectional properties of price overreaction, which have yet to be extensively explored.<sup>10</sup>

## 6. Conclusion

In this paper, we develop a model of information acquisition in a market with overconfident investors. We were able to derive implications from our model regarding the amount of information investment and the efficiency of the market as measured by both price quality and overreaction.

Our paper shows that irrationality on the part of investors does not necessarily make markets less efficient. In our model, overconfidence actually improves price quality under certain circumstances. In particular, overconfidence can generate information acquisition, whose effect on price quality can dominate the mispricing caused by this psychological bias.

We find that overconfidence generally improves market efficiency over rationality provided overconfidence is not too high because it introduces information into the market while having a comparatively small effect in generating mispricing. In addition, a market with very high overconfidence can also have superior price quality to a rational market when there is a high amount of private information acquired relative to publicly available information.

Finally, we were able to use our model to derive implications regarding the empirical properties of overreaction in prices. In particular, one distinct feature of our model is that more private information can either increase or decrease overreaction depending on relevant parameters. These implications can potentially serve as novel empirical tests of our behavioral theory of overreaction based on its cross-sectional properties, which have yet to be extensively studied.

We see this paper as a preliminary step and not the final verdict in the analysis of overconfidence, information acquisition, and mispricing. Considerable work remains in the analysis of these issues including the proposed empirical work, modifications to the model, and so on. One important topic, for instance, left absent from our paper is that of welfare analysis. To fully address this issue, one would need a model with explicit utility gains from higher price quality such as the aforementioned model of real investment in which more information would lead to superior investment policies and thereby higher social utility.

## Appendix A. Proofs of Propositions

### A.1. Proof of Proposition 1

Let us first conjecture a price  $p$  in the form of  $p = A\pi + B\bar{s} = (A + B)\tilde{x} + A\tilde{\eta} + B\bar{e}$ , and then solve the individual's maximization problem for  $\theta_i(\pi, s_i, p)$  in terms of  $A$  and  $B$ . The market clearing condition is next applied to solve for  $A$  and  $B$ .

<sup>10</sup>Several papers study how cross-sectional properties related to information and trade affect momentum such as Hong, Lim, and Stein (2000), Lee and Swaminathan (2000), and others. It is unclear, however, whether this evidence helps or hurts our case because momentum can be viewed as underreaction (as in Hong and Stein, 1999) or incremental overreaction (as in Daniel, Hirshleifer, and Subrahmanyam, 1998).

Let  $\mathbf{h}_i = [\pi, s_i, p]'$  be the information the  $i$ th investor possesses at time-1. Because  $\tilde{\pi} = \tilde{x} + \tilde{\eta}$ ,  $\tilde{s}_i = \tilde{x} + \tilde{\varepsilon}_i$ , and  $p = A\pi + B\tilde{s}$ , we have the following subjective statistics for trader  $i$ :

$$\begin{aligned}
 E_i[\tilde{\pi}] &= E_i[\tilde{s}_i] = E_i[\tilde{p}] = 0, \\
 Var_i(\tilde{\pi}) &= \tau^{-1} + \delta^{-1}, \\
 Var_i(\tilde{s}_i) &= \tau^{-1} + \beta^{-1}, \\
 Var_i(\tilde{p}) &= (A + B)^2\tau^{-1} + A^2\delta^{-1} + B^2[1/(Na) - (\beta - a)/(N^2a\beta)], \\
 Cov_i(\tilde{\pi}, \tilde{s}_i) &= \tau^{-1}, \\
 Cov_i(\tilde{\pi}, \tilde{p}) &= (A + B)\tau^{-1} + A\delta^{-1}, \\
 Cov_i(\tilde{s}_i, \tilde{p}) &= Cov_i(\tilde{s}_i, A\pi + (B/N)\tilde{s}_i + [B(N - 1)/N]\tilde{s}_{-i}) = (A + B)\tau^{-1} + B/(N\beta), \text{ and} \\
 Cov_i(\tilde{x}, \mathbf{h}_i) &= \begin{bmatrix} Cov_i(\tilde{x}, \tilde{\pi}) \\ Cov_i(\tilde{x}, \tilde{s}_i) \\ Cov_i(\tilde{x}, \tilde{p}) \end{bmatrix} = \begin{bmatrix} \tau^{-1} \\ \tau^{-1} \\ (A + B)\tau^{-1} \end{bmatrix}. \tag{A.1}
 \end{aligned}$$

