Estimating Consumption Substitution between Housing and Non-Housing Goods using Macro Data

Thomas Davidoff* and Jiro Yoshida†

Abstract

The static elasticity of substitution (SES) between housing and non-housing consumption is important not only in understanding housing demand, but also in asset pricing because housing consumption influences the marginal utility of numeraire consumption. Previous estimates of the elasticity are low when micro data are used but high when macro data are used. We use aggregate time-series data to estimate SES by allowing for non-homotheticity in preferences. We obtain lower estimates of SES ranging from 0.4 to 0.9 when we allow for non-homotheticity than when we maintain homotheticity assumption. Homotheticity is rejected in the way that consumption share of housing decreases as income grows and as income is derived more from employment than from investments. We also obtain low IES ranging from 0.05 to 0.14.

JEL classification: G12, D12, R21, R30.

Keywords: LaTeX; papers with no content.

* Sauder School of Business, University of British Columbia. Email: thomas.davidoff@sauder.ubc.ca.
† Smeal College of Business, the Pennsylvania State University. Email: jiro@psu.edu.
1. Introduction

Introspection and a host of studies using microdata suggest that when housing becomes
more expensive, the fraction of a household’s expenditure allocated to housing will rise; that
is, the elasticity of substitution between housing and other goods is less than one. Past
studies (e.g. Carliner (1973)) also confirm an income elasticity less than one.

Two influential recent papers have challenged these micro-founded analyses using aggre-
gate US statistics. Piazzesi et al. (2007) observe that while the quantity of housing per capita
has increased and rents have fallen, the housing expenditure share has remained roughly con-
stant. Assuming an income elasticity of one, this supports a price elasticity of at least one in
magnitude for a representative consumer. Davis and Ortalo-Magne (2011) find a remarkable
lack of variation in a panel of US Census metropolitan area median rental expenditure to
income ratios. Given considerable heterogeneity across time and places in median incomes
and rental prices, this suggests that income and price elasticities cannot be far from one.

Recent macro-based estimate of SES is quite high (2.2 or higher by Davis and Martin
(2005)).

We ask whether methodological refinements can reconcile aggregate data with the balance
of microeconometric evidence on demand elasticities. With respect to GMM asset pricing
estimates, we follow the observation of Pakos (2003) that forcing an income elasticity of one
(as is commonly done in macro-finance studies) may bias the price elasticity away from zero.
For example, consider an economy with low supply and income and price demand elasticities
in which rising prices drive up rents and drive down the expenditure share on housing. If
the income effect is shut down, one would attribute the negative correlation between rents
and expenditure share to a high price elasticity. We find that the Pakos (2003) Generalized
Elasticity of Substitution method applied to the housing and asset price data considered in
Piazzesi et al. (2007) results in income and price elasticity estimates well below one. We
describe this analysis in more detail in Section ??.

1 Add Hanushek and Quigley (1980) to the references.
Along similar lines, suppose that there is a positive relationship between wages and rents. This could occur either through a labor market or an amenity and sorting effect. If housing demand is both income inelastic and price inelastic, the unconditional cross sectional relationships among rents, incomes, and rent-to-income ratio are ambiguous. This could explain in part the remarkable lack of variation in rent-to-price ratios across metropolitan areas with very different income profiles and rental prices found by Davis and Ortalo-Magne (2011). In Section 2 we present bivariate regressions of median rent to price ratios on metropolitan wage and price levels designed to avoid mechanical “reflection” problems. positive relationship between rental prices and rent to expenditure ratios.

It is worth emphasizing that the elasticities computed by Davis and Ortalo-Magne (2011) and are incidental to their analyses. We do not intend for this study to be read as a critique of their respective studies of the interactions among labor, financial, and housing markets. However, given the centrality of income and price elasticities of housing demand, it is worthwhile to reconcile aggregate data with earlier microeconometric studies.

2. Rental housing expenditure shares in Census Data

Davis and Ortalo-Magne (2011) report modest differences across metropolitan areas and within metropolitan areas over time in median rental expenditure to income ratios. We ask whether the absence of variation might mask significant sensitivity of the median ratio to both income and prices due to offsetting effects, and a positive correlation between income and prices.

As in Davis and Ortalo-Magne (2011), we explore the 2000 5% PUMS sample and consider household heads between 25 and 62 with more than 40 weeks of work in the 50 large metropolitan areas described used in the original study. We measure income as household income, so our ratios of rent to income are lower. We find an unconditional interquartile

\[\text{We use the IPUMS sample. Panel results using the 1980 1% sample are consistent with the cross sectional results we present below.}\]
range of median rent to income across metropolitan areas of 2.5 (around a mean of 17.8%).

Davis and Ortalo-Magne (2011) observe a problem in interpreting the correlation between the expenditure share on housing and income using Census data. The Census reports income from different sources, but not expenditures on goods other than rent. Measured income differs from true expenditures due to reporting problems and saving. Some fraction of savings will be allocated to future housing, so the ratio of rent to income may understate a properly time averaged ratio of rent to expenditures. Because savings generally rises with income, it is not clear that a negative relationship between income and the ratio of rent to income implies a static income elasticity of housing demand less than one.

The cross sectional relationship between rents and the fraction of income devoted to rent is complicated by equilibrium forces outlined in Davis and Ortalo-Magne (2011). If housing supply is less than infinitely elastic, variation in potential earnings will drive variation in rents. If both income and price elasticities are less than one in magnitude, then higher incomes and higher rents will push the expenditure share in opposite directions. The likely negative savings bias described above will exacerbate the problem of measuring a price impact on the ratio of rent to income.

To partially address these issues, we estimate two regressions:

\[
\ln \left( \frac{\text{rent}}{\text{income}} \right)_{my} = \beta_0 + \beta_1 \ln \text{price}_{my} + \beta_2 \ln \text{income}_{my} + \epsilon_{my},
\]  

(1)

In equation (1), \( m \) denotes a metropolitan area (consolidated, where applicable) and \( y \) denotes an indicator for whether or not an individual is college educated. We bifurcate the sample in light of the differences observed by Davis and Ortalo-Magne (2011). \( \frac{\text{rent}}{\text{income}}_{my} \) is the unconditional median among household heads.

Using income and rent data as regressors in (1) would be problematic for several reasons. First, measurement error would guarantee a positive bias on \( \beta_1 \) and a negative bias on \( \beta_2 \), even if rents and income are purged of demographic and hedonic means. Second, unobserved demand for rental housing among those in the Census sample would generate omitted variable
bias. To avoid some of these mechanical endogeneity problems, we estimate income and price levels among homeowners. In particular, we proxy for income with the mean (natural) log wage income of homeowners in metropolitan area \( m \) with college education indicator \( y \). We retain both wage and skill sources of income in this estimate, as we are not interested in the differential roles of these factors.

We proxy for rental price levels with the mean value of a three bedroom detached home by metropolitan area. The log value of this “representative” home should reflect log net rents to owners as well as a metropolitan-specific capitalization rate. Other than through income effects, it is plausible that renters’ choices in 2000 would be unaffected by, e.g. expected growth in rents and prices.

