Arithmetic Sequences

Supplemental Material Not Found in You Text

Math 34: Fall 2014

Do NOT print these slides!!

There are printer friendly files on the website.

September 3, 2014
Arithmetic Sequences

1. Arithmetic Sequences
   - Review
   - Real World Examples

2. General Way to Write an Arithmetic Sequence
   - Formula
   - Examples

3. Partial Sums
   - Formula
   - Examples

4. Homework
Recall an **Arithmetic Sequence** is a sequence where the *difference* between any two consecutive numbers in the sequence is constant.

\[ a_{k+1} - a_k = d \quad \text{where } d \text{ is a constant.} \]
Recall an Arithmetic Sequence is a sequence where the difference between any two consecutive numbers in the sequence is constant.

In other words: \( a_{k+1} - a_k = d \) where \( d \) is a constant.

Which of the following are Arithmetic Sequences?

1. 1, 4, 7, 10, 13, . . .

2. 2, 4, 8, 16, 32, . . .

3. -3, 7, 17, 27, . . .
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*In other words:* \( a_{k+1} - a_k = d \) where \( d \) is a constant.

Which of the following are Arithmetic Sequences?

1. 1, 4, 7, 10, 13, . . .  
   IS arithmetic, with constant difference \( d = 3 \)

2. 2, 4, 8, 16, 32, . . .

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*In other words:* \( a_{k+1} - a_k = d \) *where* \( d \) *is a constant.*

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   - IS arithmetic, with constant difference \( d = 3 \)

2. 2, 4, 8, 16, 32, . . .
   - is NOT arithmetic

3. −3, 7, 17, 27, . . .
Recall an **Arithmetic Sequence** is a sequence where the *difference* between any two consecutive numbers in the sequence is constant.

*In other words:* \( a_{k+1} - a_k = d \) where \( d \) is a constant.

**Which of the following are Arithmetic Sequences?**

- **1** 1, 4, 7, 10, 13, . . .  
  IS arithmetic, with constant difference \( d = 3 \)

- **2** 2, 4, 8, 16, 32, . . .  
  is NOT arithmetic

- **3** −3, 7, 17, 27, . . .  
  IS arithmetic, with constant difference \( d = 10 \)
Real World Examples

1. You start a new job and you’re told you salary is $29,000 for the first year, and that you’ll get a $1700 raise each year. What will your salary be in the third year? What will your salary be in 10 years? How long does it take for your salary to (at least) double?

2. A new company has a loss of $2,500 in its first month, but they expect their monthly profit to increase by $400 each month. What is their profit in the 12th month? What is their total profit/loss of the year?
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1. You start a new job and you’re told your salary is $29,000 for the first year, and that you’ll get a $1700 raise each year. What will your salary be in the third year? What will your salary be in 10 years? How long does it take for your salary to (at least) double?

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*Both these scenarios can be modeled by Arithmetic Sequences, and we will develop tools to help us answer these questions.*
Consider the Arithmetic Sequence below. Notice the first term is 5 and the common difference is 2:

5, 7, 9, 11, 13, ...
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Look at the pattern that the common difference of 2 creates.
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Look at the pattern that the common difference of 2 creates.

\begin{align*}
5, & \quad 7, \quad 9, \quad 11, \quad 13, \quad \ldots \\
5, & \quad 5+2, \quad 7+2, \quad 9+2, \quad 11+2, \quad \ldots
\end{align*}
Consider the Arithmetic Sequence below. Notice the first term is 5 and the common difference is 2:

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Look at the pattern that the common difference of 2 creates.

5, 7, 9, 11, 13, . . .

5, 5+2, 7+2, 9+2, 11+2, . . .

5, 5 + 2, 5 + (2)2, 5 + (3)2, 5 + 4(2), . . .
General Way to Write an Arithmetic Sequence

Consider the Arithmetic Sequence below. Notice the first term is 5 and the common difference is 2:

$$5, 7, 9, 11, 13, \ldots$$

Look at the pattern that the common difference of 2 creates.

$$\begin{array}{cccccc}
5, & 7, & 9, & 11, & 13, & \ldots \\
5, & 5+2, & 7+2, & 9+2, & 11+2, & \ldots \\
5, & 5 + 2, & 5 + (2)2, & 5 + (3)2, & 5 + 4(2), & \ldots \\
a_1, & a_2, & a_3, & a_4, & a_5, & \ldots 
\end{array}$$
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5, 7, 9, 11, 13, . . .