Therefore, the covariance matrix of  $\mathbf{h}_i$  is

$$\begin{aligned}
 Var_i(\mathbf{h}_i) &= \begin{pmatrix} Var_i(\tilde{\pi}) & Cov_i(\tilde{\pi}, \tilde{s}_i) & Cov_i(\tilde{\pi}, \tilde{p}) \\ Cov_i(\tilde{s}_i, \tilde{\pi}) & Var_i(\tilde{s}_i) & Cov_i(\tilde{s}_i, \tilde{p}) \\ Cov_i(\tilde{p}, \tilde{\pi}) & Cov_i(\tilde{p}, \tilde{s}_i) & Var_i(\tilde{p}) \end{pmatrix} \\
 &= \begin{pmatrix} \tau^{-1} + \delta^{-1} & \tau^{-1} & (A + B)\tau^{-1} + A\delta^{-1} \\ \tau^{-1} & (\tau^{-1} + \beta^{-1}) & (A + B)\tau^{-1} + B/(N\beta) \\ (A + B)\tau^{-1} + A\delta^{-1} & (A + B)\tau^{-1} + B/(N\beta) & (A + B)^2\tau^{-1} + A^2\delta^{-1} + B^2[1/(Na) - (\beta - a)/(N^2a\beta)] \end{pmatrix}. \tag{A.2}
 \end{aligned}$$

The conditional expectation and variance of the risky payoff at time 2 are, therefore,

$$\begin{aligned}
 E_i[\tilde{x}|\mathbf{h}_i] &= \mathbf{h}_i' \cdot \mathbf{Var}_i(\mathbf{h}_i)^{-1} \cdot \mathbf{Cov}_i(\tilde{x}, \mathbf{h}_i) \\
 &= \frac{aN(p - A\pi) + B(\delta\pi + s_i(\beta - a))}{B(\delta + a(N - 1) + \beta + \tau)} \quad \text{and} \tag{A.3}
 \end{aligned}$$

$$\begin{aligned}
 Var_i[\tilde{x}|\mathbf{h}_i] &= \tau^{-1} - Cov_i(\tilde{x}, \mathbf{h}_i)' \mathbf{Var}_i(\mathbf{h}_i) \mathbf{Cov}_i(\tilde{x}, \mathbf{h}_i) \\
 &= \frac{1}{\delta + \beta + (N - 1)a + \tau}. \tag{A.4}
 \end{aligned}$$

Trader  $i$ 's maximization problem in Eq. (5) is standard in the Grossman-Stiglitz framework (Grossman and Stiglitz, 1980) with optimal demand

$$\theta_i(\pi, s_i, p) = \frac{E_i[\tilde{x}|\mathbf{h}_i] - \mathbf{p}}{\lambda \cdot Var_i[\tilde{x}|\mathbf{h}_i]} = \zeta_1\pi + \zeta_2s_i + \zeta_3p, \tag{A.5}$$

where

$$\begin{aligned}\zeta_1 &= \left(\frac{1}{\lambda}\right) \left(\delta - a \frac{A}{B} N\right), \\ \zeta_2 &= \left(\frac{1}{\lambda}\right) (\beta - a), \quad \text{and} \\ \zeta_3 &= \left(\frac{1}{\lambda}\right) \left[\frac{aN}{B} - (\delta + a(N-1) + \beta + \tau)\right].\end{aligned}$$

Applying the market clearing condition of OEE in Eq. (6) and substituting equilibrium  $\beta = b$ , we can solve for  $A$  and  $B$  as

$$\begin{aligned}\sum_{i=1}^N \theta_i(\pi, s_i, p)|_{\beta=b} &= N(\zeta_1 \pi + \zeta_2 s_i + \zeta_3 p)|_{\beta=b} = 0 \\ \Rightarrow p &= -\frac{\zeta_1 \pi + \zeta_2 \bar{s}}{\zeta_3} \Big|_{\beta=b} \\ \Rightarrow \begin{cases} A = \delta / [b + a(N-1) + \delta + \tau] \\ B = [b + a(N-1)] / [b + a(N-1) + \delta + \tau]. \end{cases} \quad (\text{A.6})\end{aligned}$$

We obtain the following solution for  $p$ :

$$\begin{aligned}p &= A\pi + B\bar{s} \\ &= \left(\frac{\delta}{b + (N-1)a + \delta + \tau}\right) \pi + \left(\frac{b + a(N-1)}{b + (N-1)a + \delta + \tau}\right) \left(\frac{s_1 + s_2 + \cdots + s_N}{N}\right). \quad (\text{A.7})\end{aligned}$$

After plugging in  $p$ ,  $A$ , and  $B$  into Eq. (A.5), we obtain

$$\begin{aligned}\theta_i(\pi, s_i, p) &= \frac{1}{\lambda} \left[ \frac{(b-a)\delta}{b + a(N-1)} \pi + (\beta - a)s_i \right. \\ &\quad \left. + \frac{a^2(N-1) - b(\delta + \beta + \tau) + a(\delta + b - (N-1)\beta + \tau)}{b + a(N-1)} p \right]. \quad (\text{A.8})\end{aligned}$$

In equilibrium,  $\beta = b$  so that

$$\theta_i(\pi, s_i, p)|_{\beta=b} = \left(\frac{b-a}{\lambda}\right) (s_i - \bar{s}) = \frac{(\gamma-1)a}{\lambda} (s_i - \bar{s}). \quad \square \quad (\text{A.9})$$

## A.2. Proof of Proposition 2

**Lemma 1.** Let  $X$  and  $Y$  be normal random variables and let  $k$  be a constant, then

$$E[e^{-kXY}] = [\det(\mathbf{I} + \mathbf{\Omega} \cdot \mathbf{V})]^{-1/2}, \quad (\text{A.10})$$

where  $\mathbf{\Omega} = \begin{pmatrix} \text{Var}(X) & \text{Cov}(X, Y) \\ \text{Cov}(X, Y) & \text{Var}(Y) \end{pmatrix}$ ,  $\mathbf{V} = \begin{pmatrix} 0 & k \\ k & 0 \end{pmatrix}$  and  $\mathbf{I}$  is the  $2 \times 2$  identity matrix.



