Problem 1
If the followings statements are true, answer "TRUE". If not, give a brief explanation why.
(1) If $f : \mathbb{C} \to \mathbb{C}$ is complex differentiable at $a$, so is the function $g(z) := \overline{f(z)}$.
(2) The set $\{z \in \mathbb{C} : |z^2 - 3| < 1\}$ is a domain.
(3) The set $\{z \in \mathbb{C} : |z| < 1 \land |z + 1| < \sqrt{2}\}$ is a star-shaped domain.
(4) The function $f(z) = -1/z^4$ has an anti-derivative in $D = \mathbb{C}^\ast$.
(5) $\int_\alpha \frac{2z - 1}{z^2 - 8z + 15} \, dz = 0$, where $\alpha$ is the unit circle around 0.

Problem 2
Compute the real and imaginary part of $(1 + i)/\sqrt{2}$ for $k \in \mathbb{Z}$.

Problem 3
Determine in which points the function $f(x + iy) = \sin^2(x + y) + i\cos^2(x + y)$ is complex differentiable and compute the derivative at those points.

Problem 4
Compute
$$\int_\alpha \frac{1}{|z|^2} \, dz,$$
where $\alpha$ is the circle of radius 3 around 0.