The algebraic multigrid (AMG) method refers to a class of multilevel iterative methods based on a hierarchy of subspaces that are constructed algebraically by only using the underlying algebraic matrix. The main advantage of AMG, in comparison with the classic geometric multigrid method, is its convenience in practical applications.

In addition to the geometric multigrid (GMG) and single-grid multilevel method, Algebraic multigrid (AMG) methods were designed in an attempt to start from the algebraic linear system directly.

Recently, Xu and Zikatanov ([1]) provides an overview of AMG methods for solving large-scale systems of equations, such as those from discretizations of partial differential equations and a unified framework and theory that can be used to derive and analyse different algebraic multigrid methods in a coherent manner. AMG is often understood as the acronym of "algebraic multigrid", but it can also be understood as "abstract multigrid". Indeed, this paper demonstrates how and why an algebraic multigrid method can be better understood at a more abstract level. Given a smoother R for a matrix A, such as Gauss–Seidel or Jacobi, they prove that the optimal coarse space of dimension n_c is the span of the eigenvectors corresponding to the first eigenvectors $\bar{R}A$. They also prove that this optimal coarse space can be obtained via a constrained trace-minimization problem for a matrix associated with $\bar{R}A$, and demonstrate that coarse spaces of most existing AMG methods can be viewed as approximate solutions of this trace-minimization problem. Furthermore, they provide a general approach to the construction of quasi-optimal coarse spaces, and prove that under appropriate assumptions the resulting two-level AMG method for the underlying linear system converges uniformly with respect to the size of the problem, the coefficient variation and the anisotropy. Their theory applies to most existing multigrid methods, including the standard geometric multigrid method, classical AMG, energy-minimization AMG, unsmoothed and smoothed aggregation AMG and spectral AMGe.

References

[1] J. Xu and L. Zikatanov, *Algebraic multigrid methods* , Acta Numerica, 26 (2017), pp. 591–721.