Two-Sided Matching in International Markets

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Disclaimer

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Objectives

- **Enduring issues:** nature, severity, and welfare implications of trade frictions.
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• **Imperfect approaches:** fixed costs, beachhead costs, iceberg costs.
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- **Imperfect approaches**: fixed costs, beachhead costs, iceberg costs.

- **This paper**: dynamic search model with buyer-seller matching in international markets.
  - retailers intermediate between exporters and consumers
  - endogenous search and matching effort on both sides of the market.
  - many-to-many matching with discrete numbers of business partners.
  - characterize market equilibria in steady state and transition.
Applications

- Measure the value of international business relationships
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- **Measure international search/matching costs**, assess their role in inhibiting trade

Characterize welfare and market structure effects of globalization shocks. Effects of changes in the global population of sellers (the "China shock") and effects of reductions in search/matching costs (the "internet shock"). Assess impact of key players on equilibria (granularity).
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- Measure incidence of search/matching costs across firm types and states.
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- Assess impact of key players on equilibria (granularity)
Some related papers

- **Firm dynamics with customer accumulation** (Albornoz et al., 2012; Drozd and Nosal, 2012; Eaton et al., 2015; Piveteau, 2015; Fitzgerald et al., 2016; Arkolakis, 2016)

- Trade with search and matching frictions in product markets (Rauch and Watson, 2003; Rauch and Watson, 2004; Drozd and Nosal, 2012; Antras and Costinot, 2011; Fernandez, 2012; Eaton et al. 2015)

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- Dynamic models of trading networks (Chaney, 2014; Lim, 2016)
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• **Models of firm-to-firm trade** (Antras and Shor, 2012; Bernard, Moxnes, and Saito, 2015; Sugita, Teshima, and Siera, 2014; Carballo, Ottaviano, and Volpe-Martincus, 2016; Bernard, Moxnes, and Ullveit-Moe, 2016)
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- **Dynamic models of trading networks** (Chaney, 2014; Lim, 2016)
• **Fit to customs records** on U.S. apparel imports.
  • shipment values, dates, and HS10 codes
  • identifies **buyers** (importing retailers/wholesalers) and **sellers** (foreign exporters)

• **Why U.S. apparel?**
  • retailing is a relatively simple technology.
  • buyers are mostly retailers and local distributors, while sellers are mostly foreign firms
  • natural experiment: major new players in the market and Multifibre Arrangement phase-out
U.S. apparel consumption and imports

Consumption and imports over the years from 1990 to 2015, showing a general increase in both categories.
Number of apparel exporters to U.S.
Number of apparel exporters to U.S., by country

The chart shows the number of apparel exporters to the U.S. by country from 1996 to 2010. The countries included are Mexico, El Salvador, Honduras, India, Pakistan, Bangladesh, Vietnam, Cambodia, Indonesia, and China. The number of sellers is measured in thousands (Num. of Sellers (1,000)).
Client counts are roughly Pareto-distributed across buyers
Client counts are nearly Pareto-distributed across sellers.
Imports per seller and buyer connections

- Buyers with many suppliers import less per supplier
Network dynamics: relationship duration

- Match durations are nearly a Poisson process
## Transition of Number of Sellers for each Buyer

<table>
<thead>
<tr>
<th>$t$ \ $t+1$</th>
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<td>0.11</td>
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<td>4</td>
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<td>0.10</td>
<td>0.12</td>
<td>0.12</td>
<td>0.11</td>
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<tr>
<td>5</td>
<td>0.17</td>
<td>0.06</td>
<td>0.08</td>
<td>0.09</td>
<td>0.11</td>
<td>0.10</td>
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### Transition of Number of Buyers for each Seller

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<td>0.02</td>
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<td>0.19</td>
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<td>0.23</td>
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<td>0.05</td>
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<tr>
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<td>0.13</td>
<td>0.15</td>
<td>0.18</td>
<td>0.18</td>
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<td>4</td>
<td>0.10</td>
<td>0.10</td>
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<td>0.10</td>
<td>0.13</td>
<td>0.14</td>
<td>0.13</td>
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## Age and Partner Counts

(relative to entry year)

<table>
<thead>
<tr>
<th>Age</th>
<th># Sellers</th>
<th>Cumulative #</th>
<th># Buyers</th>
<th>Cumulative #</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.59</td>
<td>1.59</td>
<td>0.21</td>
<td>1.22</td>
</tr>
<tr>
<td>3</td>
<td>0.79</td>
<td>2.77</td>
<td>0.29</td>
<td>2.41</td>
</tr>
<tr>
<td>4</td>
<td>1.23</td>
<td>4.89</td>
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<td>4.48</td>
</tr>
</tbody>
</table>

