Geometric Sequences

Supplemental Material Not Found in Your Text

Math 34: Fall 2014

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September 22, 2014
Geometric Sequences

1. Geometric Sequences
   - Motivating Examples
   - Review

2. Formula for Geo. Seq.

3. Examples
   - Compound Interest
   - Real World Example

4. Partial Sums
   - Formula
   - Example

5. Homework
Geometric Sequences will help us answer the following:

- An interest-free loan of $12,000 requires monthly payments of 15% of the unpaid balance. What is the unpaid or outstanding balance after 18 payments?

- Suppose a business makes a $1,000 profit in its first month and has its monthly profit increase by 10% each month for the next 2 years. How much profit will the business earn in its 24th month? How much profit total profit will the business have earned at the end of 2 years?
A Geometric Sequence is a sequence where the ratio between any two consecutive numbers in the sequence is a constant.

In other words: $a_{k+1}/a_k = r$ where $r$ is a constant.
A **Geometric Sequence** is a sequence where the *ratio* between any two consecutive numbers in the sequence is a constant.

In other words: \( a_{k+1}/a_k = r \) where \( r \) is a constant.

**Examples of Geometric Sequences:**

- \( 1, 4, 16, 64, \ldots \)
- \( 32, 16, 8, 4, 2, 1, \frac{1}{2}, \frac{1}{4}, \ldots \)
A **Geometric Sequence** is a sequence where the *ratio* between any two consecutive numbers in the sequence is a constant.

*In other words:* \( a_{k+1}/a_k = r \) *where* \( r \) *is a constant.*

**Examples of Geometric Sequences:**

- 1, 4, 16, 64, \ldots
  
  \[ \frac{4}{1} = 4, \quad \text{and} \quad \frac{16}{4} = 4, \quad \text{and} \quad \frac{64}{16} = 4 \]

- 32, 16, 8, 4, 2, 1, \( \frac{1}{2} \), \( \frac{1}{4} \), \ldots
A **Geometric Sequence** is a sequence where the *ratio* between any two consecutive numbers in the sequence is a constant.

*In other words:* \( a_{k+1}/a_k = r \) *where* \( r \) *is a constant.*

**Examples of Geometric Sequences:**

- \( 1, 4, 16, 64, \ldots \) has common ratio \( r = 4 \)
  
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  \frac{4}{1} = 4, \quad \text{and} \quad \frac{16}{4} = 4, \quad \text{and} \quad \frac{64}{16} = 4
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- \( 32, 16, 8, 4, 2, 1, \frac{1}{2}, \frac{1}{4}, \ldots \)
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- 32, 16, 8, 4, 2, 1, \( \frac{1}{2} \), \( \frac{1}{4} \), ...  
  \[
  \frac{16}{32} = \frac{1}{2}, \quad \text{and} \quad \frac{8}{16} = \frac{1}{2}, \quad \text{and} \quad \frac{4}{8} = \frac{1}{2}, \text{etc}
  \]
A Geometric Sequence is a sequence where the ratio between any two consecutive numbers in the sequence is a constant.

In other words: \( a_{k+1}/a_k = r \) where \( r \) is a constant.

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  \frac{16}{32} = \frac{1}{2}, \quad \text{and} \quad \frac{8}{16} = \frac{1}{2}, \quad \text{and} \quad \frac{4}{8} = \frac{1}{2}, \quad \text{etc}
  \]
Finding a Formula for a Geometric Sequence

Consider the Geometric Sequence: 1, 4, 16, 64, …

<table>
<thead>
<tr>
<th>Index (Order)</th>
<th>Sequence Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>62</td>
</tr>
</tbody>
</table>
Finding a Formula for a Geometric Sequence

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<tbody>
<tr>
<td>0</td>
<td>1 = 1</td>
</tr>
<tr>
<td>1</td>
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So we can write a formula for the \((n+1)\)st term:

\[ a_n = 1(4)^n \]

where the index starts with \(n = 0\).
Finding a Formula for a Geometric Sequence

- Consider the Geometric Sequence: 1, 4, 16, 64, . . .

