Homework 8 for MATH 185
Due: Wednesday March 21, 3:10 pm in class

Problem 1
Give two new proofs of the fundamental theorem of algebra:

(a) Using the open mapping theorem. (Show that the image $P(\mathbb{C})$ of a complex polynomial $P$ is open and closed.)

(b) Using the mininmal modulus principle.

Problem 2
Let $T$ be a Möbius transformation, i.e. $T$ is of the form

$$T(z) = \frac{az + b}{cz + d},$$

where $a, b, c, d \in \mathbb{C}$ such that $ad - bc \neq 0$.

(a) Show that $T$ can be written as a composition $T = T_4 \circ T_3 \circ T_2 \circ T_1$, where $T_1$ and $T_4$ are translations, $T_2$ is an inversion, and $T_3$ is a rotation-dilation.

(b) Use part (a) to show that if $L \subset \mathbb{C}$ is a straight line and $S \subset \mathbb{C}$ is a circle, then $T(L)$ is either a straight line or a circle, and $T(S)$ is either a straight line or a circle.