- Buyers and sellers sample many business partners over their life cycles
- Average duration of relationships 1.24 years
Micro features of the apparel market: types of agents

- **Manufacturers**: use labor, materials and other inputs to make apparel
- **Wholesalers and retailers**: acquire apparel from manufacturers or sourcing firms
- **Consumers**: purchase apparel from retailers through brick-and-mortar stores and web sites
Our modeling abstractions

• We won’t distinguish commercial apparel importers (buyers) by type: wholesalers, retailers, sourcing firms, and other.
  • Many major importers are both wholesalers and retailers.
  • Most wholesale/retailers have their own sourcing divisions (McFarland et al., 2012; Lu, 2016).

• We will assume buyers do not own their suppliers’ production facilities.
  • With a few exceptions (e.g., Hanes), this is a good approximation to the data.

• We will not distinguish between web and brick-and-mortar outlets.
  • Wholesale/retailers often outsource brick-and-mortar retailing (e.g., Macy’s carries Polo line)
  • Most branded merchandise sells for the the same price (given barcode), in both types of outlets (Cavallo, 2016)
• As a group, domestic **consumers** spend a fixed amount $E$ on product category of interest (apparel).

• Measure-$M^B$ continuum of **buyers** (retailers), indexed by $b$, offer various, possibly overlapping, menus of varieties.

• **Sellers** (exporters), indexed by $x$, each supply a distinct variety to a finite number of stores.

• Consumer demand represented by nested CES utility function

\[ C = \left[ \int_{b \in M^B} (\mu_b C_b)^{\frac{\eta-1}{\eta}} \right]^\frac{\eta}{\eta-1}, \quad C_b = \left[ \sum_{x \in J_b} (\xi_x C_x^b)^{\frac{\alpha-1}{\alpha}} \right]^\frac{\alpha}{\alpha-1} \]
• Exogenous marginal cost of a buyer-seller pair: \( c_{xb} \)

• Optimal mark-up (Hottman et al., 2015, Atkeson and Burstein 2008):

\[
\frac{p_{xb} - c_{xb}}{p_{xb}} = \frac{1}{\eta}
\]

• Share of variety \( x \) in sales of retailer \( b \):

\[
h_{x|b} = \frac{\tilde{c}^{1-\alpha}_{xb}}{\sum_{x \in J_b} \tilde{c}^{1-\alpha}_{xb}}
\]

where

\[
\tilde{c}_{xb} = \frac{c_{xb}}{\tilde{\zeta}_x}
\]
Revenue flow

- **Bargaining** (Stole and Zweibel, 1996; Brugemann et al., 2016)
  - Surplus from match divided up according to Nash bargaining game
  - buyer bargains continuously with all sellers, treating each as marginal supplier
  - fixed share of surplus ($\beta$) for each seller implies same fixed share of profit flow

- **Implied payment flow** from buyer $b$ to seller $x$:

\[
\ln r_{xbt} = \left(\frac{\alpha - \eta}{\alpha - 1}\right) \ln h_{x|b,t} + \ln \left(\frac{\mu_b}{\bar{c}_{xb}}\right)^{\eta - 1} + d_t
\]

\[
= \left(\frac{\alpha - \eta}{\alpha - 1}\right) \ln h_{x|b,t} + (\eta - 1) \ln \mu_b - (\eta - 1) \ln \left(\frac{c_{xb}}{\bar{\zeta}_x}\right) + d_t
\]

\[
\underbrace{\ln \left(\frac{c_{xb}}{\bar{\zeta}_x}\right)}_{v_{xb}}
\]

- When $\alpha > \eta$, diminishing returns to adding clients.
Matching hazards

- **buyer (seller) visibility levels**
  
  \[ \sigma^B, \sigma^S \]

- chosen continuously by each agent to maximize expected present value of market participation. [Details]

- **visibility choices depend upon**
  
  - own type: \( \mu_b \in \{\mu^i\}_{i=1,l}, \quad \xi_x \in \{\xi^j\}_{j=1,J} \)
  
  - current portfolio of business relationships: \( s = (s_1, s_2, \ldots, s_J) \) for buyers, \( n \) for sellers
  
