Combinatory Analysis 2008: Partitions, q-series, and Applications

Penn State University

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Talk Titles and Abstracts

Krishnaswami Alladi

Two fundamental partition statistics: hook lengths and successive ranks

Abstract: Dyson defined the rank of a partition as the largest part minus the number of parts. The rank is famous because of its role in the combinatorial explanation of Ramanujan's partition congruences mod 5 and 7. More generally, one can consider successive ranks or hook differences in Ferrers graphs. Andrews et-al established some very appealing Rogers-Ramanujan type partition theorems by imposing restrictions on the sizes of successive ranks. I have studied in some detail the successive hook lengths (in contrast to successive ranks) and obtained weighted partition identities providing new connections between fundamental partition functions. For example, by combining the study of hook lengths and successive ranks, one can get new proofs and insights (both combinatorial and q-theoretic) on the Andrews et-al theorems on partitions with restricted hook differences. I will conclude by discussing some recent results on basis partitions which are the minimal partitions associated with successive ranks.

George Andrews

Parity in Partitions Identities

Abstract: The role of parity in partition identities dates back to Euler's theorem relating partitions with distinct parts and partitions with odd parts. More recently, the Goellnitz-Gordon identities essentially introduced parity into the Rogers-Ramanujan identities. In this talk, we shall discuss a number of new results in which the role of parity is further developed in the Rogers-Ramanujan identities and their generalizations. We shall also introduce "parity indices" which not only interact with the Rogers-Ramanujan identities but also throw light on mock theta functions and the little q-Jacobi polynomials.

Richard Askey

Problems for George Andrews and Others

Abstract: I have a backlog of problems I have been unable to solve, and will describe some of them.

Bruce Berndt

Analysis in Ramanujan's Lost Notebook

Abstract: We discuss some recently examined entries falling under the purview of either classical analysis or classical analytic number theory in Ramanujan's Lost Notebook. Warning: It is highly likely that this lecture will not mention any q.

Kathrin Bringmann

Rank-Crank Type PDEs and Non-holomorphic Jacobi Forms

Abstract: In this talk I will show how Rank-Crank type PDEs (first found by Atkin and Garvan) occur naturally in the framework of non-holomorphic Jacobi forms and give an infinite family of such differential equations. As an application I obtain an infinite family of congruences for odd Durfee symbols, a partition statistic introduced by George Andrews. This is joint work with Sander Zwegers.

Zhu Cao

Integer Matrix Exact Covering System and Product Identities for Theta Funtions

Abstract: In this talk, we derive a general approach for establishing q-series identities. Using this method, we can show that certain product identities for theta funtions correspond to integer matrix exact covering system. Many identities can be shown as special cases of this method.

Hei-Chi Chan

An Analog of Ramanujan's "Most Beautiful Identity" and the Cubic Continued Fraction

Abstract: In this talk, we will discuss an analog of Ramanujan's "Most Beautiful Identity" that is derived from Ramanujan's cubic continued fraction. We will also discuss certain applications of this identity which involve the congruence properties of the partition function, $a(n)$, defined by $\sum_n a(n)q^n = \prod_n (1-q^n)^{-1}(1-q^{2n})^{-1}$. The method involved is quite general and is applicable to other continued fractions. For example, the same method shows how Jacobi's 'rather obscure' 8-fold identity (emphaequatio identica satis abstrusa) can be derived from an identity of a Ramanujan-Selberg continued fraction. We will also report some recent results from the joint work with Shaun Cooper.

Song Heng Chan

A New Proof of Winquist's Identity

Abstract: We shall present a new proof of Winquist's identity.

Sylvie Corteel

Plane overpartitions

Abstract: I will describe a natural object which generating function is

$$\prod_{i\geq 1}\left(\frac{1+q^i}{1-q^i}\right)^i.$$

This is joint work with C. Savelief and M. Vuletic.

Atul Dixit

A Transformation Formula Involving the Gamma and Riemann Zeta Functions in Ramanujan's Lost Notebook

Abstract: In 'The Lost Notebook and Other Unpublished Papers' of Ramanujan are present some manuscripts of Ramanujan in the handwriting of G. N. Watson which are 'copied from loose papers'. We present a proof of a beautiful formula of Ramanujan in one of these manuscripts, namely a transformation formula involving the Gamma function and Riemann Zeta function. This formula elegantly yields a modular relation. This is joint work with Bruce C. Berndt.