**Proof.** Let  $\mathbf{Z} = \begin{bmatrix} X \\ Y \end{bmatrix}$ , then

$$\begin{aligned} E[e^{-kXY}] &= E[e^{-(1/2)\mathbf{Z}'\mathbf{V}\mathbf{Z}}] \\ &= \frac{1}{\sqrt{(2\pi)^2 \det(\mathbf{\Omega})}} \int_{\mathcal{R}^2} [e^{-(1/2)\mathbf{Z}'\mathbf{V}\mathbf{Z}} \cdot e^{-(1/2)\mathbf{Z}'\mathbf{\Omega}^{-1}\mathbf{Z}}] d\mathbf{Z} \\ &= \frac{1}{\sqrt{(2\pi)^2 \det(\mathbf{\Omega})}} \int_{\mathcal{R}^2} [e^{-(1/2)\mathbf{Z}'(\mathbf{\Omega}^{-1} + \mathbf{V})\mathbf{Z}}] d\mathbf{Z} \\ &= \frac{1}{\sqrt{(2\pi)^2 \det(\mathbf{\Omega})}} \cdot \sqrt{\frac{(2\pi)^2}{\det(\mathbf{\Omega}^{-1} + \mathbf{V})}} \\ &= [\det(\mathbf{I} + \mathbf{\Omega} \cdot \mathbf{V})]^{-(1/2)}. \end{aligned}$$

End of the proof of Lemma 1.  $\square$

Using the expression of the time-1 individual demand  $\theta_i(\pi, s_i, p)$  from the proof of Proposition 1, we can write investor  $i$ 's wealth as the profit of investing  $\theta_i(\tilde{x} - \tilde{p})$  minus the information cost  $c$ :

$$\begin{aligned} \tilde{w}_i &= \left(\frac{1}{\lambda}\right) \underbrace{(\Psi_1\pi + \Psi_2s_i + \Psi_3p)}_X \underbrace{(\tilde{x} - \tilde{p})}_Y - c \\ &= \left(\frac{1}{\lambda}\right) \underbrace{\Psi' \mathbf{h}_i}_X \underbrace{(\tilde{x} - \tilde{p})}_Y - c, \end{aligned} \quad (\text{A.11})$$

where

$$\Psi = \begin{bmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \end{bmatrix} = \begin{bmatrix} [(b-a)\delta]/[b+a(N-1)] \\ (\beta-a) \\ (a^2(N-1) - b(\delta+\beta+\tau) + a(\delta+b-(N-1)\beta+\tau))/[b+a(N-1)] \end{bmatrix} \quad (\text{A.12})$$

and, as defined in the proof of Proposition 1,  $\mathbf{h}_i = [\pi, \mathbf{s}_i, \mathbf{p}]'$  is the information set available for investor  $i$ .

According to Eq. (A.11) and Lemma 1,

$$\begin{aligned} E[U(w)] &= -e^{\lambda c} E \left[ \exp \left( -\frac{1}{2} (X \quad Y) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} \right) \right] \\ &= -e^{\lambda c} [\det(\mathbf{I} + \mathbf{\Omega V})], \end{aligned} \quad (\text{A.13})$$

where  $\mathbf{V} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ ,  $X = \Psi_1\pi + \Psi_2s_i + \Psi_3p$ ,  $Y = \tilde{x} - \tilde{p}$ , and  $\mathbf{\Omega}$  is the  $2 \times 2$  covariance matrix of  $X$  and  $Y$ . We can show that

$$\text{Var}_i(X) = \Psi' \text{Var}_i(\mathbf{h}_i) \Psi, \quad (\text{A.14})$$

$$\text{Var}_i(Y) = \text{Var}(\tilde{x}) - 2 \text{Cov}(\tilde{x}, \tilde{p}) + \text{Var}(\tilde{p}), \quad (\text{A.15})$$

and

$$\text{Cov}(X, Y) = \Psi' \text{Cov}(\tilde{x}, \mathbf{h}_i) - \Psi' \text{Var}_i(\mathbf{h}_i) \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \quad (\text{A.16})$$