  - market tightness: match arrival rate per unit of visibility \( (\theta^B, \theta^S) \). [Details]
  
  - costs of visibility (search costs)

- **match arrival hazards**
  
  \[ \sigma^B_i(s)\theta^B, \sigma^S_j(n)\theta^S \]
Search (visibility) costs

- **Buyers’ search costs** depend on chosen visibility $\sigma^B$ and number of business partners, $s$:

$$k_B \left( \sigma^B, s \right) = \frac{k_0^B \left( \sigma^B \right)^{\nu_B}}{(s + 1)^{\gamma^B}}$$

- **Sellers’ search costs**, depend on chosen visibility $\sigma^S$ and number of business partners, $n$:

$$k_S \left( \sigma^S, n \right) = \frac{k_0^S \left( \sigma^S \right)^{\nu_S}}{(n + 1)^{\gamma^S}}$$

- Things to note
  - convexity: $\nu_B, \nu_S > 1$
  - network effects: $\gamma^B, \gamma^S > 0$
Steady states and transition dynamics

- Given $\sigma_i^B(s)$ and $\sigma_j^S(n)$, a matching function, and the exogenous match separation hazard, $\delta$, equations of motion for the measures of each type of agent ($M_i^B(s)$, $M_j^S(n)$) are implied.

- Imposing stationary ($\dot{M}_i^B(s) = \dot{M}_j^S(n) = 0$) determines equilibrium network structure.

- Can solve for transition between steady states using techniques from Acemoglu and Hawkins (2015)
Transfer function estimates

<table>
<thead>
<tr>
<th></th>
<th>1st Stage IV-FE</th>
<th></th>
<th>2nd stage IV-FE</th>
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<tbody>
<tr>
<td></td>
<td>(dep. var.: ( \ln h_{x</td>
<td>b,t} ))</td>
<td></td>
</tr>
<tr>
<td>( \ln \hat{h}_{x</td>
<td>b,t} )</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>( \ln \bar{r}_{-x</td>
<td>b,t} )</td>
<td>–0.239</td>
<td>(0.002)</td>
</tr>
<tr>
<td>( \ln \bar{r}_{-b</td>
<td>x} )</td>
<td>–</td>
<td>–0.381</td>
</tr>
<tr>
<td>instrument</td>
<td>–</td>
<td>–</td>
<td>(0.011)</td>
</tr>
<tr>
<td>match effects</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>year effects</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.8896</td>
<td>0.9045</td>
<td>0.8615</td>
</tr>
<tr>
<td>obs.</td>
<td>771,200</td>
<td>1,256,000</td>
<td>771,200</td>
</tr>
</tbody>
</table>

\[
\ln r_{xbt} = \left( \frac{\alpha - \eta}{\alpha - 1} \right) \ln h_{x|b,t} + \nu_{xb} + d_t + \epsilon_{xbt}
\]
• Fix $\rho = 0.05$ and $\beta = 0.5$

• Assume $\alpha = 4.35$ (Hottman, et al., 2015).

• Infer $\eta = 3.70$ from $\frac{\alpha - \eta}{\alpha - 1} = 0.193$

• Infer from match effects and estimate of $\eta$ that $\text{var}(\ln \mu_x) = 0.71$ and $\text{var}(\ln \xi_b) \approx 0.24$

• Discretize distributions following Kennan (2006).
Calibrating remaining parameters

- Calibrate search cost parameters to minimize sum of Euclidean distances (model versus data) for:
  - Buyer and seller degree distributions
  - Buyer and seller sizes (partner counts) over their life cycles
  - Within-buyer share of largest seller versus number of sellers
  - Partner transition matrices
• Imposing symmetry on cost function convexity parameters
  \( \nu_B = \nu_S = 3.099 \)

• Imposing symmetry on cost function network parameters
  \( \gamma^B = \gamma^S = 2.870 \)

• Cost function scalars: \( k_0^B = 2.578 \), and \( k_0^S = 0.465 \)

• Separation hazard inferred from separation rate that best matches match duration distribution, imposing Poisson process: 0.81
  
  • Adjusting for buyer and seller deaths,
    \( \delta = 0.81 - 0.07 - 0.15 = 0.59 \).
Matching moments: client distributions

- Target is estimated quadratic approximation to client distributions.

\[
\ln(1 - F(c)) = \beta_0 + \beta_1 \ln c + \beta_2 (\ln c)^2 + \epsilon
\]