Kimmo Eriksson

Partitions and Cultural Evolution

Abstract: Having said goodbye to pure mathematics, I am working at the interdisciplinary Center for the Study of Cultural Evolution in Stockholm. I thought I would never see another integer partition in my research - but I was wrong. I will discuss a couple of ways in which, unexpectedly, integer partitions have entered the picture in my work at the Center.

Amanda Folsom

The *spt*-function of Andrews

Abstract: Recently, Andrews introduced the function $s(n) = spt(n)$ which counts the number of smallest parts among the integer partitions of n. We show that its generating function satisfies an identity analogous to Ramanujan's mock theta identities. As a consequence, we are able to completely determine the parity of $s(n)$. Using another type of identity, one based on Hecke operators, we obtain a complete multiplicative theory for $s(n)$ modulo 3. These congruences confirm unpublished conjectures of Garvan and Sellers. Our methods generalize to all integral moduli. This is joint work with Ken Ono.

Frank Garvan

Yet Even More Partition Congruences

Abstract: We consider the problem of congruences for the rank of partitions, the crank of partitions and Andrews' smallest parts partition function. How common are such congruences? Are they related?

Chadwick Gugg

Modular Identities Involving Powers of the Rogers-Ramanujan Functions

Abstract: In his Lost Notebook, Ramanujan recorded 40 elegant modular relations for the Rogers-Ramanujan functions. The first one stands out as unique, in that it is the only one involving powers of the Rogers-Ramanujan functions. In this talk, we provide further examples of identities involving powers of the Rogers-Ramanujan functions. Connections are made to continued fractions, partitions, and analogues of the Rogers-Ramanujan functions.

Michael Hirschhorn

Partitions with even parts distinct

Abstract: Ramanujan showed that $p(n)$ is divisible by 5 at least one fifth of the time. I shall outline a result of George Andrews, James Sellers and myself that $ped(n)$ is divisible by 6 at least one sixth of the time, where $ped(n)$ is the number of partitions of n with even parts distinct.

Brian Hopkins

Garden of Eden Partitions for General Column-to-Row Operations

Abstract: Bulgarian solitaire can be defined by moving the first column of a partition's Ferrers diagram to a row. Partitions with no preimage under this operation are called Garden of Eden partitions; they have been counted both analytically and combinatorially. The Bulgarian solitaire operation can be generalized to moving multiple columns to rows, until finally the effect is equivalent to conjugation. For each of these operations, there are GE-partitions. We present a nice two-variable generating function that determines the number of these GE-partitions; for a fixed number of columns being moved, we can produce formulas in terms of standard partition numbers. This is joint work with Louis Kolitsch.

Tim Huber

Distribution of Zeros for Laurent Coefficients of Jacobian Elliptic Functions

Abstract: In 1970, Rankin and Swinnerton-Dyer gave a simple and elegant proof that the τ-zeros of Eisenstein series for $SL(2,\mathbb{Z})$ all lie on the unit circle between $\tau = i$ and $\tau = e^{2\pi i/3}$. Recently, H. Nozaki added to our knowledge of the distribution of these zeros by proving that the zeros of Eisenstein series of index n interlace with those of index $n+12$. The Eisenstein series arise from the Laurent expansion of the Weierstrass $\wp$-function about zero, or equivalently, from the squares of certain Jacobian elliptic functions. I conjecture that the distribution of zeros for the classical Eisenstein series is characteristic of the more general class of functions arising from Laurent coefficients for the twelve Jacobian elliptic functions and their squares. In particular, the zeros of these series are located along certain arcs of the boundaries for their respective fundamental domains, and the zeros of successive series interlace. In this talk I will discuss preliminary results on the zeros of the aforementioned series and the behavior of their divisor polynomials.

Mourad Ismail

Nonlinear Difference Equations and q-orthogonal polynomials

Abstract: We show how structure relations for orthogonal polynomials lead to nonlinear difference equations some having the Painleve property. Several examples will be given.