Using these expressions and substituting  $\text{Var}_i(\mathbf{h}_i)$  and  $\text{Cov}_i(\tilde{x}, \mathbf{h}_i)$  with their values in Eqs. (A.1) and (A.2), we can rewrite and simplify the right-hand side of Eq. (A.13) as  $E[U(w)] = -e^{\lambda c} f(a, b, \beta)$ , where

$$\begin{aligned} [f(a, b, \beta)]^2 &= \frac{aN^2\beta[b + a(N-1) + \delta + \tau]^2}{[\beta + a(N-1) + \delta + \tau][(b + a(N-1))^2(a + \beta(N-1)) + aN^2\beta(\delta + \tau)]} \\ &= \frac{a\beta}{(\delta + \tau)^2} N^2 \left( \frac{b + a(N-1)}{\delta + \tau} + 1 \right)^2 \left( \frac{\beta + a(N-1)}{\delta + \tau} + 1 \right)^{-1} \\ &\quad \times \left[ \left( \frac{b + a(N-1)}{\delta + \tau} \right)^2 \left( \frac{a + \beta(N-1)}{\delta + \tau} \right) + \frac{a\beta}{(\delta + \tau)^2} N^2 \right]^{-1}. \quad \square \quad (\text{A.17}) \end{aligned}$$

### A.3. Proof of Proposition 3

We prove the monotonicity properties for  $\gamma$  and  $\lambda$  first, then show the asymptotics. We prove monotonicity for equilibrium information investment,  $c^*$ .

#### A.3.1. Monotonicity for $\gamma$

From Eq. (11), the first-order condition (FOC) for the time 0 optimization problem can be written as

$$\frac{\partial \log f(a, b, \beta)}{\partial \beta} \cdot \frac{\partial \beta}{\partial c} + \lambda = 0. \quad (\text{A.18})$$

Let  $\text{FOC}^*$  denote the left-hand side of the above equation in equilibrium, i.e., when  $\beta = b$ :

$$\text{FOC}^* = \frac{\partial \log f(a, b, \beta)}{\partial \beta} \cdot \frac{\partial \beta}{\partial c} + \lambda|_{\beta=b}. \quad (\text{A.19})$$

Using the implicit function theorem, we show that

$$\frac{dc^*}{d\gamma} = -\frac{d\text{FOC}^*}{d\gamma} \left( \frac{d\text{FOC}^*}{dc} \right)^{-1} > 0 \quad (\text{A.20})$$

for any interior solutions of the time 0 optimization problem.

For the corner solution of  $c^*(\gamma_1) = 0$ , we have  $c^*(\gamma_0) = c^*(\gamma_1) \leq c^*(\gamma_2)$ ,  $\forall \gamma_0 < \gamma_1 < \gamma_2$  because of Eq. (A.20).

The following two claims prove Eq. (A.20).

**Claim 1.**  $d\text{FOC}^*/d\gamma < 0$ .

**Proof.** First, we have the following relations:

$$\frac{\partial \beta}{\partial \gamma} = \alpha, \quad (\text{A.21})$$

$$\frac{\partial^2 \beta}{\partial c \partial \gamma} = \frac{\partial \alpha}{\partial c}, \quad (\text{A.22})$$

$$\frac{\partial \beta}{\partial c} = \gamma \frac{\partial \alpha}{\partial c}, \quad \text{and} \quad (\text{A.23})$$

$$\frac{\partial^2 \beta}{\partial c^2} = \gamma \frac{\partial^2 \alpha}{\partial c^2}. \quad (\text{A.24})$$

From Eq. (A.19), we can write  $dFOC^*/d\gamma$  as

$$\begin{aligned} \frac{dFOC^*}{d\gamma} &= \left( \frac{\partial^2 \log f}{\partial \beta^2} + \frac{\partial^2 \log f}{\partial \beta \partial b} \right) \cdot \frac{\partial \beta}{\partial \gamma} \cdot \frac{\partial \beta}{\partial c} + \frac{\partial \log f}{\partial \beta} \cdot \frac{\partial^2 \beta}{\partial c \partial \gamma} \\ &= \left( \frac{\partial^2 \log f}{\partial \beta^2} + \frac{\partial^2 \log f}{\partial \beta \partial b} \right) \cdot \alpha \gamma \frac{\partial \alpha}{\partial c} + \frac{\partial \log f}{\partial \beta} \cdot \frac{\partial \alpha}{\partial c} \\ &= \left[ \left( \frac{\partial^2 \log f}{\partial \beta^2} + \frac{\partial^2 \log f}{\partial \beta \partial b} \right) \cdot b + \frac{\partial \log f}{\partial \beta} \right] \cdot \underbrace{\frac{\partial \alpha}{\partial c}}_{>0}. \end{aligned} \quad (\text{A.25})$$

From the expression of  $f(a, b, \beta)$  in Proposition 2, the terms inside the square bracket can be calculated in terms of  $N$ ,  $a$ ,  $\gamma$ ,  $\delta$ , and  $\tau$  by using the equilibrium condition  $\beta = b = \gamma a$ :

$$\begin{aligned} &\left[ \left( \frac{\partial^2 \log f}{\partial \beta^2} + \frac{\partial^2 \log f}{\partial \beta \partial b} \right) \cdot b + \frac{\partial \log f}{\partial \beta} \right] \\ &= \frac{N-1}{\Omega_6} \{ -a^3(N+\gamma-1)^4 \Omega_1 \\ &\quad - a^2(N+\gamma-1)^2(\delta+\tau)(\Omega_2+\Omega_3) - a(\delta+\tau)^2 \Omega_4 - (\delta+\tau)^3 \Omega_5 \}, \end{aligned} \quad (\text{A.26})$$

where under the conditions  $N \geq 2$  and  $\gamma \geq 1$ , all  $\Omega$ 's in Eq. (A.26) are positive.