Our results are as follows are summarized in Table 1. We find that the elasticity of the rental share with respect to home values is significantly positive at approximately .2 for both groups of .22 of the rental share of income of approximately .22 for both. The elasticity of the ratio with respect to comparably educated homeowners’ wage level is approximately twice as high for non-college graduates than for college graduates.

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<tr>
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<tbody>
<tr>
<td></td>
<td>No College</td>
<td>College Grad or more</td>
</tr>
<tr>
<td>Intercept</td>
<td>-3.60**</td>
<td>-5.09**</td>
</tr>
<tr>
<td></td>
<td>(1.07)</td>
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<tr>
<td>ln homeowner wage income</td>
<td>-0.32**</td>
<td>-0.15</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.14)</td>
</tr>
<tr>
<td>ln home value</td>
<td>0.22**</td>
<td>.21**</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(.04)</td>
</tr>
<tr>
<td>Obs.</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>R-Squared</td>
<td>.56</td>
<td>.48</td>
</tr>
</tbody>
</table>
3. Data

We use aggregate time-series data for consumption of housing and non-housing, a relative price of housing services, and asset returns. We use two different data sets for housing consumption: housing services and housing stock. We adopt the beginning-of-period convention, following Campbell (2003) and Yogo (2006). For example, the consumption during the first quarter of 2000 is treated as if all the consumption took place at the beginning of that quarter. Prices and stocks are all measured at the beginning of each period. The use of end-of-period convention will not alter our results.

Relative price of housing consumption: There are two data series available for price of housing services: 1) Personal Consumption Expenditure (PCE) price index for housing taken from National Income and Products Accounts (NIPA) published by the Bureau of Economic Analysis (BEA) and 2) Consumer Price Index (CPI) for shelter published by the Bureau of Labor Statistics (BLS). Both statistics currently use the rental equivalence approach to estimating the shelter costs of homeowners. The owners’ equivalent rent is obtained from corresponding market rental value of homes that have the same size, quality and type. In equilibrium of frictionless markets, owners’ equivalent rent is equal to the user cost of housing. The rental equivalence approach is less prone to model errors, especially errors on expected future appreciation of housing asset prices, than the user cost approach.

We use NIPA-based PCE price index for housing (Table 2.3.4 - Price Indexes for Personal Consumption Expenditures, Line 14) because CPI for shelter is discontinuous in 1983 due to a major methodological change. Prior to 1983, BLS adopted the "asset price" approach by which the shelter costs consist of five elements: 1) home purchase cost, 2) mortgage interest cost, 3) property taxes, 4) homeowner insurance charges and 5) maintenance and repair costs. The asset price approach is irrelevant especially because housing asset prices for purchase are directly used for price of housing services. The CPI shelter price is driven by large changes in housing asset prices in the 70’s and the early 80’s.\footnote{There are other differences between two indexes. First, CPI is a Laspeyres index (i.e. it is constructed based on the relative weights of goods in the basket of goods in a given year). Second, PCE price index for housing is a Paasche index (i.e. it is constructed based on the relative weights of goods in the basket of goods in the base period).}
Figure 1 plots two data series for price of housing service. Panel A is relative price of housing deflated by non-housing prices and Panel B is the growth rates of price indexes of housing. As apparent in the figure, two series are closely related from 1983, but they exhibit a wide gap prior to 1983 (shaded region).

Second, PCE price index and CPI use very different weights on housing. As of December 2004, PCE puts 15% on shelter while CPI puts 32.7%. The gap is partly due to different coverages of items; CPI covers only about three quarters of PCE price index. CPI captures out-of-pocket expenditures made by households while PCE also includes expenditures made on behalf of households (e.g. medical care paid by employers and governments). See Poole et al. (2005), Moyer (2006) and McCully (2006) for detailed discussion.
We deflate NIPA-based PCE price index of housing by that of non-housing. Non-housing includes non-durables, durables, and services but housing. Since non-housing defined in this study is not directly reported in NIPA tables, price index for non-housing is calculated by dividing the nominal expenditure by the real expenditure of non-housing items.

**Non-housing consumption**: Non-housing consumption data is per capita real consumption expenditure on non-housing. To obtain the data series, first, nominal personal consumption expenditure is taken from NIPA Table 2.3.5. (Personal Consumption Expenditures by Major Type of Product) published by the BEA. Then the real personal consumption expenditure in 2000 dollar is calculated for each product type by using the chain-type price indexes that are taken from NIPA Table 2.3.4. (Price Indexes for Personal Consumption Expenditures by Major Type of Product) We subtract real housing services from real total consumption expenditure. The data is converted to per capita basis by dividing by the population published by the Census Bureau (taken from freelunch.com).

**Housing Consumption**: We use two different measures of housing consumption. The first is simply per capita real personal consumption expenditure for housing services, which
is constructed in the same way as non-housing consumption.

The second is rescaled per capita real housing stock excluding land. The underlying assumption is a linear transformation technology from housing stock to housing services. We constructed quarterly data series for housing stock by using two different methods. The first data for residential stock is the Net Stock of Fixed Reproducible Tangible Wealth for 1925-1997 reported in the SURVEY OF CURRENT BUSINESS, September 1998 by BEA. It provides annual end-of-year estimate of net residential capital. The data is splined up to 2004 by the Detailed Data for Fixed Assets and Consumer Durable Goods from BEA. Since these stock data are annual, we estimate the quarterly values by using the Private Fixed Investment data taken from NIPA Table 5.3.5. Specifically, given the beginning-of-year stock and the end-of-year stock, the net addition to the stock during the year is distributed to each quarter depending on the share of the real investment for the quarter. This method is also adopted by Yogo (2005). The data are then shifted one period to convert to beginning-of-period values. Finally, the data is rescaled by dividing by the sample mean.

The second data is estimated by the accumulation equation:

\[ H_{t+1} = (1 - \delta) H_t + I_t, \]

where \( H_t \) is real residential stock at the beginning of period \( t \), \( I_t \) is real gross residential investment during period \( t \), and \( \delta \) is constant depreciation rate. For real investment, Real Residential Private Fixed Investment in NIPA Table 5.3.5 is used. The depreciation rate is set at 1.2% per year, or 0.2987% per quarter, which is roughly the average depreciation rate computed from the Detailed Data for Fixed Assets and Consumer Durable Goods from BEA. The initial value of residential stock is set so that the rescaled initial value matches the first data series.

As shown in Figure 2, two data series roughly match while the BEA-based data exhibits more short-term variability. We adopt the BEA-based data.
Income level: We use per capita real GDP as an income measure. GDP is current income, which includes both permanent and transitory income. For our purpose to examine homotheticity, current income is relevant since it is well-known that households also respond to transitory-income shocks. Although some of previous studies use total consumption as a proxy for permanent income, use of total consumption as income level causes an econometric problem in our regression of consumption ratio. Log non-housing consumption, which enters negatively on the left-hand side, is a major source of variations in total consumption, which enters positively on the right-hand side. Therefore, a clear negative relationship between consumption ratio and ”permanent income” is observed if we use total consumption as income proxy.