William Keith

Partitions in $\mathbb{Z}_p$

Abstract: In how many ways can we write the residue class $[i]$ mod p as a sum of other residue classes mod p? Under natural restrictions, this simple question turns out to have some attractive symmetries. This talk will lay out the basics and some early structural and enumerative results, and comment on some connections to other questions in number theory.

Byungchan Kim

A Crank Analog for a Certain Kind of Partition Function Arising from the Cubic Continued Fraction

Abstract: In a series of papers, H.-C. Chan has studied congruence properties of a certain kind of partition function that arises from the cubic continued fraction. This partition function, $a(n)$ is defined by

$$\sum_{n=0}^{\infty} a(n)q^n = \frac{1}{(q;q)_\infty (q^2;q^2)_\infty}.$$

In particular, he proved that $a(3n+2) \equiv 0 \pmod 3$. As Chan mentioned in his paper, it is natural to ask if there exists an analog of the rank or the crank for the ordinary partition function that provides a combinatorial explanation of the above congruence. Here, we will

define a crank analog for $a(n)$ and prove that

$$M'(0,3,3n+2) \equiv M'(1,3,3n+2) \equiv M'(2,3,3n+2) \pmod 3,$$

for all nonnegative integers n, where $M'(m,N,n)$ is the number of partitions of n with crank $\equiv m \pmod N$. Next, using the theory of modular forms, we will investigate further congruences of $a(n)$

Christian Krattenthaler

Punching a hole into a theorem of George Andrews

Abstract: Almost 30 years ago, George Andrews proved MacMahon's conjecture on the enumeration of symmetric plane partitions. In joint work with Mihai Ciucu, I managed "to punch a hole" into this theorem, thereby obtaining a more general result. I shall explain how we should understand this statement, and what this has to do with electrostatics.

Brandt Kronholm

Palindromic Congruence Properties of $p(n,m)$

Abstract: $p(n,m)$ is the function which enumerates the number of partitions of n into exactly m parts. In a previous publication [PAMS,133 (2005), 2891-2895] the speaker established and gave an explicit formula for an infinite family of Ramanujan-like congruences for $p(n,m)$. The goal of this talk is to show that almost all of these congruences occur in pairs and those that do not are easily identified. Moreover, preliminary to our main result, we will show that modulo 5, the first 26 values of $p(n,4)$ determine all subsequent values of $p(n,4)$ modulo 5. This result will be generalized.

Kagan Kursungoz

Parity Indices and Andrews-Gordon Identities

Abstract: In his recent paper "Parity in Partition Identities", Andrews first considered various parity questions related to Andrews-Gordon Identities. He then defined parity indices of partitions. We will demostrate a reconciliation of these two themes, which helps solve one of the open problems Andrews listed at the end of the mentioned paper.

Joon Yop Lee

Reordering Methods in q-series

Abstract: Partition bijection is a combinatorial method which proves q-series identities with a bijection. Many q-series identities were proved combinatorially, but the number of these identities is very small, compared to the number of whole identities.

In this talk, introducing new partition bijections which will be called the 1st and the 2nd reordering method, some q-series identities will be proved by these two.

James Lepowsky

Partition Theory as an Inspiration for Vertex-Algebraic Structure

Abstract: I will sketch how the Rogers-Ramanujan and Gordon-Andrews identities have played, and continue to play, an important role in the development of the theory of vertex operator algebras. Among the recent and current developments I will discuss joint work with Corina Calinescu, Stefano Capparelli and Antun Milas on new vertex-algebraic structure underlying the Rogers-Ramanujan and Rogers-Selberg recursions.

David Little

A New Combinatorial Approach to q-Series Identities

Abstract: In this talk, we present a new combinatorial context for q-series identities. By simply rewriting many partition theoretic identities, we can make them more amenable to tilings of an infinitely long board. The advantage of this approach is that beautiful yet complex bijections relating two different collections of partitions are replaced by simple constructions involving the same collection of tilings. Classical results of Euler, Sylvester, Lebesgue, Rogers, and Ramanujan will be discussed. This is joint work with James Sellers.

Jeremy Lovejoy

Andrews' generalization of Selberg's q-difference equations

Abstract: Andrews' generalization of Selberg's q-difference equations is one of the foundations of modern partition theory. The talk will trace the history of this topic, from Andrews' proof of Basil Gordon's combinatorial generalization of the Rogers-Ramanujan identities up to some recent advances.