$$\Omega_1 = (2 - 2N + N^2)(\gamma - 1)^2 + (2N - 2)2\gamma > 0, \quad (\text{A.27})$$

$$\Omega_2 = (\gamma - 1)^2[(N - 2)(\gamma - 1)^2 + \gamma^2 + 6N - 1] > 0, \quad (\text{A.28})$$

$$\Omega_3 = 3N^3(\gamma^2 + 1) + N^2(\gamma - 1)(7 + 2\gamma^2 - 3\gamma) > 0, \quad (\text{A.29})$$

$$\begin{aligned} \Omega_4 &= 4N(\gamma - 1)^3 + (\gamma - 1)^4 + 3N^4(1 + \gamma^2) \\ &\quad + 4N^3(\gamma - 1)(2 + \gamma^2) + 2N^2(\gamma - 1)^2(4 + \gamma^2) > 0, \end{aligned} \quad (\text{A.30})$$

$$\Omega_5 = N^2(N - 1 + (N + 1)\gamma^2) > 0, \quad (\text{A.31})$$

$$\begin{aligned} \Omega_6 &= 2[a(N + \gamma - 1) + (\delta + \tau)]^2 \\ &\quad \times [a(N + \gamma - 1)^2(1 + (N - 1)\gamma) + N^2\gamma(\delta + \tau)]^2 > 0. \end{aligned} \quad (\text{A.32})$$

This proves that Eq. (A.26) is negative and  $(dFOC^*/d\gamma) < 0$ .  $\square$

**Claim 2.**  $dFOC^*/dc > 0$ .

**Proof.** From Eq. (A.19), we can write  $dFOC^*/dc$  as

$$\begin{aligned} \frac{dFOC^*}{dc} &= \left( \frac{\partial^2 \log f}{\partial \beta^2} + \frac{\partial^2 \log f}{\partial \beta \partial b} \right) \cdot \left( \frac{\partial \beta}{\partial c} \right)^2 + \frac{\partial^2 \log f}{\partial a \partial \beta} \cdot \frac{\partial \alpha}{\partial c} \cdot \frac{\partial \beta}{\partial c} + \frac{\partial \log f}{\partial \beta} \cdot \frac{\partial^2 \beta}{\partial c^2} \\ &= \left[ \frac{\partial^2 \log f}{\partial \beta^2} \cdot \left( \frac{\partial \beta}{\partial c} \right)^2 + \frac{\partial \log f}{\partial \beta} \cdot \frac{\partial^2 \beta}{\partial c^2} \right] \\ &\quad + \frac{\partial^2 \log f}{\partial \beta \partial b} \cdot \left( \frac{\partial \beta}{\partial c} \right)^2 + \frac{\partial^2 \log f}{\partial a \partial \beta} \cdot \frac{\partial \alpha}{\partial c} \cdot \frac{\partial \beta}{\partial c}. \end{aligned} \quad (A.33)$$

For any interior solution, the concavity of the objective function gives

$$\frac{\partial^2 \log f}{\partial \beta^2} \cdot \left( \frac{\partial \beta}{\partial c} \right)^2 + \frac{\partial \log f}{\partial \beta} \cdot \frac{\partial^2 \beta}{\partial c^2} > 0. \quad (A.34)$$

Plugging in  $f(a, b, \beta)$  from Proposition 2, we get

$$\frac{\partial^2 \log f}{\partial \beta \partial b} = \frac{a^2[b + a(N-1)]N^2(\delta + \tau)}{[b + a(N-1)]^2[a + b(N-1)] + abN^2(\delta + \tau)^2} > 0 \quad (A.35)$$

and

$$\begin{aligned} \frac{\partial^2 \log f}{\partial a \partial b} &= \frac{N-1}{\Omega_6} \{a^3(N + \gamma - 1)^4 \Omega_7 \\ &\quad + 2a^2(N + \gamma - 1)^2(\delta + \tau) \Omega_8 + a(\delta + \tau)^2 \Omega_9 + (\delta + \tau)^3 \Omega_{10}\} > 0 \end{aligned} \quad (A.36)$$

because under the conditions  $N \geq 2$  and  $\gamma \geq 1$ ,  $\Omega_7, \Omega_8, \Omega_9$ , and  $\Omega_{10}$  are all positive.

$$\Omega_7 = (2 - 2N + N^2)(\gamma^2 + 1) + 4(N-1)\gamma > 0, \quad (A.37)$$

$$\Omega_8 = 3N(\gamma - 1)^2 + (\gamma - 1)^3 + N^2[(N-1)(2 + \gamma^2) + 5\gamma - 2] > 0, \quad (A.38)$$

$$\begin{aligned} \Omega_9 &= 12N^3(\gamma - 1) + 10N^2(\gamma - 1)^2 + 4N(\gamma - 1)^3 \\ &\quad + (\gamma - 1)^4 + N^4(5 + \gamma^2) > 0, \end{aligned} \quad (A.39)$$

$$\Omega_{10} = 2N^2(N + \gamma - 1) > 0. \quad (A.40)$$

Therefore,  $dFOC^*/dc$  in Eq. (A.33) is greater than 0.