Income share of investors: Employees’ income is ”Compensation of employees, received” (line 2) in NIPA Table 2.1-Personal Income and Its Disposition, published by BEA. Investors’ income is the sum of 1) Proprietors’ income with inventory valuation and capital consumption adjustments (line 9), 2) Rental income of persons with capital consumption
adjustment (line 12), and 3) Personal income receipts on assets (line 13), in the same table. The income share of investor is investors’ income divided by the sum of both types of income.

As a preliminary check of time-series properties of the data, we first test unit-root by Augmented Dicky-Fuller test with four lags and with intercept and trend terms. All series are $I(1)$; the null hypotheses of unit-root are not rejected for levels but are rejected for first-differences at one percent level. Second, we check for cointegration by Johansen’s LR test. For each of consumption ratio equation, housing demand equation and housing expenditure equation, any cointegration is rejected at 5% level. Therefore, we use log-differenced series for estimation.

**Asset returns**: Market return is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks taken from Kenneth French’s web site. Risk-free rate is three month constant maturity Treasury yield.

### 4. Estimating SES with Linear Regression Models

#### 4.1. Elasticities of Consumption Ratio and Housing Demand

Consider a static consumption choice of a household:

\[
\max_{C,H} u(C, H) \quad \text{s.t.} \quad C + PH = Y,
\]

where $P$ is relative price of housing services and $Y$ is income. The CES utility function is

\[
u(C, H) = \left[ (1 - \alpha) C^{1-1/\rho} + \alpha H^{1-1/\rho} \right]^{1/(1-1/\rho)},
\]

where $C$ is consumption of non-housing good and $H$ is consumption of housing services, $\alpha \in (0,1)$ is a constant, and $\rho \geq 0$ is SES between two goods. The CES utility function nests the Cobb-Douglas utility function when $\rho = 1$.

Assuming an interior solution, the relative price of housing services and housing demand
are, respectively,

\[ P = \frac{\partial u/\partial H}{\partial u/\partial C} = \frac{\alpha}{1 - \alpha} \left( \frac{H}{C} \right)^{-\frac{1}{\rho}}, \]

\[ H = Y \left[ P + \left( \frac{1 - \alpha}{\alpha} P \right)^{\rho} \right]^{-1}. \]

Price and income elasticities of consumption ratio \((\varepsilon_{P}^{H/C}, \varepsilon_{Y}^{H/C})\) and housing demand \((\varepsilon_{p}^{H}, \varepsilon_{w}^{H})\) at a particular price level are, respectively,

\[ \varepsilon_{P}^{H/C} \equiv \frac{\partial \ln (H/C)}{\partial \ln P} = -\rho \leq 0, \]

\[ \varepsilon_{Y}^{H/C} \equiv \frac{\partial \ln (H/C)}{\partial \ln Y} = 0, \]

\[ \varepsilon_{p}^{H} \equiv \frac{\partial \ln H}{\partial \ln P} = -1 - \frac{(\rho - 1) \left( \frac{1 - \alpha}{\alpha} \right)^{\rho} P^{\rho-1}}{\left( \frac{1 - \alpha}{\alpha} \right)^{\rho} P^{\rho-1} + 1} < 0, \]

\[ \varepsilon_{w}^{H} \equiv \frac{\partial \ln H}{\partial \ln Y} = 1, \]

Elasticities of consumption ratio \((\varepsilon_{P}^{H/C} \text{ and } \varepsilon_{Y}^{H/C})\) are particularly simple. Empirical relationship between log consumption ratio and log housing prices (i.e. \(\varepsilon_{P}^{H/C}\)) directly indicates SES.\(^4\) We can also use coefficient on log income \((\varepsilon_{Y}^{H/C})\) for a test of homotheticity.

Price elasticity of housing demand \((\varepsilon_{p}^{H})\) is more complicated and depends on price level and utility share of housing. Although price elasticity is in general a non-monotonic function of SES, their levels are determined by whether SES is greater than or smaller than one. If SES is greater than one \((\rho > 1)\), price elasticity of housing demand is also greater than one in absolute value. \((\text{sgn} \left( |\varepsilon_{p}^{H}| - 1 \right) = \text{sgn} \left( \rho - 1 \right))\). In this case, price elasticity of housing expenditure is negative: A higher price of housing services is associated with a lower expenditure on housing.

We can derive a reduced-form empirical equations based on the CES model above. By taking logs of \((3)\) and differentiating, we obtain

\[ d \ln (H/C)_{t} = -\rho d \ln P_{t}. \]

\(^4\)Although this elasticity is for Marshalian (uncompensated) demand, income compensation will not alter the consumption ratio under the CES utility assumption.
intercept, effect of income, and effect of interest rate, we obtain an empirical equation of consumption ratio:

\[
\frac{d \ln (H/C)}{dt} = a + b d \ln P_t + c d \ln Y_t + d d r_t + \mu_t, 
\]

where \( r_t \) are interest rates and \( \mu_t \) are disturbances. This functional form is flexible in the sense that it is capable of attaining any value of each variable (Lau (1986)). \( a = c = 0 \) and \( b = -\rho \) according to the CES model. Estimating consumption-ratio equation is relatively unique in this research. Vast majority of previous studies focus on housing demand and expenditure for the sake of demand analyses of housing. In the current research, our focus is to obtain SES for the sake of asset pricing analysis.

Furthermore, we allow for the possibility that different consumers have different parameter values in their utility functions. We focus on the source of household income. Income distribution among different types of consumers affects aggregate demand unless income expansion paths for all consumers are parallel straight lines.\(^5\) We simply hypothesize that the economy consists of two types of representative consumers: investors and employees. If investors and employees have different SES, we expect to observe time-variations in consumption ratio as composition of two types of consumers change over time. The model is shown in Appendix 3. We empirically examine whether income share of non-employee income affects consumption ratio of housing, independent of income levels, by adding a term of income share of investors, \( w^i \):

\[
\frac{d \ln (H/C)}{dt} = a + b d \ln P_t + c d \ln Y_t + d d \ln w^i_t + c d r_t + \mu_t. 
\]

(5)

After obtaining the sign of coefficient \( d \) in equation (5), we can infer relative level of SES for the two types of consumers by assuming a common value of \( \alpha \). Although \( \alpha \) cannot be

\(^5\)The condition is equivalent to indirect utility function of any consumer \( k \) having the Gorman form: \( v_k (p, Y_k) = a_k (p) + b (p) Y_k \).
identified in this regression, it is estimated in the GMM estimation in the following section. As shown in Appendix 3, if \( P (1 - \alpha^k) / \alpha^k > 1 \) for \( k = e, i \), a positive value of coefficient \( d \) implies that investor has a lower SES than employee does (\( \rho^i < \rho^e \)), and vice versa.