Karl Mahlburg

Asymptotics for partitions without sequences

Abstract: A partition that does not contain any consecutive integers as parts is known as a partition without sequences. The generating function for such partitions is the product of a mock theta function and an infinite series, and the analytic properties of such products are not well understood. We obtain an asymptotic expansion for the number of partitions without sequences of n using a generalized version of the Hardy-Ramanujan circle method. The expansion is particularly interesting because the "error integrals" in the modular transformation of the mock theta function component make a nontrivial contribution to the exponential

main term.

Riad Masri

The Error Term in Rademacher's Formula for the Partition Function

Abstract: We will explain how the equidistribution of Galois orbits of Heegner points on the modular curve $X_0(6)$ can be used to sharpen Lehmer's bound on the error term in Rademacher's formula for the partition function. This is joint work with Amanda Folsom.

Stephen Milne

A nonterminating q-Dougall Summation Theorem for Basic Hypergeometric Series in $U(n)$

Abstract: In this talk we extend important classical one-variable summations and transformations of Bailey to multiple basic hypergeometric series very-well-poised on unitary groups $U(n+1)$. In particular, we derive multivariable generalizations of Bailey's 3-term transformation formula for ${}_8\phi_7$ series, and Bailey's nonterminating q-Dougall summation formula. As pointed out by Michael Schlosser, our nonterminating $U(n+1)$ q-Dougall summation formula yields a natural multivariable extension of Jacobi's classical identity for eighth powers of theta functions. All of this work is a consequence of the nonterminating $U(n+1)$ q-Whipple transformation formula of Milne and Newcomb.

Rishi Nath

On Diagonal Hooks of Symmetric Partitions

Abstract: A procedure to obtain the diagonal hook lengths of a symmetric partition given just the associated symmetric p-core and p-quotient will be given. Related results on p-core partitions will be discussed. This research is relevant to the analysis of a recent conjecture of Navarro involving Galois automorphisms in the case of the alternating groups.

Ken Ono

q-series and topology

Abstract: Here we discuss one of the mock theta functions of Ramanujan, and its role in the differential topology of 4-manifolds. We will discuss its role in the topology of 4-manifolds, in the context of Donaldson, and Seiberg-Moore-Witten Theory.

Michael Rowell

A New General Conjugate Bailey Pair

Abstract: We introduce a new general conjugate Bailey pair which bridges the gap between Bailey and Slater's work and the work done recently by Andrews and Warnaar. With this new general pair we are able to find many useful conjugate Bailey pairs similar to those of Andrews and Warnaar. Time permitting, we will show how these new pairs can be used to find results related to sums of triangular numbers, indefinite quadratic forms and partition identities.

Jose Plinio Santos

New Two Line Arrays Representing Partitions

Abstract: By making use of a new two line representation for partitions we get new combinatorial interpretations for unrestricted partitions and many identities including the Rogers-Ramanujan identities. One of the representations for unrestricted partitions has the property of giving also a complete description for the conjugate partition. This new notation can be used to prove important partitions identities.

Carla Savage

Symmetrically Constrained Compositions

Abstract: We consider the problem of counting the number of compositions of an integer n into k nonnegative parts $n = p_1 + p_2 + \cdots + p_k$, where the parts must satisfy a set of linear constraints that are symmetric in the p_i. Simple examples include integer-sided triangles (p_1, p_2, p_3) satisfying $p_i + p_j \geq p_k$; and pairs (p_1, p_2) where $2p_1 \geq p_2$ and $2p_2 \geq p_1$.

Andrews, Paule, and Riese identified a generalization of these families that they enumerated with the help of MacMahon's partition analysis. In this work, we formulate a further generalization and show how to reduce the enumeration problem to computing permutation statistics. In cases where those statistics can be computed, nice enumeration formulas emerge. This reports on joint work with Sylvie Corteel, Ira Gessel, Sunyoung Lee, and Herbert Wilf.

Anne Schilling

Affine Crystals and q-series

Abstract: In the 1980s, George Andrews and his collaborators Rodney Baxter and Peter Forrester showed a connection between certain statistical mechanical models such as the Eight-vertex SOS model and generalized Rogers-Ramanujan-type identities.