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5, 5 + 2, 5 + (2)2, 5 + (3)2, 5 + 4(2), . . .

We notice the pattern for this sequence $a_n = 5 + (n - 1)2$
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\end{align*}
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\[
\begin{align*}
a_1, & \quad a_2, \quad a_3, \quad a_4, \quad a_5, \quad \ldots \\
\end{align*}
\]

We notice the pattern for this sequence \( a_n = 5 + (n - 1)2 \)

We also see that \( a_n = a_{n-1} + 2 \) (each term is 2 more than the previous term)
Way to Write a Formula for an Arithmetic Sequence: Given that \( a_1, a_2, a_3, \ldots \) is an arithmetic sequence with common difference \( d \),

We can rewrite the sequence as

\[
    a_n = a_1 + (n - 1)d
\]

where the index starts at \( n = 1 \).

Here \( a_1 \) is the first term of the sequence (a constant) and \( d \) is the common difference (also a constant).
Examples (Arithmetic Sequences)

Given the Arithmetic Sequence $-10, -4, 2, 8, \ldots$

1. Find the fifth term in the sequence.

   Since the first 4 terms are given, and we can see the common difference is $d = 6$, we can see the 5th term 6 more than the 4th term.  
   
   2. Find the seventh term in the sequence. 

   $a_6 = a_5 + 6 = 14 + 6 = 20$ and $a_7 = a_6 + 6 = 20 + 6 = 26$

3. Find the 20th term in the sequence.

   We'd rather not do this out to $a_{20}$, so let's use the formula.  

   Since our sequence is an arithmetic with first term $a_1 = -10$ and common difference $d = 6$, we can rewrite it as: 
   
   $a_n = -10 + (n-1)6$ with starting term $n = 1$

   This mean the 20th term is: 

   $a_{20} = -10 + (20-1)6 = 104$

4. Find a formula for the $n^{th}$ term in the sequence.

   Done above because shortcuts are awesome
Examples (Arithmetic Sequences)

Given the Arithmetic Sequence $-10, -4, 2, 8, \ldots$

1. Find the fifth term in the sequence.
   
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   Since the first 4 terms are given, and we can see the common difference is \(d = 6\), we can see the 5th term 6 more than the 4th term. i.e. \(a_5 = a_4 + 6 = 8 + 6 = 14\)

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   \[a_n = a_1 + (n - 1)d\]

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Examples (Arithmetic Sequences)

Given the Arithmetic Sequence $-10, -4, 2, 8, \ldots$

1. **Find the fifth term in the sequence.**
   
   \[a_5 = a_4 + 6 = 8 + 6 = 14\]

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   Since our sequence is an arithmetic with first term \(a_1 = -10\) and common difference \(d = 6\) we can rewrite it as:
   
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Given the Arithmetic Sequence $-10, -4, 2, 8, \ldots$

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   *Since the first 4 terms are given, and we can see the common difference is $d = 6$, we can see the 5th term 6 more than 4th term.*
   
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common difference $d = 6$ we can rewrite it as:
$a_n = -10 + (n - 1)6$ with starting term $n = 1$

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   This means the $20^{th}$ term is: $a_{20} = -10 + (20 - 1)6 = 104$

4. Find a formula for the $n^{th}$ term in the sequence.
   
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Joan invests $3,000 in an account that pays 2% simple interest. Determine how much money is in her account after each of the first 5 years.
Examples, Real World Arithmetic Sequences (Number 1)

Joan invests $3,000 in an account that pays 2% simple interest. Determine how much money is in her account after each of the first 5 years.

- Using \( I = PRT \) formula for simple interest.
Examples, Real World Arithmetic Sequences (Number 1)

Joan invests $3,000 in an account that pays 2% simple interest. Determine how much money is in her account after each of the first 5 years.