<table>
<thead>
<tr>
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<th>buyers per seller</th>
<th>sellers per buyer</th>
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<tbody>
<tr>
<td>(\beta_1)</td>
<td>-1.898</td>
<td>-1.208</td>
</tr>
<tr>
<td>(\beta_2)</td>
<td>-0.200</td>
<td>0.0106</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.997</td>
<td>0.987</td>
</tr>
</tbody>
</table>

- \(\beta_1\): 
  - Buyers per seller data: -1.898
  - Buyers per seller model: -1.765
  - Sellers per buyer data: -1.208
  - Sellers per buyer model: -0.975

- \(\beta_2\): 
  - Buyers per seller: -0.200
  - Sellers per buyer: 0.0106

- \(R^2\): 
  - Buyers per seller: 0.997
  - Sellers per buyer: 0.987

- \(\epsilon\): 
  - Buyers per seller: 0.989
  - Sellers per buyer: 0.998
Matching moments: buyer and seller life cycles

<table>
<thead>
<tr>
<th>Sellers per buyer (relative to birth year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>age</td>
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<tr>
<td>-----</td>
</tr>
<tr>
<td>2</td>
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<td>3</td>
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</table>

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<tr>
<td>age</td>
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<td>2</td>
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<tr>
<td>3</td>
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<tr>
<td>4</td>
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Matching moments: within buyer share of largest seller

<table>
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<th>Data</th>
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<td>1.000</td>
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### Matching moments: sellers per buyer transition probs.

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<th>simulated</th>
<th>data</th>
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<td>probs: sellers per buyer</td>
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<td>$n \rightarrow n \rightarrow n \rightarrow$</td>
<td>$n \rightarrow n \rightarrow n \rightarrow$</td>
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<tr>
<td>$n-1$</td>
<td>$n$</td>
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<td>4</td>
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<td>5</td>
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<td>0.273</td>
<td>0.434</td>
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Preliminary model implications based on Colombian footwear imports

- Cost of search
- Value of business relationships
- Welfare effects of scaling down search costs by 30 percent
Large buyers spend disproportionately more on matching. To stay large, they must replace suppliers at a rapid rate.

Overall, search costs are 7.8% of export revenues—could account for a substantial fraction of unexplained trade frictions.
Buyer versus seller search costs

- Sellers bear most of the matching costs, especially at large firms.
For a high-$\mu$ retailer with 30 suppliers, value of portfolio, with replacement: $540,000; without replacement: $2,300,000.
• Reductions in search cost reduce value of customer connections, but help high-$\mu$, small $s$ retailers.
Counterfactual: 30% reduction in ko

- Takes 6-7 years to transit to new steady state.
Most adjustment comes in the right-hand tail—large firms search relatively more intensively, crowding out small firms.
Observed change in degree distribution
2006-09 versus 2010-12
• Redo experiments in previous slides for U.S. apparel

• **Explore the role of luck** in determining buyer and seller success.

• **Possible decomposition of changes in network structure**
  
  • choose change in search cost ($k_0$) to match trade expansion and shift in degree dist., given assumptions about mass of sellers ($M^S$).
  
  • how much is due to changes in efficiency of search/matching $k_0$ versus $M^S$?
\[ \pi_i^T(s) = \frac{E}{P\eta} \left( \frac{P(s)^{1-\eta}}{P^{-\eta}} \right) \mu_i^{\eta-1} \]

\[ = \frac{E}{\eta P^{1-\eta}} \left[ \sum_j s_j \left( \frac{\eta}{\eta - 1} \right)^{1-\alpha} \tilde{c}_{ji}^{1-\alpha} \right]^{\frac{\eta-1}{\alpha-1}} \mu_i^{\eta-1} \]

\[ \frac{\partial \pi_i^T(s)}{\partial s_j} = \frac{1}{\alpha - 1} \left( \frac{\eta}{\eta - 1} \right)^{-\eta} E \frac{1}{P^{1-\eta}} \left[ \sum_j s_j \tilde{c}_{ji}^{1-\alpha} \right]^{\frac{\alpha-\eta}{1-\alpha}} \tilde{c}_{ji}^{1-\alpha} \mu_i^{\eta-1} \]

\[ = (h_{j|i})^{\frac{\alpha-\eta}{\alpha-1}} \frac{E}{P^{1-\eta}} \left( \frac{\eta}{\eta - 1} \right)^{-\eta} \left( \frac{\mu_i}{\tilde{c}_{ji}} \right)^{\eta-1} \left[ \frac{1}{\alpha - 1} \right] \]