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<td>1 = 1 1 = 1 = 1(4)^0</td>
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<td>4 = 1(4) = 1(4) = 1(4)^1</td>
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So we can write a formula for the \((n + 1)^{st}\) term: 
\[ a_n = 1(4)^n \] where the index starts with \( n = 0 \)
The Formula for a Geometric Sequence

- A geometric sequence can be written as

\[ a_0, a_0(r), a_0(r)^2, \ldots, a_0(n-1) \]

- The \((n+1)^{st}\) term of a geometric sequence is

\[ a_n = a_0 r^n \]

\(a_0\) is the first term in the sequence

\(r\) is the common ratio \((r = \frac{a_1}{a_0} = \frac{a_2}{a_1} = \ldots)\)
A **geometric sequence** can be written as

\[ a_0, a_0(r), a_0(r)^2, \ldots, a_0(n - 1) \]

- **Note that we start the index with 0, so...**
  - The first term is \( a_0 \),
  - The second term is \( a_1 \),
  - The third term is \( a_2 \),
  - etc.
- Each term is \( r \) times the previous term:

\[ a_k = a_{k-1} \cdot r \]
Geometric Sequence Example:

For the Geometric Sequence 9, 12, 16, 21\(\frac{1}{3}\), …

1. What is the common ratio?

2. What is the first term in the sequence?

3. What is the fifth term in the sequence?

4. Find a formula for \(a_n\). (the \(n + 1^{st}\) term in the sequence)

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2. What is the first term in the sequence? $a_0 = 9$

3. What is the fifth term in the sequence? $a_4 = 28.4$

4. Find a formula for $a_n$. (the $n + 1^{st}$ term in the sequence)
   \[
   a_n = 9 \left( \frac{4}{3} \right)^n
   \]

5. What is the $9^{th}$ term in the sequence? (Round to 4 decimal places)
   \[
   a_8 = 9 \left( \frac{4}{3} \right)^8 = 89.785
   \]

6. What is the $20^{th}$ term in the sequence? (Round to 4 decimal places)
   \[
   a_{19} = 9 \left( \frac{4}{3} \right)^{19} = 2128.5238
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   We know \( S_n = a_0(r)^n \)

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5. What is the 9th term in the sequence? (Round to 4 decimal places)
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4. Find a formula for $a_n$. (the $n + 1^{st}$ term in the sequence)  
   We know $S_n = a_0(r)^n$
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5. What is the 9\(^{th}\) term in the sequence? (Round to 4 decimal places)
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   \(a_n = 9(\frac{4}{3})^n\)
   
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5. What is the 9\(^{\text{th}}\) term in the sequence? (Round to 4 decimal places)  
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2. What is the first term in the sequence?  
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   $a_n = 9\left(\frac{4}{3}\right)^n$

   We know $S_n = a_0(r)^n$

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5. What is the $9^{th}$ term in the sequence? (Round to 4 decimal places)  
   $9^{th}$ term is $a_8$.  
   $a_8 = 9\left(\frac{4}{3}\right)^8 = 89.8985$

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6. What is the 20\(^{th}\) term in the sequence?  (Round to 4 decimal places)
   20\(^{th}\) term is \(a_{19}\).
   \[
   a_{19} = 9(\frac{4}{3})^{19} = 2128.5238
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Recall that the compound interest formula is:

$$FV = PV(1 + i)^n$$
Recall that the compound interest formula is:

\[ FV = PV(1 + i)^n \]

Geometric Interest Formula:

\[ a_n = a_0(r)^n \]
Geometric Sequences and Compound Interest

- Recall that the compound interest formula is:
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- Geometric Interest Formula:
  \[ a_n = a_0(r)^n \]
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Geometric Interest Formula:

\[ a_n = a_0(r)^n \]

**Compound Interest is a Geometric Sequence:**

The first term: \( a_0 = PV \)

The common ratio: \( r = (1 + i) \)
Revisiting Our First Compound Interest Example: (From 3.1 Notes)

- Suppose Olaf invests $5,000 in an investment that pays 6% interest compounded annually. How much does he have at the end of each of the first 5 years?
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- Suppose Olaf invests $5,000 in an investment that pays 6% interest compounded annually. How much does he have at the end of each of the first 5 years?