Claims 1 and 2 suffice to prove that  $c^*$  is nondecreasing with respect to  $\gamma$ .  $\square$

### A.3.2. Monotonicity for $\lambda$

For interior solutions of  $c^*$ ,

$$\frac{dc^*}{d\lambda} = -\frac{dFOC^*}{d\lambda} \cdot \left( \frac{dFOC^*}{dc} \right)^{-1}. \quad (A.41)$$

$dFOC^*/dc > 0$  for interior minimum  $c^*$  and  $dFOC^*/d\lambda = 1$  from Eq. (A.19). Therefore,  $dc^*/d\lambda < 0$ .

Similarly, for the corner solution of  $c^*(\lambda_1) = 0$ , we have  $c^*(\lambda_0) = c^*(\lambda_1) \leq c^*(\lambda_2), \forall \lambda_0 > \lambda_1 > \lambda_2$  because  $(dc^*/d\lambda) < 0$  as we have just proved.  $\square$

### A.3.3. Proof of asymptotic cases

For the asymptotics, we offer the following heuristic proofs. We first rewrite the equilibrium first-order condition as

$$-\left(\frac{1}{\lambda}\right) \frac{\partial \log f(a, b, \beta)}{\partial \beta} \cdot \frac{\partial \beta}{\partial c} \Big|_{\beta=b} = 1, \quad (\text{A.42})$$

where the left-hand term is the marginal dollar benefit of information cost, which should be equal to the marginal cost of 1 at an interior optimum. We denote this equilibrium marginal benefit as  $m_b^*$ , which we rewrite in terms of  $N$ ,  $\delta$ ,  $\tau$ ,  $a$ , and  $\gamma$ :

$$\begin{aligned} m_b^* &= -\left(\frac{1}{\lambda}\right) \frac{\partial \log f(a, b, \beta)}{\partial \beta} \cdot \frac{\partial \beta}{\partial c} \Big|_{\beta=b} \\ &= \left(\frac{1}{\lambda}\right) \cdot \frac{(N-1)(\gamma-1)[a(1+\gamma)(N+\gamma-1)^2 + (N\gamma + N + \gamma - 1)(\delta + \tau)]}{2(a(N+\gamma-1) + (\delta + \tau))[a(N+\gamma-1)^2(\gamma(N-1) + 1) + N^2\gamma(\delta + \tau)]} \cdot \frac{\partial \alpha}{\partial c}. \end{aligned} \quad (\text{A.43})$$

We start with the proof for  $\gamma$ . For any  $a$ ,  $\lim_{\gamma \rightarrow 1} m_b^* = 0$ . Because the marginal benefit of information goes to zero in this limit, the left boundary of  $c^* = 0$  is attained as  $\gamma \rightarrow 1$  and the investor collects zero information.

In the limit as  $\gamma \rightarrow \infty$ ,  $\lim_{\gamma \rightarrow \infty} m_b^* = [1/(2a\lambda)] \cdot (\partial \alpha / \partial c)$ . Hence,  $c_\infty^*$  is the solution to the following equation, which does not depend on  $N$ ,  $\delta$ , or  $\tau$  as noted in Section 4.3:

$$\frac{1}{2a\lambda} \cdot \frac{\partial \alpha}{\partial c} = 1. \quad (\text{A.44})$$

In the limit as  $\lambda \rightarrow 0$ ,  $m_b^* \rightarrow \infty$  for any  $a$ . Because the marginal benefit of information goes to infinity,  $c^*$  approaches infinity and the investor collects an arbitrarily large amount of information.

Because  $\lim_{\lambda \rightarrow \infty} m_b^* = 0$ ,  $c^* \rightarrow 0$  as  $\lambda \rightarrow \infty$  by the previous argument.  $\square$

### A.4. Proof of Proposition 4

We first prove two claims:

**Claim 3.**  $dFOC^*/d\tau > 0$  and  $dFOC^*/d\delta > 0$ .

**Proof.**

$$\frac{dFOC^*}{d\tau} = \frac{\partial \beta}{\partial c} \cdot \frac{\partial^2 \log f}{\partial \beta \partial \tau} = \frac{\partial \beta}{\partial c} \frac{(N-1)(\gamma-1)}{2} \cdot \frac{\Omega_{11}\Omega_{12}}{\Omega_{13}} > 0 \quad (\text{A.45})$$

because  $(\partial \beta)/(\partial c)$ ,  $\Omega_{11} = a(N+\gamma-1)^2 + N(\delta + \tau)$ ,  $\Omega_{12} = a(N+\gamma-1)^2[N+1+(N-1)\gamma] + N(\delta + \tau)(N\gamma + N + \gamma - 1)$ , and  $\Omega_{13} = [a(N+\gamma-1) + \delta + \tau]^2[a(N+\gamma-1)^2(N\gamma - \gamma + 1) + N^2\gamma(\delta + \tau)]^2$  are all positive values.