- Using \( I = PRT \) formula for simple interest.
  
  \[
  P = $3,000 \\
  R = 0.02 \\
  T = \text{(depends which year we’re talking about)}
  \]

<table>
<thead>
<tr>
<th>Year</th>
<th>Interest ((I = PRT))</th>
<th>Total In Account</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$3000 \cdot 0.02 \cdot 1 = $60</td>
<td>$3000 + $60 = $3060</td>
</tr>
<tr>
<td>2</td>
<td>$3000 \cdot 0.02 \cdot 2 = $120</td>
<td>$3120</td>
</tr>
<tr>
<td>3</td>
<td>$3000 \cdot 0.02 \cdot 3 = $180</td>
<td>$3180</td>
</tr>
<tr>
<td>4</td>
<td>$3000 \cdot 0.02 \cdot 4 = $240</td>
<td>$3240</td>
</tr>
<tr>
<td>5</td>
<td>$3000 \cdot 0.02 \cdot 5 = $300</td>
<td>$3300</td>
</tr>
</tbody>
</table>
Examples, Real World Arithmetic Sequences (Number 1)

Joan invests $3,000 in an account that pays 2% simple interest. Determine how much money is in her account after each of the first 5 years.

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  $T = (depends\ which\ year\ we’re\ talking\ about)$

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<td>$3000 + 60 = 3060$</td>
</tr>
<tr>
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<td>$3000 \cdot 0.02 \cdot 2 = 120$</td>
<td>$3000 + 120 = 3120$</td>
</tr>
<tr>
<td>3</td>
<td>$3000 \cdot 0.02 \cdot 3 = 180$</td>
<td>$3000 + 180 = 3180$</td>
</tr>
<tr>
<td>4</td>
<td>$3000 \cdot 0.02 \cdot 4 = 240$</td>
<td>$3000 + 240 = 3240$</td>
</tr>
<tr>
<td>5</td>
<td>$3000 \cdot 0.02 \cdot 5 = 300$</td>
<td>$3000 + 300 = 3300$</td>
</tr>
</tbody>
</table>
Examples, Real World Arithmetic Sequences (Number 1) Cont.

- Another way to think about it:

  - The simple interest from each year is $3000 \cdot 0.02 \cdot 1 = $60,
  - so each year Joan has $60 more than the previous year.

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So the list of how much money Joan has in the account each year is:

$3060, $3120, $3180, $3240, $3300, 

It looks like an Arithmetic Sequence with first term $a_1 = $3060 and constant difference $d = $60.

So the amount of money in the account at (the end of) year $n$ is:

$$a_n = 3000 + (n - 1) \cdot 60$$
Examples, Real World Arithmetic Sequences (Number 1) Cont.

- Another way to think about it:

  The simple interest from each year is $3000 \cdot 0.02 \cdot 1 = 60$, so each year Joan has $60$ more than the previous year.
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Examples, Real World Arithmetic Sequences (Number 1) Cont.

Another way to think about it:

The simple interest from each year is \(3000 \cdot 0.02 \cdot 1 = 60\), so each year Joan has $60 more than the previous year.

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Another way to think about it:

The simple interest from each year is $3000 \cdot 0.02 \cdot 1 = \$60$, so each year Joan has $\$60$ more than the previous year.

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$\$3060, \$3120, \$3180, \$3240, \$3300, \ldots$
Another way to think about it: The simple interest from each year is $3000 \cdot 0.02 \cdot 1 = $60, so each year Joan has $60 more than the previous year.

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So the list of how much money Joan has in the account each year is:

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*The simple interest from each year is* $3000 \cdot 0.02 \cdot 1 = $60, *so each year Joan has $60 more than the previous year.*

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So the list of how much money Joan has in the account each year is:

$3060, 3120, 3180, 3240, 3300, \ldots$

It looks like an Arithmetic Sequence with first term $a_1 = 3060$ and constant difference $d = 60$

So the amount of money in the account at (the end of) year $n$ is: $a_n = 3000 + (n - 1)\cdot 60$
Examples, Real World Arithmetic Sequences (Number 2)

You start a new job and you’re told you salary is $29,000 for the first year, and that you’ll get a $1700 raise each year. What will your salary be in the third year? What will your salary be in 10 years? How long does it take for your salary to (at least) double?
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You start a new job and you’re told you salary is $29,000 for the first year, and that you’ll get a $1700 raise each year. What will your salary be in the third year? What will your salary be in 10 years? How long does it take for your salary to (at least) double?