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<tbody>
<tr>
<td>1</td>
<td>$5000 \times 0.06 \times 1 = $300</td>
<td>$5300.00</td>
</tr>
<tr>
<td>2</td>
<td>$5300 \times 0.06 \times 1 = $318</td>
<td>$5618.00</td>
</tr>
<tr>
<td>3</td>
<td>$5618 \times 0.06 \times 1 = $337.08</td>
<td>$5955.08</td>
</tr>
<tr>
<td>4</td>
<td>$5955.08 \times 0.06 \times 1 = $357.30</td>
<td>$6312.38</td>
</tr>
<tr>
<td>5</td>
<td>$6312.38 \times 0.06 \times 1 = $378.74</td>
<td>$6691.12</td>
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</tr>
<tr>
<td>2</td>
<td>$5300 \times 0.06 \times 1 = $318</td>
<td>$5618.00</td>
</tr>
<tr>
<td>3</td>
<td>$5618 \times 0.06 \times 1 = $337.08</td>
<td>$5955.08</td>
</tr>
<tr>
<td>4</td>
<td>$5955.08 \times 0.06 \times 1 = $357.30</td>
<td>$6312.38</td>
</tr>
<tr>
<td>5</td>
<td>$6312.38 \times 0.06 \times 1 = $378.74</td>
<td>$6691.12</td>
</tr>
</tbody>
</table>

- So 5000, 5300, 5618, 5955.08, 6312.38, 6691.12, ... is a geometric sequence,
Revisiting Our First Compound Interest Example: (From 3.1 Notes)

Suppose Olaf invests $5,000 in an investment that pays 6% interest compounded annually. How much does he have at the end of each of the first 5 years?

<table>
<thead>
<tr>
<th>Year</th>
<th>Interest earned that year</th>
<th>Balance at end of year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$5000 \times 0.06 \times 1 = $300</td>
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So $5000, 5300, 5618, 5955.08, 6312.38, 6691.12, \ldots$ is a geometric sequence,

- with $a_0 = $5300
- and $r = 1.06$
Real World Examples 1

An interest-free loan of $12,000 requires monthly payments of 15% of the unpaid balance. What is the unpaid or outstanding balance after 18 payments?

- Let’s Work the Remaining Balance for a few months:
An interest-free loan of $12,000 requires monthly payments of 15% of the unpaid balance. What is the unpaid or outstanding balance after 18 payments?

- Let’s Work the Remaining Balance for a few months:
  - Your starting balance (after 0 months) is
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- Let’s Work the Remaining Balance for a few months:
  - Your starting balance (after 0 months) is 12,000
  - After 1 month, you owe a payment:
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Let’s Work the Remaining Balance for a few months:

- Your starting balance (after 0 months) is $12,000
- After 1 month, you owe a payment:
  
  Your first payment is 15% of $12,000
  
  \[0.15 \times 12000 = 1800\]
Real World Examples 1

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  - \(0.15 \times 12000 = 1800\)
  - Remaining Balance (after 1 month):
    - \(12,000 - 1800 = 10,200\)
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Another way to think of this, if you paid of 15% of 12000, you have 85% of 12000 remaining
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  - Another way to think of this, if you paid off 15% of 12000, you have 85% of 12000 remaining
- Your second payment is 15% of $12,000
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  - After 1 month, you owe a payment:
    - Your first payment is 15% of $12,000
    - $0.15 \times 12000 = 1800
    - Remaining Balance (after 1 month):
      - $12,000 - 1800 = 10,200
      - *Another way to think of this, if you paid of 15% of 12000, you have 85% of 12000 remaining*
  - Your second payment is 15% of $12,000
    - $0.15 \times 10,200 = 1530$
    - Remaining balance (after 2 months):
      - $10200 - 1530 = 8670
Real World Examples 1

An interest-free loan of $12,000 requires monthly payments of 15% of the unpaid balance. What is the unpaid or outstanding balance after 18 payments?