For public precision  $\delta$ , the expression is identical to the previous one for  $\tau$ .

$$\frac{dFOC^*}{d\delta} = \frac{\partial \beta}{\partial c} \cdot \frac{\partial^2 \log f}{\partial \beta \partial \delta} = \frac{\partial \beta}{\partial c} \frac{(N-1)(\gamma-1)}{2} \cdot \frac{\Omega_{11}\Omega_{12}}{\Omega_{13}} > 0, \quad (\text{A.46})$$

which is the identical expression for  $dFOC^*/d\tau$ .  $\square$

**Claim 4.**  $dFOC^*/d\xi < 0$ .

**Proof.** Because  $\beta = b = \gamma a = \gamma \xi \phi(c)$ , we have the following relations:

$$\frac{\partial \alpha}{\partial \xi} = \phi, \quad (\text{A.47})$$

$$\frac{\partial \beta}{\partial c} = \gamma \xi \phi', \quad (\text{A.48})$$

$$\frac{\partial \alpha}{\partial \xi} \cdot \frac{\partial \beta}{\partial c} = \phi \cdot \gamma \xi \phi' = \beta \phi', \quad \text{and} \quad (\text{A.49})$$

$$\frac{\partial^2 \beta}{\partial c \partial \xi} = \gamma \phi'. \quad (\text{A.50})$$

Using Eq. (A.19) and relations above, we show

$$\begin{aligned} \frac{dFOC^*}{d\xi} &= \left( \frac{\partial^2 \log f}{\partial \beta^2} + \frac{\partial^2 \log f}{\partial \beta \partial b} \right) \cdot \gamma \frac{\partial \alpha}{\partial \xi} \cdot \frac{\partial \beta}{\partial c} + \frac{\partial \log f}{\partial \beta} \cdot \frac{\partial^2 \beta}{\partial c \partial \xi} + \frac{\partial^2 \log f}{\partial \beta \partial a} \cdot \frac{\partial \alpha}{\partial \xi} \cdot \frac{\partial \beta}{\partial c} \\ &= \left[ \gamma \cdot \frac{\partial \log f}{\partial \beta} + \beta \cdot \frac{\partial^2 \log f}{\partial \beta \partial a} + \gamma \beta \left( \frac{\partial^2 \log f}{\partial \beta^2} + \frac{\partial^2 \log f}{\partial \beta \partial b} \right) \right] \cdot \phi' \\ &= - \left[ \frac{\gamma(\gamma - 1)(N - 1)(\delta + \tau)}{2} \right] \left( \frac{\Omega_{11}\Omega_{12}}{\Omega_{13}} \right) \cdot \phi' \\ &< 0, \end{aligned} \quad (\text{A.51})$$

where  $\Omega_{11}$ ,  $\Omega_{12}$ , and  $\Omega_{13}$  are defined in Claim 3.  $\square$

Again, we prove monotonicity for  $\xi$ ,  $\delta$ , and  $\tau$  first, then argue the asymptotic results.

Based on Claim 2, Claim 3, and Claim 4, we have

$$\frac{dc^*}{d\xi} = - \frac{dFOC^*}{d\xi} \cdot \left( \frac{dFOC^*}{dc} \right)^{-1} > 0, \quad (\text{A.52})$$

$$\frac{dc^*}{d\tau} = - \frac{dFOC^*}{d\tau} \cdot \left( \frac{dFOC^*}{dc} \right)^{-1} < 0, \quad (\text{A.53})$$

and

$$\frac{dc^*}{d\delta} = - \frac{dFOC^*}{d\delta} \cdot \left( \frac{dFOC^*}{dc} \right)^{-1} < 0 \quad (\text{A.54})$$

for interior solutions of  $c^*$ .

For a corner solution of  $c^*(\xi_1) = 0$ , we have  $c^*(\xi_0) = c^*(\xi_1) \leq c^*(\xi_2)$ ,  $\forall \xi_0 < \xi_1 < \xi_2$  because  $dc^*/d\xi > 0$ . The same argument also applies for  $\delta$  and  $\tau$ .