We also estimate price and income elasticities of housing demand. By taking logs of (4) and differentiating, we obtain \( d \ln H = \varepsilon^H_P d \ln P + d \ln Y \). A simple way to reflect price elasticity being a function of price is to include a term of squared log price in empirical equation. Together with the terms of income share and interest rate, the empirical equation becomes

\[
d \ln H_t = a + b_0 d \ln P_t + b_1 d \ln^2 P_t + c d \ln Y_t + dd \ln w_t + edr_t + \mu_t. \tag{6}
\]

According to the CES model, \( a = 0, b_0 < 0, b_1 \leq 0 \) and \( c = 1 \).

By adding \( d \ln P_t \) to both sides of equation (6), we can obtain an empirical equation of housing expenditure in log-linear form. Although previous studies often derive linear expenditure equations from the Stone-Geary utility function, the linearity of expenditure comes at the expense of restrictions on SES in the Stone-Geary utility function. We will examine the Stone-Geary utility function in the next section.

4.2. Result

Table 1 shows the estimation result of consumption ratio equation (5). Panel A is based on housing services data taken from NIPA and Panel B is based on housing stock data. In each panel, we estimate three different versions. In the third specification, we include 10-year Treasury yield in order to account for potential intertemporal effects on consumption ratio. 3-month Treasury yield generates almost identical results, which is not reported for brevity.

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6Since price elasticity is negative, \( b_0 + b_1 \ln P_{H,i} < 0 \). In addition, \( \partial \varepsilon^H_P / \partial P_H = - (\rho - 1)^2 \left( \frac{1 - \alpha}{\alpha} \right)^\rho P_H^{\rho - 2} \left[ \left( \frac{1 - \alpha}{\alpha} \right)^\rho P_H^{\rho - 1} + 1 \right]^{-2} \leq 0 \). Therefore \( b_0 < 0 \) and \( b_1 \leq 0 \).
Table 1 — Estimated Elasticities of Consumption Ratio

Dependent Variable = $d\ln (H_t/C_t)$


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<tbody>
<tr>
<td>Constant</td>
<td>$-0.003$</td>
<td>$-0.003$</td>
<td>$-0.003$</td>
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<tr>
<td></td>
<td>($0.001$)</td>
<td>($0.001$)</td>
<td>($0.001$)</td>
</tr>
<tr>
<td>$d\ln$ (Housing Prices)</td>
<td>$-0.682$</td>
<td>$-0.617$</td>
<td>$-0.617$</td>
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<tr>
<td></td>
<td>($0.138$)</td>
<td>($0.090$)</td>
<td>($0.084$)</td>
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<tr>
<td>$d\ln$ (Income Level)</td>
<td>$-0.511$</td>
<td>$-0.521$</td>
<td>$-0.521$</td>
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<tr>
<td></td>
<td>($0.065$)</td>
<td>($0.065$)</td>
<td>($0.065$)</td>
</tr>
<tr>
<td>$d\ln$ (Income Composition)</td>
<td>$0.083$</td>
<td>$0.080$</td>
<td>$0.080$</td>
</tr>
<tr>
<td></td>
<td>($0.038$)</td>
<td>($0.040$)</td>
<td>($0.040$)</td>
</tr>
<tr>
<td>$d$ (10y Treasury Yield)</td>
<td>$0.001$</td>
<td>$0.001$</td>
<td>$0.001$</td>
</tr>
<tr>
<td></td>
<td>($0.001$)</td>
<td>($0.001$)</td>
<td>($0.001$)</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.182</td>
<td>0.454</td>
<td>0.461</td>
</tr>
<tr>
<td>DW</td>
<td>1.68</td>
<td>2.02</td>
<td>2.09</td>
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Panel B: $H_t =$ BEA-based Housing Stock (1953:2-2004:1, $N = 204$)

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<tr>
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<tr>
<td></td>
<td>($0.001$)</td>
<td>($0.001$)</td>
<td>($0.001$)</td>
</tr>
<tr>
<td>$d\ln$ (Housing Prices)</td>
<td>$-0.554$</td>
<td>$-0.445$</td>
<td>$-0.474$</td>
</tr>
<tr>
<td></td>
<td>($0.141$)</td>
<td>($0.100$)</td>
<td>($0.111$)</td>
</tr>
<tr>
<td>$d\ln$ (Income Level)</td>
<td>$-0.500$</td>
<td>$-0.485$</td>
<td>$-0.485$</td>
</tr>
<tr>
<td></td>
<td>($0.075$)</td>
<td>($0.080$)</td>
<td>($0.080$)</td>
</tr>
<tr>
<td>$d\ln$ (Income Composition)</td>
<td>$0.064$</td>
<td>$0.074$</td>
<td>$0.074$</td>
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<tr>
<td></td>
<td>($0.072$)</td>
<td>($0.076$)</td>
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</tr>
<tr>
<td>$d$ (10y Treasury Yield)</td>
<td>$-0.001$</td>
<td>$-0.001$</td>
<td>$-0.001$</td>
</tr>
<tr>
<td></td>
<td>($0.001$)</td>
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Notes: Significance levels: *: 10 percent; **: 5 percent; ***: 1 percent. In parentheses are Newey-West heteroscedasticity and autocorrelation consistent standard errors.
The price elasticity, or negative of SES, is $-0.682$ with NIPA-based housing services data, and $-0.554$ with BEA-based housing stock data if we do not include the income-related variables (Column (1)). The estimates have smaller absolute values with higher precisions when we include the income variables in column (2) and (3). The adjusted-R^2 also significantly improves when income variables are included. For the full model in column (3), the price elasticity is $-0.617$ with NIPA-based housing services data, and $-0.474$ with BEA-based housing stock data. All estimates are significantly below unity at one percent level.

The income elasticity of consumption ratio is about $-0.5$ for each specification, and is significantly less than zero at one percent level. A higher income is, ceteris paribus, associated with a smaller share of housing consumption. Although we obtain mixed results on signs in our GMM estimation, here we obtain consistently negative estimates for both data sets. The result indicates that consumers have non-homothetic preferences, and in particular, income elasticity of housing demand is lower than that of non-housing. This is consistent with observed pattern that the ratio of housing expenditure to income is decreasing in income levels, as reported by Green and Malpezzi (2003).

The coefficients on income composition (i.e. income share of investors) are positive, and statistically significant at 5 percent level when we use housing services data. The consumption share of housing increases as income is derived more from investments. The GMM estimates of $\alpha$ are about 0.41 with the GES specification. Then $P (1 - \alpha) / \alpha > 1$ in sample, and a positive coefficient implies that investors have a lower SES than employees. If employees are more likely to be renters, then the result is consistent with the findings in the previous literature that renters exhibit a higher price elasticity of housing demand.

Interest rate does not affect consumption share significantly. Possibly, appropriate interest rates are incorporated into market rent and interest rate itself does not have a direct effect on housing demand.