<table>
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<tr>
<th>Month(s)</th>
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<tr>
<td>0</td>
<td>$12,000</td>
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<td>1</td>
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An interest-free loan of $12,000 requires monthly payments of 15% of the unpaid balance. What is the unpaid or outstanding balance after 18 payments?

\[
\begin{array}{|c|c|}
\hline
\text{(after)} & \text{Unpaid Balance} \\
\text{Month(s)} & \\
0 & \$12,000 \\
1 & \$10,200 \\
2 & \$8,670 \\
\hline
\end{array}
\]

\[
\frac{a_1}{a_0} = \frac{10,200}{12,000} = 0.85
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\[ \frac{a_2}{a_1} = \frac{8670}{10200} = 0.85 \]
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So this is a geometric sequence:

- \( a_0 = 12000 \)
- \( r = 0.85 \)
- So \( a_n = 12,000(0.85)^n \)
Real World Examples 1

An interest-free loan of $12,000 requires monthly payments of 15% of the unpaid balance. What is the unpaid or outstanding balance after 18 payments?

- So \( a_n = 12,000(0.85)^n \)

- Since \( a_1 \) is balance remaining after month 1, and \( a_2 \) is balance remaining after month 2 ...

- Balance remaining after 18 months is....
An interest-free loan of $12,000 requires monthly payments of 15% of the unpaid balance. What is the unpaid or outstanding balance after 18 payments?

- So $a_n = 12,000(0.85)^n$

- Since $a_1$ is balance remaining after month 1, and $a_2$ is balance remaining after month 2, ....

- Balance remaining after 18 months is... $a_{18} =$
Real World Examples 1

An interest-free loan of $12,000 requires monthly payments of 15% of the unpaid balance. What is the unpaid or outstanding balance after 18 payments?

- So \( a_n = 12,000(0.85)^n \)

- Since \( a_1 \) is balance remaining after month 1, and \( a_2 \) is balance remaining after month 2 ....

- Balance remaining after 18 months is....
  \[ a_{18} = 12000(0.85)^{18} = 643.76 \]
Finding Formula for the Sum of the First $n$ Terms of a Geometric Sequence:

- Find the $n^{th}$ partial sum of the geometric series

\[
\begin{align*}
\sum_{k=0}^{n-1} a_0 \cdot r^k &= a_0 + a_0r + a_0r^2 + \cdots + a_0r^{n-1} \\
&= a_0 \left( \frac{1 - r^n}{1 - r} \right)
\end{align*}
\]
Finding Formula for the Sum of the First $n$ Terms of a Geometric Sequence:

- Find the $n^{th}$ partial sum of the geometric series

\[
\begin{align*}
S_n &= a_0 + a_1 + a_2 + \ldots + a_{n-1} \\
S_n &= a_0 + a_0r + a_0(r)^2 + \ldots + a_0r^{n-1}
\end{align*}
\]
Finding Formula for the Sum of the First $n$ Terms of a Geometric Sequence:

- **Find the $n^{th}$ partial sum of the geometric series**

  \[ S_n = a_0 + a_1 + a_2 + \ldots + a_{n-1} \]

- **Now for a clever trick**

  \[ r(S_n) = r \left( a_0 + a_0r + a_0(r)^2 + \ldots + a_0r^{n-1} \right) \]

  \[ r(S_n) = a_0r + a_0rr + a_0(r)^2r + \ldots + a_0r^{n-1}r \]

  \[ r(S_n) = a_0r + a_0(r)^2 + a_0(r)^3 + \ldots + a_0r^n \]
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\]

- Subtract the 2 Equations....
Finding Formula for the Sum of the First \( n \) Terms of a Geometric Sequence:

- **Subtract the 2 Equations....**

\[
S_n = a_0 + a_0 r + a_0 (r)^2 + \ldots + a_0 r^{n-1} \\
\quad - r(S_n) = -(a_0 r + a_0 (r)^2 + a_0 (r)^3 + \ldots + a_0 r^n) \\
\]

\[
S_n - rS_n = a_0 - a_0 r^n
\]
Finding Formula for the Sum of the First $n$ Terms of a Geometric Sequence:

- **Subtract the 2 Equations....**

\[
S_n = a_0 + a_0r + a_0(r^2) + \ldots + a_0r^{n-1} \\
-rS_n = -\left(a_0r + a_0(r^2) + a_0(r^3) + \ldots + a_0r^n\right)
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\[
S_n - rS_n = a_0 - a_0r^n
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- **Now a bit of Algebra**
Finding Formula for the Sum of the First $n$ Terms of a Geometric Sequence:

- Subtract the 2 Equations....

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\]

\[
S_n - rS_n = a_0
\]

- Now a bit of Algebra

\[
S_n - rS_n = a_0 - a_0r^n
\]

\[
S_n(1 - r) = a_0(1 - r^n)
\]

\[
\frac{S_n(1 - r)}{(1 - r)} = \frac{a_0(1 - r^n)}{1 - r}
\]
Finding Formula for the Sum of the First $n$ Terms of a Geometric Sequence:

- **Subtract the 2 Equations....**

$$S_n = a_0 + a_0 r + a_0 (r)^2 + \ldots + a_0 (r)^{n-1}$$

$$- \left( -a_0 r + a_0 (r)^2 + a_0 (r)^3 + \ldots + a_0 (r)^n \right)$$

$$S_n - rS_n = a_0$$

- **Now a bit of Algebra**

$$S_n - rS_n = a_0 - a_0 r^n$$

$$S_n(1 - r) = a_0(1 - r^n)$$

$$\frac{S_n(1 - r)}{(1 - r)} = \frac{a_0(1 - r^n)}{1 - r}$$

$$S_n = \frac{a_0(1 - r^n)}{1 - r}$$
Formula for Partial Sum of Geometric Sequence

The sum of the first $n$ terms of a geometric sequence with first term $a_0$ and common ratio $r$ is

$$S_n = \frac{a_0(1 - r^n)}{1 - r}$$

As long as $r$ is not equal to 1.
Partial Sum of Geometric Sequence Example:

Find the $20^{th}$ partial sum of the Geometric Sequence $1000, 1200, 1440, 1728, \ldots$
Partial Sum of Geometric Sequence Example:

- Find the $20^{th}$ partial sum of the Geometric Sequence $1000, 1200, 1440, 1728, \ldots$
  - Identify $a_0$
  - Identify $r$
  - Plug into $S_n$ formula.

$$S_n = a_0 \left(1 - r^n\right)$$

$$S_{20} = 1000 \left(1 - (1.2)^{20}\right)$$

$$S_{20} \approx 186,687.999622 = \$186,687.99$$
Partial Sum of Geometric Sequence Example:

- Find the 20\textsuperscript{th} partial sum of the Geometric Sequence 1000, 1200, 1440, 1728, \ldots
  - Identify \( a_0 = 1000 \)
  - Identify \( r \)
  - Plug into \( S_n \) formula.
Partial Sum of Geometric Sequence Example:

Find the $20^{th}$ partial sum of the Geometric Sequence 1000, 1200, 1440, 1728, …

- Identify $a_0 = 1000$

- Identify $r \frac{1200}{1000} = 1.2$

- Plug into $S_n$ formula.
Partial Sum of Geometric Sequence Example:

- Find the $20^{th}$ partial sum of the Geometric Sequence $1000, 1200, 1440, 1728, \ldots$
  - Identify $a_0 = 1000$
  - Identify $r = \frac{1200}{1000} = 1.2, \frac{1440}{1200} = 1.2$
  - Plug into $S_n$ formula.