So, assuming sellers incur some fraction \( \lambda \) of their match’s marginal costs and get share \( \beta \) of the rent flow:

\[ r_i^T(s) = (h_{j|i})^{\frac{\alpha-\eta}{\alpha-1}} \frac{E}{P^{1-\eta}} \left( \frac{\eta}{\eta - 1} \right)^{-\eta} \left( \frac{\mu_i}{\tilde{c}_{ji}} \right)^{\eta-1} \left[ \frac{\beta}{\alpha - 1} + \lambda \right] \]
Value functions

Flow value of a type-\(i\) buyer currently in state \(s\):

\[
\rho V_i^B(s) = \pi_i^B(s) - k_s^B(\sigma_i^B(s)) + \sigma_i^B(s) \sum_j \theta^B P_j^S \left[ V_i^B(s + \ell_j) - V_i^B(s) \right] \\
+ \delta \sum_j s_j \left[ V_i^B(s - \ell_j) - V_i^B(s) \right]
\]

Flow value to a type-\(j\) seller of match with a type-\(i\) buyer in state \(s\):

\[
\rho V_{ji}^S(s) = \tau_{ji}(s) + \sigma_i^B(s) \sum_k \theta^B P_k^S \left[ V_{ji}^S(s + \ell_k) - V_{ji}^S(s) \right] \\
+ \delta \sum_{k=1}^J (s_k - 1_{k=j}) \left[ V_{ji}^S(s - \ell_k) - V_{ji}^S(s) \right]
\]

where \(\theta^B\) is arrival hazard of sellers per unit of buyer visibility and remaining definitions appear on the next slide.
First order conditions

- Definitions
  - \( P_i^B(s) = H_i^B(s) / H^B \): relative visibility of type-\( i \) buyers in state \( s \)
  - \( P_j^S = H_j^S / H^S \): relative visibility of type-\( j \) sellers
  - \( \ell_j \): \( 1 \times J \) vector of zeros except for \( j^{th} \) element, which is 1.

- First-order conditions:

\[
\frac{\partial k^B}{\partial \sigma_i^B} (\sigma_i^B, s) = \sum_j \theta^B P_j^S \left[ V_i^B(s + \ell_j) - V_i^B(s) \right]
\]

\[
\frac{\partial k^S}{\partial \sigma_j^S} (\sigma_j^S, n) = \sum_i \sum_{s \in S} \theta^S P_i^B(s) V_{ji}^S(s + \ell_j).
\]
Market tightness

- Visibility of type $j$ sellers who are matched with $n$ buyers:
  \[ H^S_j(n) = \sigma^S_j(n) M^S_j(n) \]
- Overall visibility of sellers to buyers:
  \[ H^S = \sum_{j=1}^{J} \sum_{n=0}^{n_{\text{max}}} H^S_j(n) \]
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Market tightness

• Visibility of type \( j \) sellers who are matched with \( n \) buyers:
  \[
  H_j^S(n) = \sigma_j^S(n) M_j^S(n)
  \]

• Overall visibility of sellers to buyers:
  \[
  H^S = \sum_{j=1}^{J} \sum_{n=0}^{n_{\text{max}}} H_j^S(n)
  \]

• Visibility of type \( i \) buyers who are matched with \( s \) sellers:
  \[
  H_i^B(s) = \sigma_i^B(s) M_i^B(s)
  \]

• Overall visibility of buyers to sellers:
  \[
  H^B = \sum_{i=1}^{I} \sum_{s=0}^{s_{\text{max}}} H_i^B(s)
  \]

• Exogenous:
  \[
  M_i^B = \sum_{s=1}^{s_{\text{max}}} M_i^B(s), \quad M_j^S = \sum_{n=1}^{n_{\text{max}}} M_j^S(n)
  \]
Market tightness, continued

- Measure of match flow per unit time (Petrongolo and Pissarides, 2001):
  \[ X = f(H^S, H^B) = H^B \left[ 1 - (1 - \frac{1}{H^B})^{H^S} \right] \approx H^B \left[ 1 - e^{-H^S/H^B} \right] \]

- Match flow per unit of buyer visibility, seller visibility:
  \[ \theta^B = \frac{f(H^S, H^B)}{H^B}, \quad \theta^S = \frac{f(H^S, H^B)}{H^S} \]

