Syllabus for Math 597A, Fall 2009
Advanced Topics in Functional Analysis

The plan for the course is the following. I will begin with 10-12 lectures which rapidly review some of the basic results of functional analysis (roughly those that appear in Chapters 1-4 of Rudin's book). Following that there will be three "topic" sections of roughly 6 lectures each, which will be largely independent of each other. The planned "topic" sections are

- Distribution Theory: the basic notions of distribution theory will be briefly reviewed and then we'll go on to prove two rather deeper results, the Ehrenpreis-Malgrange theorem on the existence of fundamental solutions for constant coefficient partial differential equations, and the Schwarz Kernels Theorem which gives a general model for all continuous linear operators on function spaces. Some of this is in Rudin's Chapter 8; I don't really know a nice textbook reference for the Kernels Theorem but I will try to locate one.
- Nonlinear Methods in Banach Spaces: we will review calculus in Banach spaces, including the inverse and implicit function theorems. If time permits (it probably won't) I will try to say something about why this machinery doesn't work as it stands in more general spaces, and about the Nash-Moser theorem which is a far more delicate result which does work in Frechet spaces. Then we'll discuss fixed-point results of Schauder type and their application to existence theorems for suitable nonlinear PDE (see Gilbarg and Trudinger). The fixed point theorems are in Rudin Chapter 5; calculus on Banach spaces can be found in many places including Dieudonne, *Foundations of Modern Analysis*
- Spectral Theory for operators on Hilbert space, including unbounded operators, with a little discussion of semigroups if we have time to work this in (but we probably won't have time for anything very general here.) This is in Chapters 12 and 13 of Rudin.

Prerequisites for the course are a subset of those topics covered in the Analysis C qualifying exam course (Math 503), namely:
1. Hilbert spaces. Definitions and examples. Projections onto a closed, convexsets. Orthonormal bases. Operators. Adjoints.
2. Compact operators. Compact and Hilbert-Schmidt operators. Spectral theorem.
4. Topological vector spaces. Review of general topological spaces. Open sets, continuity and convergence. Topological vector spaces. Linear functionals and
convex sets. Weak topologies. Alaoglu's theorem. Spaces of smooth functions.
5. Distributions and partial differential equations. Distributions and operations on distributions. Convolutions. The Fourier transform. Schwartz space
and tempered distributions.

Please do not hesitate to contact me with any questions or concerns.

Instructors	John Roe (roe@math.psu.edu)
Text	Rudin, *Functional Analysis*, second edition (optional). Online lecture notes will be provided.
Meetings	1:00 – 2:15 Tuesdays and Thursdays, 106 McAllister
Office Hours	By appointment in 204 McAllister Building.
Mode of Assessment	Five homework assignments (see schedule below). There will be no final exam. Details will be announced.
Academic Integrity	All Penn State Policies regarding ethics and honorable behavior apply to this course (see http://www.psu.edu/ufs/policies). Academic integrity is the pursuit of scholarly activity in an open, honest and responsible manner. Academic integrity is a basic guiding principle for all academic activity at The Pennsylvania State

	University, and all members of the University community are expected to act in accordance with this principle. Consistent with this expectation, the University's Code of Conduct states that all students should act with personal integrity, respect other students' dignity, rights and property, and help create and maintain an environment in which all can succeed through the fruits of their efforts. Academic integrity includes a commitment by all members of the University community not to engage in or tolerate acts of falsification, misrepresentation or deception. Such acts of dishonesty violate the fundamental ethical principles of the University community and compromise the worth of work completed by others. To protect the rights and maintain the trust of honest students and support appropriate behavior, faculty and administrators should regularly communicate high standards of integrity and reinforce them by taking reasonable steps to anticipate and deter acts of dishonesty in all assignments

Course Calendar (Tentative)

- August 25: class begins
- September 8, 10: no class meetings
- September 15: first homework due
- October 6: second homework due, and start distribution theory section
- October 27: third homework due, and start nonlinear/Banach section
- November 17: fourth homework due, and start spectral theory section
- December 10: last day of class, final homework due