Statistical fit and properties of residuals are much better for the consumption ratio equa-
tion than housing demand equation (Table 2). For example, adjusted R-squared is 0.461 and the Durbin-Watson statistic is 2.09 when housing service data is used. These statistics are much lower for the housing demand equation.

Table 2 shows the estimation result of housing demand equation \(\text{(6)}\). For potential endogeneity bias caused by simultaneous equations of supply and demand, we conduct Wu-Hausman endogeneity test. The instrumental variables of the supply-side equation is construction cost and housing starts. Data for construction cost is Price of Residential Investments taken from NIPA Table 1.5.4, deflated by non-housing price index. Housing starts are New Residential Construction for Total privately owned, published by the U.S. Census Bureau. We strongly reject the null that the endogeneity problem affects the estimates. Therefore, we use OLS rather than 2SLS for the sake of efficiency.
Table 2 — Estimated Elasticities of Housing Demand

Dependent Variable = $d \ln H_t$

$H_t = \text{NIPA-based Housing Services (1953:2-2006:4, } N = 215)$

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.006</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>***</td>
<td>***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>$d \ln (\text{Housing Prices})$</td>
<td>$-0.310$</td>
<td>$-0.306$</td>
</tr>
<tr>
<td></td>
<td>***</td>
<td>***</td>
</tr>
<tr>
<td></td>
<td>(0.057)</td>
<td>(0.056)</td>
</tr>
<tr>
<td>$d \ln (\text{Housing Prices})^2$</td>
<td>0.308</td>
<td>$-0.066$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.552)</td>
<td>(3.566)</td>
</tr>
<tr>
<td>$d \ln (\text{Income Level})$</td>
<td>0.083</td>
<td>0.076</td>
</tr>
<tr>
<td></td>
<td>***</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.031)</td>
</tr>
<tr>
<td>$d \ln (\text{Income Composition})$</td>
<td>0.030</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.024)</td>
</tr>
<tr>
<td>$d (10y \text{ Treasury Yield})$</td>
<td>$-$</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.001</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.200</td>
<td>0.191</td>
</tr>
<tr>
<td>DW</td>
<td>0.94</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Notes: Significance levels: *: 10 percent; **: 5 percent; ***: 1 percent. In parentheses are Newey-West heteroskedasticity and autocorrelation consistent standard errors.

The mean price elasticity is about $-0.3$ based on the mean value of $d \ln(\text{Housing Prices})$. It is significantly greater than $-1$ at one percent level. Although there is no closed form solution for $\rho$ in terms of $\varepsilon^H_P$, we can conclude that $\rho$ is significantly below one. We obtain
the same result of a low SES as in GMM estimation. In the previous studies using cross-sectional regressions, the price elasticity range from $-0.3$ to $-0.8$ (e.g. Mayo (1981)). Our result is at the low end of the range in absolute value. Price elasticity of less than one in absolute value implies that housing expenditure is greater when price of housing services is high, which agrees with our casual observation.

Income elasticity of housing demand is very low about 0.08. A low income elasticity of housing demand is consistent with negative income elasticity of consumption ratio reported in Table 1. It is also consistent with the results from previous studies that income elasticity is well below one. However, previous estimates of income elasticity range from 0.3 to 0.8. Our result is far below previous estimates.

Income composition and interest rate do not enter significantly; the same result as in consumption ratio estimation.

5. Linear Expenditure Equation Based on the Stone-Geary Utility Function

Another departure from the plain CES utility function for non-homothetic preferences is introducing subsistence levels. The Stone-Geary utility function is a modified Cobb-Douglas utility function with subsistence levels:

$$U(C, H) = (C - \bar{C})^{1-\alpha} (H - \bar{H})^\alpha, \ C > \bar{C}, \ H > \bar{H},$$

where $\bar{C}, \ \bar{H}$, are subsistence levels of $C$ and $H$, respectively, and $\alpha \in (0, 1)$.

Demand for housing services $(H)$ from the static problem is

$$H = \frac{\alpha (Y - \bar{C})}{P} + (1 - \alpha) \bar{H}.\quad (8)$$

The demand is the sum of the minimum level of housing consumption $((1 - \alpha) \bar{H})$ and the
usual demand with Cobb-Douglas utility except for income adjustment for minimum level of non-durable consumption \((\alpha (Y - \bar{C}) / P)\). Housing expenditure is written as a linear function of income and price:

\[
PH = -\alpha \bar{C} + \alpha Y + (1 - \alpha) \bar{H} P.
\]

We specify empirical equation for housing expenditure by including income share of investors and interest rate, and taking first differences:

\[
d(P_t H_t) = a + bdP_t + cdY_t + ddw_t^i + edr_t + \mu_t. \tag{9}
\]

Then coefficients \(a, b\) and \(c\) just identify parameters \(\alpha, \bar{C}\) and \(\bar{H}\).

The price elasticity of housing demand \(\varepsilon_P^H\) is

\[
\varepsilon_P^H \equiv \frac{\partial H}{\partial P} \frac{P}{H} = -\frac{\alpha (Y - \bar{C})}{PH} = - \left[ 1 + \frac{(1 - \alpha) PH}{\alpha (Y - \bar{C})} \right]^{-1}.
\]

If there is no subsistence levels in housing consumption (i.e. \(\bar{H} = 0\)), price elasticity of housing demand is negative one as with simple Cobb-Douglas utility function. With \(\bar{H} > 0\), price elasticity is strictly less than one in absolute value \((|\varepsilon_P^H| < 1)\). This is understood by recognizing that price elasticity of \(H - (1 - \alpha) \bar{H}\) is always negative one, irrespective of price or income\(^7\). Sufficiently large percentage change in \(H - (1 - \alpha) \bar{H}\) is achieved by a smaller response of \(H\).

\(^7\)This is easily confirmed from \((8)\).
Regarding homotheticity, income elasticity of consumption ratio is shown to be

\[
\varepsilon_{H/C}^Y \equiv \frac{\partial \ln (H/C)}{\partial \ln Y} = \alpha \left( \frac{PH}{Y} \right)^{-1} - (1 - \alpha) \left( \frac{C}{Y} \right)^{-1}
\]

\[
= \left[ \alpha \left( \frac{PH}{Y} \right)^{-1} - (1 - \alpha) \left( \frac{C}{Y} \right)^{-1} \right] \frac{\bar{C}P\bar{H}}{(Y - C)\bar{C}}.
\]

The sign of income elasticity of consumption ratio is determined by

\[
sgn \left( \varepsilon_{H/C}^Y \right) = sgn \left( \frac{\alpha}{1 - \alpha} - \frac{PH}{C} \right)
\]

\[
= sgn \left( \frac{\alpha}{1 - \alpha} - \frac{P\bar{H}}{\bar{C}} \right). \tag{10}
\]

Again, if there is no subsistence level (i.e. if the utility function is simple Cobb-Douglas one), income elasticity of consumption ratio is zero since expenditure share of housing \((PH/Y)\) and that of non-housing \((C/Y)\) are just \(\alpha\) and \(1 - \alpha\), irrespective of income level. With a positive subsistence level, expenditure share is different from the Cobb-Douglas case, but the share approaches to the Cobb-Douglas case as income grows. Therefore, if current expenditure share of housing is greater than \(\alpha\), or equivalently, the ratio of minimum housing expenditure \((P\bar{H})\) to minimum non-housing expenditure \((\bar{C})\) is greater than \(\alpha / (1 - \alpha)\), consumption share of housing decreases as income grows; income elasticity of consumption ratio is negative.