We now turn to the asymptotics. From the expression in Eq. (A.43) for the marginal benefit of information cost,  $m_b^* \rightarrow 0$  in limits as  $\xi \rightarrow \infty$ , as  $\delta \rightarrow \infty$ , as  $\tau \rightarrow \infty$ , and as  $N \rightarrow \infty$ . Hence, by the argument in the previous asymptotic proofs,  $c^*$  approaches zero in these limits.  $\square$

### A.5. Proof of Propositions 5 and 6

Because  $p = (A + B)\tilde{x} + A\tilde{\eta} + B\tilde{\varepsilon}$ , where  $A$  and  $B$  are coefficients defined in Proposition 1, MSE is given by

$$\begin{aligned}\text{MSE} &= E[(\tilde{x} - \tilde{p})^2] \\ &= E[((1 - A - B)^2\tilde{x} - A\tilde{\eta} - B\tilde{\varepsilon})^2] \\ &= (1 - A - B)^2 \frac{1}{\tau} + A^2 \frac{1}{\delta} + B^2 \frac{1}{aN} \\ &= \frac{a(N + \gamma - 1)^2 + N(\delta + \tau)}{N[a(\gamma + N - 1) + \delta + \tau]^2}.\end{aligned}\quad (\text{A.55})$$

Therefore, the limit for MSE as  $\gamma \rightarrow \infty$  is

$$\text{MSE}|_{\gamma \rightarrow \infty} = \frac{1}{a_{\infty}N}, \quad (\text{A.56})$$

where  $a_{\infty}$  is the asymptotic information precision as  $\gamma \rightarrow \infty$ .

Finally, we find equilibrium MSE at  $\gamma = 1$ . In Proposition 3, we show that when  $\gamma = 1$ ,  $c^* = 0$ . Because  $\alpha(0, \xi) = 0$ , we also have  $a = 0$  when  $\gamma = 1$ .

$$\text{MSE}|_{\gamma=1} = \text{MSE}|_{\gamma=1, a=0} = \frac{1}{\delta + \tau}. \quad \square \quad (\text{A.57})$$

### A.6. Proof of Proposition 7

The derivative of equilibrium MSE at  $\gamma = 1$  is

$$\left. \frac{d\text{MSE}}{d\gamma} \right|_{\gamma=1} = \left. \frac{\partial \text{MSE}}{\partial \gamma} \right|_{\gamma=1} + \left. \frac{\partial \text{MSE}}{\partial a} \cdot \frac{da}{d\gamma} \right|_{\gamma=1}. \quad (\text{A.58})$$

From the expression for MSE in Eq. (A.55), we show that

$$\left. \frac{\partial \text{MSE}}{\partial \gamma} \right|_{\gamma=1} = 0 \quad (\text{A.59})$$

and

$$\left. \frac{\partial \text{MSE}}{\partial a} \right|_{\gamma=1} = -\frac{N}{(\delta + \tau)^2} < 0, \quad (\text{A.60})$$

where we substitute  $a = 0$  at  $\gamma = 1$  in equilibrium because of Proposition 3 and the property  $\alpha(0, \xi) = 0$ . Because  $a$  is an increasing function of  $c^*$ , i.e.,  $a = \alpha(c^*, \xi)$ ,  $a$  has the same monotonicity properties as  $c^*$ . Because  $(dc^*/d\gamma) \geq 0$  from Proposition 3,  $(da/d\gamma) \geq 0$  as well. Therefore,  $(d\text{MSE}/d\gamma)|_{\gamma=1} \leq 0$ .  $\square$



### A.7. Proof of Eq. (20)

The numerator can be rewritten as

$$\begin{aligned}
 -E[\tilde{p}(\tilde{x} - \tilde{p})] &= -E[\tilde{p}(\tilde{x} - \tilde{p}_r + \tilde{p}_r - \tilde{p})] \\
 &= -\underbrace{E[\tilde{p}(\tilde{x} - \tilde{p}_r)]}_{=0} + E[\tilde{p}(\tilde{p}_r - \tilde{p})] \\
 &= E[\tilde{p}(\tilde{p} - \tilde{p}_r)]
 \end{aligned} \tag{A.61}$$

by the orthogonality of the rational error term to prior information. The denominator can be rewritten as follows from Eq. (18):

$$\begin{aligned}
 (1/2)(E[(\tilde{x} - \tilde{p})^2] + E[\tilde{p}^2]) &= (1/2)(2E[\tilde{p}(\tilde{p} - \tilde{p}_r)] + E[\tilde{x}^2]) \\
 &= E[\tilde{p}(\tilde{p} - \tilde{p}_r)] + (1/2)E[\tilde{x}^2].
 \end{aligned} \tag{A.62}$$

Hence, dividing the two expressions yields

$$\begin{aligned}
 \frac{-E[\tilde{p}(\tilde{x} - \tilde{p})]}{(1/2)(E[(\tilde{x} - \tilde{p})^2] + E[\tilde{p}^2])} &= \frac{E[\tilde{p}(\tilde{p} - \tilde{p}_r)]}{E[\tilde{p}(\tilde{p} - \tilde{p}_r)] + (1/2)E[\tilde{x}^2]} \\
 &= \left(1 + \frac{E[\tilde{x}^2]}{2E[\tilde{p}(\tilde{p} - \tilde{p}_r)]}\right)^{-1} \\
 &= (\Gamma^{-1} + 1)^{-1}. \quad \square
 \end{aligned} \tag{A.63}$$

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