$$S_n = \frac{a_0(1 - r^n)}{1 - r}$$

$$S_{20} = \frac{1000(1 - (1.2)^{20})}{1 - 1.2}$$

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    \[
    \frac{1200}{1000} = 1.2 \quad \frac{1440}{1200} = 1.2
    \]
  
  - Plug into $S_n$ formula.
    \[
    S_n = \frac{a_0(1-r^n)}{1-r}
    \]

  - $S_{20} = \frac{1000(1-(1.2)^{20})}{1-1.2}$
  
  - $S_{20} \approx 186,687.999622 \approx 186,688$
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- Identify $a_0 = 1000$
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\[
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\[
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  \frac{1200}{1000} = 1.2 \quad \frac{1440}{1200} = 1.2
  \]

  - Plug into $S_n$ formula.

  \[
  S_n = \frac{a_0(1-r^n)}{1-r}
  \]

  \[
  S_n = \frac{1000(1-(1.2)^n)}{1-(1.2)}
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  \]
Real World Examples 2:

Suppose a business makes a $1,000 profit in its first month and has its monthly profit increase by 10% each month for the next 2 years. How much profit will the business earn in its 24th month? How much profit total profit will the business have earned at the end of 2 years?
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- Work out a few months: Remember, we need the index to start at 0.
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<td>$1,000</td>
</tr>
<tr>
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<td>1</td>
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- Realize this is a Geometric Series
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- Realize this is a Geometric Series
  - Identify $a_0$
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  \[ S_{24} = \frac{1000(1 - 1.1^{24})}{1 - 1.1} = \$79543.02 \]
Shortcuts to Finding $r$

- Sometimes there is an easier way to find $r$ rather than working out several terms and checking the ratio.

- If each term in the sequence is a certain percent *more* than the previous term:
  \[ r = 1 + p \]
  (where $p$ is the percent, converted to a decimal)

- If each term in the sequence is a certain percent *less* than the previous term:
  \[ r = 1 - p \]
  (where $p$ is the percent, converted to a decimal)
You start advertising your dog grooming business on a new social network called Woofer. Your advertising cost in January $100. Since the social network is growing in popularity, your advertising cost in February are 8% higher. You assume this pattern will continue, and each month your advertising cost will be 8% higher than the previous month.

1. What are your advertising costs in December?

Let's denote the advertising cost in January as $a_0 = 100$. The common ratio $r = 1.08$. The advertising cost in December ($a_{11}$) can be calculated as:

$$a_{11} = 100(1.08)^{11} = 233.16$$

2. How much will you spend in advertising over the year?

This is a partial sum, we want $S_{12}$, the sum of the first 12 terms.

$$S_{12} = 100(1 - (1.08)^{11})/(1 - 1.08) = 1664.55$$
Real World Examples 3

You start advertising your dog grooming business on a new social network called Woofer. Your advertising cost in January $100. Since the social network is growing in popularity, your advertising cost in February are 8% higher. You assume this pattern will continue, and each month your advertising cost will be 8% higher than the previous month.

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1. What are your advertising costs in December?
   This will be a Geo Series
   
   \[ a_0 = \text{advert. costs in Jan}, \quad a_2 = \text{advert. costs in Feb} \ldots \]
   
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A business has a profit of $25,000 in the first year and then loses 5% each year for the next seven years.

1. What is the business’ profit in the 4th year?

2. What is the total profit after 7 years?
Real World Examples 4

A business has a profit of $25,000 in the first year and then loses 5% each year for the next seven years. 
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1. **What is the business’ profit in the 4th year?**
   - We’ll answer with a term of the sequence
   - Since the profit in the first year is $a_0$, the profit in the 4th year is $a_3$

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   a_3 = 25000(0.95)^3 = 21434.38
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   $a_3 = 25000(0.95)^3 = 21434.38$

2. What is the total profit after 7 years?
   Since this is about adding up the profit in each of the first 7 years...
   We answer with $S_7$
   $S_7 = \frac{25000(1-0.95^7)}{(1-0.95)} = 150,831.35$
Homework

It is NOT in your book.

It IS at the end of the printout on the course website.