- Fill in the table indicating your salary in the first several years:

<table>
<thead>
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<th>Year</th>
<th>Salary in indicated year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
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<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>
You start a new job and you’re told you salary is $29,000 for the first year, and that you’ll get a $1700 raise each year. What will your salary be in the third year? What will your salary be in 10 years? How long does it take for your salary to (at least) double?

Fill in the table indicating your salary in the first several years:

<table>
<thead>
<tr>
<th>Year</th>
<th>Salary in indicated year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$29,000</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
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<tr>
<td>1</td>
<td>$29,000</td>
</tr>
<tr>
<td>2</td>
<td>$30,700</td>
</tr>
<tr>
<td>3</td>
<td>$32,400</td>
</tr>
<tr>
<td>4</td>
<td>$34,100</td>
</tr>
</tbody>
</table>
Examples, Real World Arithmetic Sequences (Number 2) Cont.

- Notice that the list of your salaries year by year look like an Arithmetic Sequence. Identify the common difference, and the first term:

- Write a formula for $a_n$ (your salary in year $n$).

- What will your salary be in the third year?

- What will your salary be in 10 years?
Examples, Real World Arithmetic Sequences (Number 2) Cont.

- Notice that the list of your salaries year by year look like an Arithmetic Sequence. Identify the common difference, and the first term:
  \[ a_1 = \$29,000 \]
  \[ d = \$1700 \]

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- What will your salary be in the third year?

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\[ a_1 = \$29,000 \]
\[ d = \$1700 \]

Write a formula for \( a_n \) (your salary in year \( n \)).

\[ a_n = \$29,000 + (n - 1)\$1700 \]

Where \( n \) is measured in years, and \( a_n \) is your salary in year \( n \) (measured in dollars)

What will your salary be in the third year?

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- What will your salary be in the third year?
  \[ a_3 = \$29,000 + (3 - 1)\$1700 = \$32,400 \]

- What will your salary be in 10 years?
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\[ a_1 = \$29,000 \]
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Write a formula for \( a_n \) (your salary in year \( n \)).

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What will your salary be in the third year?

\[ a_3 = \$29,000 + (3 - 1)\$1700 = \$32,400 \]

What will your salary be in 10 years?

\[ a_{10} = \$29,000 + (10 - 1)\$1700 = \$44,300 \]
Examples, Real World Arithmetic Sequences (Number 2) Cont.

- How long does it take for your salary to (at least) double?
Examples, Real World Arithmetic Sequences (Number 2) Cont.

- How long does it take for your salary to (at least) double?

  *Double your (starting) salary is $58,000*
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  \[
  $58,000 = $29,000 + (n - 1)$1700
  \]
Examples, Real World Arithmetic Sequences (Number 2) Cont.

- How long does it take for your salary to (at least) double?

  *Double your (starting) salary is $58,000*

  This is a value for $a_n$

  \[
  \begin{align*}
  58,000 &= 29,000 + (n - 1) \times 1700 \\
  58,000 - 29,000 &= (n - 1) \times 1700 \\
  29,000 &= n \times 1700 - 1700 \\
  29,000 &= n \times 1700 \\
  n &\approx 18.0588, \text{ We must round up to 19 years.}
  \end{align*}
  \]
Examples, Real World Arithmetic Sequences (Number 2) Cont.

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  \end{align*}
  \]

  So \(n \approx 18.0588\), We must round up to 19 years.

  Double Check:

  \[
  \begin{align*}
  a_{18} & = $29,000 + (18 - 1)$1700 = $57,900 \text{ (less than double)} \\
  a_{19} & = $29,000 + (19 - 1)$1700 = $59,600 \text{ (more than double)}
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  So it takes 19 year for your salary to double.
Examples, Real World Arithmetic Sequences (Number 2) Cont.