Empirically, after estimating parameters of the Stone-Geary utility function \((\alpha, \bar{C}, \bar{H})\) from the linear expenditure equation, we can infer the sign of income elasticity of consumption ratio by combining with average price of housing services.

Table 3 shows estimation result of linear expenditure equation \([9]\). In estimating this equation we use \(C_t + P_tH_t\) as a measure of income level in order to satisfy static budget constraint. Coefficients for \(d\) (Housing Prices) and \(d\) (Income Level) are BLUE for \((1 - \alpha) \bar{H}\) and \(\alpha\), respectively. In order to obtain the estimate of \(a = -\alpha \bar{C}\), we take the estimates from
the first column of Table 3, and calculate the sample mean of

\[ P_t H_t - \begin{bmatrix} 0.010t + 1.859 (Housing Prices)_t \\ +0.031 (Income Level)_t + 0.293 (Income Composition)_t \end{bmatrix}, \]

which is $-1.011$ with standard deviation of $0.039$. Then, our estimates of parameter values are $\alpha = 0.031, \bar{C} = 32.990, \bar{H} = 1.917$, all of which are significantly different from zero at one percent level. Non-zero subsistence levels of consumption implies again non-homothetic preferences.

Table 3 — Estimated Elasticities of Housing Expenditure

<table>
<thead>
<tr>
<th>Dependent Variable = $d(P_t H_t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_t =$ NIPA-based Housing Services (1953:2-2006:4, $N = 215$)</td>
</tr>
</tbody>
</table>

\[
\begin{array}{l|cc|cc}
 & (1) & & (2) & \\
\hline
\text{Constant} & 0.010 & *** & 0.010 & *** \\
 & (0.001) & (0.001) \\
\hline
\text{d (Housing Prices)} & 1.859 & *** & 1.909 & *** \\
 & (0.222) & (0.227) \\
\hline
\text{d (Income Level)} & 0.031 & *** & 0.029 & *** \\
 & (0.001) & (0.006) \\
\hline
\text{d (Income Composition)} & 0.293 & & 0.259 & \\
 & (0.209) & (0.214) \\
\hline
\text{d (10y Treasury Yield)} & - & & 0.001 & \\
 & - & & (0.002) & \\
\hline
\text{Adjusted } R^2 & 0.613 & & 0.615 & \\
\hline
\text{DW} & 1.11 & & 1.12 & \\
\end{array}
\]
Using our parameter estimates, we obtain \( P\bar{H}/\bar{C} = 0.052 > 0.032 = \alpha/(1 - \alpha) \). We infer from (10) that income elasticity of consumption ratio \( (\varepsilon_{Y/C}) \) is negative; i.e. consumption share of housing declines as income grows, also with the Stone-Geary utility function. However, estimates of \( \bar{C} \) and \( \bar{H} \) are unreasonably high due to low value of \( \alpha \), given that the means of \( C \) and \( H \) are 13 and 2.5, respectively. Low \( \alpha \) is obtained because housing expenditure \( (P_tH_t) \) is much smoother than income level \( (C_t + P_tH_t) \). For instance, if \( \alpha = 0.1 \), \( \bar{C} \) and \( \bar{H} \) become 10.1 and 2.1, respectively.

6. Estimating SES and IES with the GES utility function

6.1. Model

Following Pakos (2003), we use the generalized elasticity of substitution (GES) utility function, which allows for non-homothetic property. The function is a modified CES function with an additional parameter \( \eta \):

\[
    u(C, H) = \left[ (1 - \alpha) C^{1-1/\rho} + \alpha H^{1-\eta/\rho} \right]^{1/(1-1/\rho)},
\]

where \( C_t \) is the consumption of non-housing good and \( H_t \) is the consumption of housing services at time \( t \). \( \alpha \in (0, 1) \), and \( \rho \geq 0 \) is SES between two goods.

It nests the CES utility function as a special case of \( \eta = 1 \). The CES utility function, which is a workhorse in the asset pricing research, exhibits homothetic property so that any change in consumption ratio must be induced by a price change. The CES utility function further nests the Cobb-Douglas utility function \( (\rho = 1) \) and the log-linear utility function.
\(\rho = \theta = 1\).

If \(\eta\) is different from one, preferences are non-homothetic. As shown in Appendix 2, \(\eta < 1\) implies that the consumption ratio of housing services to non-housing rises as income grows: consumption share of housing is positively related to income level.

In this section, we derive the first-order conditions that serve as moment conditions in the estimation procedure. We will present estimation results both for the CES utility case with \(\eta = 1\) and for the GES utility function with no restriction on \(\eta\).

The consumer’s dynamic problem is

\[
\max_{\{C_t, H_t, q_t\}} E_0 \left[ \sum_{t=0}^{\infty} \beta^t \Omega [u(C_t, H_t)] \right]
\]

s.t. \(\forall t: C_t + P_t H_t + S_t = Y_t + S_{t-1} (q'_{t-1} R_t)\), \(q'_{t} = 1\),

where \(E_t[\cdot]\) is the expectation conditional on the information set at time \(t\), \(\beta\) is the consumer’s subjective discount factor, \(\Omega[X] \equiv X^{(1-1/\theta)} / (1 - 1/\theta)\), \(\theta\) is the IES, \(P_t\) is the relative price of housing services at time \(t\), and \(S_t\) is the consumer’s asset holdings at time \(t\), \(Y_t\) is exogenous labor income at time \(t\), \(q_t\) is \(N\)-vector of portfolio weights based on the information set at time \(t\), and \(R_t\) is \(N\)-vector of gross return to each asset in the portfolio at time \(t\), \(\iota\) is \(N\)-vector of ones. Therefore, \(S_{t-1} (q'_{t-1} R_t)\) is the total return to the consumer’s portfolio at time \(t\) in units of the numeraire good.