- Share of matches involving buyers of type \( i \) with \( s \) sellers, sellers of type \( j \) with \( n \) buyers:
  \[ P_i^B(s) = H_i^B(s) / H^B, \quad P_j^S(n) = H_j^S(n) / H^S \]
Equations of motion

- Suppressing the state vector \( s \), the measure of type—\( i \) buyers with \( s_j \) type—\( j \) sellers evolves according to:

\[
\dot{M}_i^B(s_j) = \sigma_i^B(s_j - 1) \theta^B M_i^B(s_j - 1) + \delta(s_j + 1) M_i^B(s_j + 1) \\
- \left( \sigma_i^B(s_j) \theta^B M_i^B(s_j) + \delta s M_i^B(s_j) \right).
\]

\( s = 1, \ldots, s_{\text{max}}; \ i = 1, \ldots, I \)

\[
\dot{M}_i^B(0) = \delta M_i^B(1) - \sigma_i^B(0) \theta^B M_i^B(0) \quad i = 1, \ldots, I
\]

- For measures of sellers, replace \( B \) with \( S \), \( i \) with \( j \), and \( s \) with \( n \) (number of buyers).
Dealing with clustering

- Some retailers specialize in plastic/rubber top sports shoes; others men’s leather shoes. Ditto for suppliers.
- Remedy: type-\(i\) buyer and type-\(j\) seller are compatible with probability
  \[d_{ij} \in [0, 1]\]
- Expected success rates for type-\(i\) buyer; type-\(j\) seller
  \[
a^B_i = \frac{\sum_j \sum_{n=0}^{\infty} d_{ij} \sigma^S_j (n) P^S_j (n)}{\sum_j \sum_{n=0}^{\infty} \sigma^S_j (n) P^S_j (n)}
  \]
  \[
a^S_j = \frac{\sum_i \sum_{s=0}^{\infty} d_{ij} \sigma^B_i (s) P^B_i (s)}{\sum_i \sum_{s=0}^{\infty} \sigma^B_i (s) P^B_i (s)}
  \]
Dealing with clustering, continued

• For type-\(i\) buyer with \(s\) suppliers, hazard of finding another compatible seller is

\[
\sigma_{i}^{B}(s)a_{i}^{B}\theta^{B}
\]

• For a type-\(j\) seller with \(n\) buyers, the hazard of finding another compatible buyer is

\[
\sigma_{j}^{S}(n)a_{j}^{S}\theta^{S}
\]

• Complication: \(a_{i}^{B}\) and \(a_{j}^{S}\) depend on \(d_{ij}\), which cannot be directly observed.
Dealing with clustering, continued

- Probability that a randomly matching seller encounters a type-\(i\) buyer; randomly matching buyer encounters a type-\(j\) seller:

\[
\phi^B_i = \frac{\sum_{s=0}^{\infty} \sigma_i^B(s) P_i^B(s)}{\sum_{i'} \sum_{s=0}^{\infty} \sigma_{i'}^B(s) P_{i'}^B(s)},
\]

\[
\phi^S_j = \frac{\sum_{n=0}^{\infty} \sigma_j^S(n) P_b^S(j)}{\sum_{j'} \sum_{n=0}^{\infty} \sigma_{j'}^S(n) P_{j'}^S(n)}.
\]

- Letting \(f_{ij}\) be the observed joint distribution of matches,

\[
f_{ij} = \phi_i^B \phi_j^S d_{ij}
\]

\[
d_{ij} = \frac{f_{ij}}{\phi_i^B \phi_j^S}
\]
Substituting this $d_{ij}$ expression back into the success rate functions yields:

$$a_i^B = \frac{\sum_j \sum_{n=0}^{\infty} d_{ij} \sigma_j^S(n) P_j^S(n)}{\sum_{j'} \sum_{n=0}^{\infty} \sigma_{j'}^S(n) P_{j'}^S(n)} = \frac{\sum_j f_{ij}}{\phi_i^B}$$

and

$$a_j^S = \frac{\sum_i \sum_{s=0}^{\infty} d_{ij} \sigma_i^B(s) P_i^B(s)}{\sum_{i'} \sum_{s=0}^{\infty} \sigma_{i'}^B(s) P_{i'}^B(s)} = \frac{\sum_i f_{ij}}{\phi_j^S}$$

Implication: $a_i^B$ and $a_j^S$ are additional objects to update, but given $f_{ij}$, there are no additional parameters involved.