In this talk I want to show how a generalization of the sum side of these identities is related to affine crystal, in particular Kirillov-Reshetikhin crystals, that I have recently studied with Masato Okado and Ghislain Fourier.

Carsten Schneider

George E. Andrews' Alternative Approach of Stembridge's TSPP Theorem

Abstract: Stembridge proved the formula for the number of totally symmetric plane partitions (TSPP) by a masterful use of the combinatorics of Pfaffians. But this is not the whole story of the TSPP problem: Already in these days, Andrews was able to reduce the TSPP problem by an ingenious determinant setting to a collection of hypergeometric multi-sum identities. The remaining task to verify these identities was out of scope in that time. In joint cooperation with Andrews and Paule we finally managed to prove these identities with the help of new and efficient multi-sum algorithms. In this talk we will report on this project which finally led us to an alternative, computer assisted proof of Stembridge's TSPP theorem. Recently, out methods have found applications in quantum field theory.

James Sellers

Enumeration of the degree sequences of non-separable graphs and connected graphs

Abstract: In 1962, S. L. Hakimi proved necessary and sufficient conditions for a given sequence of positive integers $d_1, d_2, \ldots, d_n$ to be the degree sequence of a non-separable graph or that of a connected graph. Our goal in this talk is to utilize Hakimi's results to provide generating functions for the functions $d_{ns}(2m)$ and $d_c(2m)$, the number of degree sequences with degree sum $2m$ representable by non-separable graphs and connected graphs (respectively). From these generating functions, we prove nice formulas for $d_{ns}(2m)$ and $d_c(2m)$ which are simple linear combinations of the values of $p(j)$, the number of integer partitions of j. The proofs are elementary and the talk will be accessible to a wide audience. This is joint work with Øystein Rødseth and Helge Tverberg, both from the University of Bergen, Norway.

Andrew Sills

Computer Algebra and Rademacher Type Partition Formulas

Abstract: One of the most impressive and useful contributions to twentieth century number theory was the circle method of Ramanujan, Hardy, and Littlewood, with subsequent improvements by Rademacher. The application of the circle method to the problem of finding a convergent series representation for $p(n)$, the number of partitions of n involves a number of nontrivial calculations and delicate estimates, some of which are amenable to automation in a computer algebra system such as Mathematica. I will share Rademacher-type formulas for various restricted partition functions which were obtained with the aid of the computer.

Richard Stanley

Hook lengths and contents of partitions

Abstract: Hook lengths and contents are important invariants associated with a partition of a nonnegative integer. They have deep connections with Young tableaux, the representation theory of the symmetric group, symmetric functions, and related subjects. We will survey some results concerning these numbers and will then discuss a proof of a conjecture of Guoniu Han which generalizes the well-known formula $n! = \sum_\lambda f_\lambda^2$, where λ ranges over all partitions of n and f_λ is the number of standard Young tableaux of shape λ.

Sam Vandervelde

Everyone Loves a Bijection

Abstract: It is common knowledge that the number of partitions of n into odd parts is equal to the number of partitions of n into distinct parts. However, it seems to be less well-known that this quantity is also equal to the number of partitions of n into one triangular part followed by even parts. (A triangular part has size 0, 1, 3, 6, 10, ... Furthermore, note that [10]-6-4-4-2 and [6]-10-4-4-2 are distinct such partitions of 26.) The Jacobi triple product supplies an immediate, if unilluminating, confirmation of this claim. We propose a more intriguing approach involving tilings of Young tableaux by dominoes. In particular, we describe a bijection which establishes a (hopefully) new partition identity, which states that the number of "balanced" partitions of 2n into distinct parts is equal to the number of partitions of n. (A balanced partition is one in which the odd parts occupy as many even positions as odd positions when the parts are written in descending order. Thus 9-4-1 is not a balanced partition of 14, while 7-4-2-1 is balanced.)

Ole Warnaar

Long Live the King!

Abstract: George Andrews is often referred to by his aficionados as the king of q. In this talk I will prove the incredible health and prosperity of George's kingdom by showing some 21st century q-mathematics.

Ae Ja Yee

Alternating permutations and half descents

Abstract: Inversions and descents are well known statistics on permutations. In this talk, we introduce half descents and discuss the generating functions of alternating permutations with half decent-inversion weight.