The competitive equilibrium agrees with the optimal allocation of the social planner in this complete-markets economy with the representative consumer. The planner’s dynamic programming consists of the following Bellman equation and the accumulation equation.

\[
\forall t: V_t (S_{t-1}) = \max_{C_t, H_t} \{ \Omega [u(C_t, H_t)] + \beta E_t [V_{t+1} (S_t)] \}
\]

s.t. \(\forall t: S_t = Y_t + S_{t-1} (q'_{t-1} R_t) - C_t - P_t H_t,\)

24
where \( V_t(S_{t-1}) \) is the value function at time \( t \) given \( S_{t-1} \). From the first-order conditions, we obtain the following intra- and inter-temporal optimality conditions:

\[
P_t = \frac{\Omega_{H,t}}{\Omega_{C,t}} = \frac{(\rho - \eta) \alpha}{(\rho - 1)(1 - \alpha)} \left( \frac{H_t^\eta}{C_t} \right)^{-\frac{1}{\rho}}, \tag{13}
\]

\[
1 = E_t \left[ \beta \frac{\Omega_{C,t+1}}{\Omega_{C,t}} R^j_{t+1} \right], \tag{14}
\]

where \( R^j_{t+1} \) is the gross return to any particular asset \( j \) in the portfolio at time \( t + 1 \). The derivation is shown in Appendix 1. We are going to use these two conditions as moment conditions in the empirical part. The pricing kernel \( \beta \Omega_{C,t+1}/\Omega_{C,t} \) is expressed in terms of consumption ratios of two goods, or equivalently, prices of housing services.

\[
\beta \frac{\Omega_{C,t+1}}{\Omega_{C,t}} = \beta \left\{ \left( \frac{C_{t+1}}{C_t} \right) \left[ \frac{(1 - \alpha) + \alpha \left( \frac{H_{t+1}^\eta}{C_{t+1}} \right)^{1-\frac{1}{\rho}}}{(1 - \alpha) + \alpha \left( \frac{H_t^\eta}{C_t} \right)^{1-\frac{1}{\rho}}} \right]^{\frac{\theta - \rho}{1 - \rho}} \right\}^{-\frac{1}{\theta}} \tag{PK1} \]

\[
= \beta \left\{ \left( \frac{C_{t+1}}{C_t} \right) \left[ \frac{(1 - \alpha) + \alpha^\rho \left( \frac{\theta - 1}{\rho - \eta} \right)^{1-\frac{1}{\rho}} P_{t+1}^{1-\rho}}{(1 - \alpha)^\rho + \alpha^\rho \left( \frac{\theta - 1}{\rho - \eta} \right)^{1-\rho} P_t^{1-\rho}} \right]^{\frac{1}{1 - \rho}} \right\}^{-\frac{\theta - \rho}{\rho - \eta}} \tag{PK2} \]

We use both expressions in order to utilize different data sets.

### 6.2. Estimation Strategy

We estimate five parameters \( (\rho, \theta, \alpha, \beta, \eta) \) by generalized method of moments (GMM) with moment conditions (13) and (14). By using return data of \( N \) assets including risk-free asset and \( I \) instruments as conditioning information at time \( t \), we have \( I(N + 1) \) moment conditions. \( (IN \) from inter-temporal condition (14) and \( I \) from intra-temporal condition (13)) We use market return and risk-free rate as return data. As instruments, we use constant for unconditional moments, and lagged consumptions (\( C \) and \( H \)) and returns for...
conditional moments. By using three moment conditions and five instruments, we have 15 over-identifying restrictions.

We adopt the two-step (efficient) generalized method of moments (GMM), in which the identity matrix is used as the weighting matrix in the first stage and then the inverse of the asymptotic covariance matrix estimated from the first stage is used in the second stage. The second-stage weighting matrix is estimated by the Bartlett kernel of Newey and West (1987) with 6 lags.

6.3. Results

Table 4 shows parameter estimates. Panel A shows the result for housing services data and Panel B shows the result for housing stock data. In each panel, two different types of the utility function (CES and GES), and two different versions of the pricing kernel (15) and (16) are examined.
Table 4 — Estimated Parameter Values of the CES/GES utility function

Panel A: $H_t = \text{NIPA-based Housing Services}$

(1947:2-2006:3, $N = 238$)

<table>
<thead>
<tr>
<th></th>
<th>Pricing kernel 1</th>
<th>Pricing kernel 2</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CES</td>
<td>GES</td>
<td>CES</td>
<td>GES</td>
</tr>
<tr>
<td>$\rho \ (\text{SES})$</td>
<td>6.009</td>
<td>0.850</td>
<td>***</td>
<td>4.241</td>
</tr>
<tr>
<td></td>
<td>(4.014)</td>
<td>(0.027)</td>
<td>(1.610)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>$\theta \ (\text{IES})$</td>
<td>0.089</td>
<td>-0.080</td>
<td>0.128</td>
<td>0.140</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.362)</td>
<td>(0.022)</td>
<td>(0.025)</td>
</tr>
<tr>
<td>$\alpha \ (\text{weight on housing})$</td>
<td>0.400</td>
<td>2.001</td>
<td>0.372</td>
<td>0.561</td>
</tr>
<tr>
<td></td>
<td>(0.044)</td>
<td>(5.645)</td>
<td>(0.035)</td>
<td>(0.725)</td>
</tr>
<tr>
<td>$\beta \ (\text{subjective discount factor})$</td>
<td>1.035</td>
<td>1.029</td>
<td>1.018</td>
<td>1.023</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.015)</td>
<td>(0.009)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>$\eta \ (\text{non-homotheticity})$</td>
<td>$-$</td>
<td>0.841</td>
<td>***</td>
<td>$-$</td>
</tr>
<tr>
<td></td>
<td>$-$</td>
<td>(0.026)</td>
<td>$-$</td>
<td>(0.017)</td>
</tr>
</tbody>
</table>

Panel B: $H_t = \text{BEA-based Housing Stock}$

(1947:2-2004:1, $N = 227$)

<table>
<thead>
<tr>
<th></th>
<th>Pricing kernel 1</th>
<th>Pricing kernel 2</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CES</td>
<td>GES</td>
<td>CES</td>
<td>GES</td>
</tr>
<tr>
<td>$\rho \ (\text{SES})$</td>
<td>0.813</td>
<td>0.379</td>
<td>***</td>
<td>0.731</td>
</tr>
<tr>
<td></td>
<td>(0.104)</td>
<td>(0.054)</td>
<td>(0.102)</td>
<td>(0.055)</td>
</tr>
<tr>
<td>$\theta \ (\text{IES})$</td>
<td>0.041</td>
<td>0.051</td>
<td>0.107</td>
<td>0.093</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.016)</td>
<td>(0.028)</td>
<td>(0.024)</td>
</tr>
<tr>
<td>$\alpha \ (\text{weight on housing})$</td>
<td>0.473</td>
<td>0.410</td>
<td>***</td>
<td>0.474</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.012)</td>
<td>(0.003)</td>
<td>(0.012)</td>
</tr>
<tr>
<td>$\beta \ (\text{subjective discount factor})$</td>
<td>1.111</td>
<td>1.094</td>
<td>1.034</td>
<td>1.039</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.039)</td>
<td>(0.014)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>$\eta \ (\text{non-homotheticity})$</td>
<td>$-$</td>
<td>1.217</td>
<td>***</td>
<td>$-$</td>
</tr>
</tbody>
</table>
Notes: In parentheses are Newey-West heteroschedasticity and autocorrelation consistent standard errors. Significance levels of *-10 percent, **-5 percent, and ***-1 percent represent whether point estimates for the CES function are outside of confidence interval of those for GES function.

The point estimate of SES (\( \rho \)) for the CES utility is quite large at 6.009 in Panel A (Column 1). It is consistent with the range reported by Davis and Martin (2005) (\( \rho > 2.2 \)) although the estimate is imprecise. The estimates of SES diminish significantly to 0.850 when we allow for non-homotheticity by using the GES utility function. (Column 2) The estimate for GES is very precise and significantly below one at one percent level. The same pattern of reduced SES under non-homotheticity appears for the other specification. (4.241 \( \rightarrow \) 0.912 for pricing kernel 2 in Panel A) When we use housing stock data by assuming linear transformation of stock into service flows (Panel B), estimates of SES become lower. But still, upward bias for the CES specification is observed. (0.813 > 0.379 and 0.731 > 0.409 in the first row in Panel B)

Overall level of SES is roughly consistent with the results from our linear regressions: SES is well below one and is lower when we use housing stock data. We conclude that SES between housing and non-housing is below one between 0.4 and 0.9 once we allow for non-homothetic preferences.

Next, we like to test the null hypothesis that \( \rho^{CES} = \rho^{GES} \), but there is no formal test statistic for these non-linear GMM estimators. We examine three alternative statistics. The first is t-statistic for the null hypothesis that \( \rho^{GES} \) equals to the point estimate of \( \rho^{CES} \). In other words, we treat \( \rho^{CES} \) as if it were non-stochastic. By using standard errors for \( \rho^{GES} \), we reject the null hypothesis that \( \rho^{CES} = \rho^{GES} \) at one percent level for any specification. The second statistic is an analogue of Hausman statistic: \( S \equiv (\rho^{GES} - \rho^{CES})^2 / (Var(\rho^{CES}) - Var(\rho^{GES})) \). The statistics are 1.65 (PK1) and 4.28 (PK2) for Panel A, and 23.84 (PK1) and 14.05 (PK2) for Panel B. Although the appropriate

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8Hausman test utilizes the gap of two estimators with respect to consistency and efficiency. (e.g. OLS and IV estimators in testing endogeneity bias) Here, although \( \rho^{CES} \) and \( \rho^{GES} \) are distinct with respect to consistency under different hypotheses, there is no a priori difference in efficiency.
degrees of freedom for the Chi-squared distribution are not obvious, the statistics are quite large for all specifications in a usual sense. We fail to reject the null hypothesis of equality at 1% level only if degrees of freedom are no less than 8, 14, 45, and 30, respectively. We are likely to reject the null. The third statistic is the probability of $\rho^{CES} = \rho^{GES}$ based on bi-variate normal distribution with zero correlation between them. The probability of equality is 0.044 (PK1) and 0.029 (PK2) for Panel A, and 0.004 (PK1) and 0.072 (PK2) for Panel B. Judging from three statistics above, it would be safe to conclude that the null hypothesis of $\rho^{CES} = \rho^{GES}$ is rejected.

This result of upward bias in SES casts an important caveat on empirical asset pricing with multiple goods. The CES utility function, or even more restrictive, the Cobb-Douglas and log-linear utility functions are workhorse in the literature. An upward bias in the estimates of SES caused by implicit assumption of homotheticity must be taken seriously. The parameter for non-homotheticity ($\eta$) is 0.841 and 0.922 in Panel A. Both estimates are significantly different from one, supporting existence of non-homothetic property of preferences. As shown in Appendix 2, $\eta < 1$ implies that income expansion is associated with a higher consumption share of housing. In other words, the result implies that income elasticity of housing demand is greater than that of non-housing demand. However, levels of the parameter are different if housing stock is used. (Panel B) Point estimates become 1.217 and 1.179, which are larger than one. As shown in Figure 3, historical patterns of consumption ratio of housing to non-housing substantially differ by data sets. Estimates of $\eta > 1$ for housing stock are driven by long-term decline in consumption ratio of housing. We further examine non-homothetic property of preferences in Section 3 by incorporating aggregate income explicitly in the OLS framework.

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9In case of endogeneity bias, the number of potentially endogenous variables is the degrees of freedom. Here, RHS variables are common for both estimators.
Estimates of IES (θ) are quite low for both CES and GES functions. The highest estimate is 0.140 with NIPA-based housing services and pricing kernel 2 for the GES function. Low
estimates are consistent with the results from early studies. However, our estimates of IES are likely to suffer from downward biases found in more recent studies. Yoshida (2008) theoretically shows that technology shocks or preference shocks on housing services create downward bias on the estimate of IES if single good is assumed. Bansal and Yaron (2004) argue that simple estimation of IES ignoring the effect of time-varying consumption volatility create significant downward bias on the estimate of IES. Vissing-Jorgensen and Attanasio (2003) also find a downward bias on estimates of IES when limited-stock market participation is not taken into account. Chen et al. (2008) find that IES is greater than unity (1.6 ∼ 2.2) if it is estimated separately from risk aversion by Epstein-Zin recursive utility. Bansal and Yaron (2004), Vissing-Jorgensen and Attanasio (2003), and Chen et al. (2008) all find IES of well above unity after correcting for biases. If IES is greater than one after correcting for the downward biases, we have relative complementarity between two goods (i.e. IES < SES).

Estimates of α and β are around 0.45 and 1.03, respectively. Although subjective discount factor greater than one (β > 1) is violating standard assumption, it is a typical result from aggregate time-series data. (e.g. ?)

6.4. A Subsection

To justify adding a subsection here, from now on, we’ll assume

**Condition 1.** \[ 0 < \hat{\mu} < \gamma \sigma^2. \]

This condition might be useful if there was a model.

6.5. Another Subsection, With a Figure

Figures get put at the end, with a note marking where they should go in the text, like this:

\[ \text{Among the large body of literature on the EIS estimation, Hall (1988) finds it to be negative and estimates it at 0.02.} \]
6.5.1. A Subsubsection with a Proposition

Let’s put a proposition here.

**Proposition 1.** If Condition [1] is satisfied, a solution to the central planner’s problem, \( V(B, D, t) \in C^2(\mathbb{R}_+^2 \times [0, T]) \), with control \( a : [0, 1] \times [0, T] \to [-\lambda, \lambda] \) if \( \gamma > 1 \) is

\[
V(B, D, t) = -\frac{(B + D)^{1-\gamma}}{1 - \gamma} w \left( \frac{B}{B + D}, t \right). \tag{17}
\]
Appendix A. An Appendix

Here’s an appendix with an equation. Note that equation numbering continues where it left off in the main body and that the JFE wants the word “Appendix” to appear before the letter in the appendix title. This is all handled in \texttt{jfe.sty}.

\[ E = mc^2. \] \hspace{2cm} (18)

Appendix B. Another Appendix

Here’s another appendix with an equation.

\[ E = mc^2. \] \hspace{2cm} (19)

Note that this is quite similar to Equation (18) in Appendix A.
References