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58,000 &= 29,000 + (n - 1)1700 \\
58,000 - 29,000 &= (n - 1)1700 \\
29,000 &= n \cdot 1700 - 1700 \\
29,000 + 1700 &= n \cdot 1700 \\
30,700 &= n \cdot 1700 \\
\frac{30,700}{1700} &= n
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So $n \approx 18.0588$, We must round up to 19 years.

*Double Check:*

$\begin{align*}
\text{For } n = 18: \\
&= 29,000 + (18 - 1) \cdot 1700 \\
&= 29,000 + 16 \cdot 1700 \\
&= 29,000 + 27,200 \\
&= 56,200
\end{align*}$

$\begin{align*}
\text{For } n = 19: \\
&= 29,000 + (19 - 1) \cdot 1700 \\
&= 29,000 + 18 \cdot 1700 \\
&= 29,000 + 30,600 \\
&= 59,600
\end{align*}$

So it takes 19 years for your salary to double.
How long does it take for your salary to (at least) double?

*Double your (starting) salary is $58,000*

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(58,000 - 29,000) &= (n - 1) \times 1700 \\
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Remember $S_n$ is the sum of the first $n$ terms of a sequence.
Partial Sums of an Arithmetic Sequence

- Remember $S_n$ is the sum of the first $n$ terms of a sequence.
- Let’s work our a formula for the $n^{th}$ partial sum of an Arithmetic Sequence.
Here’s one way to write our Arithmetic sequence:

\[ a_1, (a_1 + d), (a_1 + 2d), (a_1 + 3d), \ldots \]
Partial Sums of an Arithmetic Sequence

- Here’s one way to write our Arithmetic sequence:
  \[ a_1, (a_1 + d), (a_1 + 2d), (a_1 + 3d), \ldots \]

- So the \( n^{th} \) Partial Sum of the Arithmetic Series can be written as:
  \[ S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + (a_1 + (n - 2)d) + (a_1 + (n - 1)d) \]
  \[ a_n \]
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- Another way to name the terms:
  \[a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + (a_1 + (n-2)d) + (a_1 + (n-1)d)\]
Partial Sums of an Arithmetic Sequence

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\[
S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + (a_1 + (n-2)d) + \underbrace{(a_1 + (n - 1)d)}_{a_n}
\]

Another way to name the terms:

\[
S_n = \underbrace{a_1}_{a_n-(n-1)d} + \underbrace{(a_1 + d) + (a_1 + 2d)}_{a_n-(n-2)d} + \underbrace{\cdots + (a_1 + (n-2)d)}_{a_n-(n-3)d} + \underbrace{(a_1 + (n - 1)d)}_{a_n-d} + \underbrace{(a_1 + (n - 1)d)}_{a_n}
\]

This gives us another way to write \( S_n \):

\[
S_n = a_n + (a_n - d) + (a_n - 2d) + \cdots + (a_n - (n-2)d) + (a_n - (n-1)d)
\]
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  \]
  \[
  = a_1 + a_2 + a_3 + \cdots + a_n
  \]

- Another way to name the terms:
  \[
  S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + (a_1 + (n-2)d) + (a_1 + (n-1)d)
  \]
  \[
  = a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + (a_n - (n-1)d) + (a_n - (n-1)d)
  \]

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  \]

- Add the two ways to write \( S_n \) together.....
Add the two ways to write $S_n$ together.....

\[
S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + (a_1 + (n-2)d) + (a_1 + (n-1)d)
\]

\[
+ S_n = a_n + (a_n - d) + (a_n - 2d) + \cdots + (a_n - (n-2)d) + (a_n - (n-1)d)
\]

\[
2S_n = (a_1 + a_n) + (a_1 + a_n) + (a_1 + a_n) + \cdots + (a_1 + a_n) + (a_1 + a_n)
\]
Partial Sums of an Arithmetic Sequence (Cont.)

Add the two ways to write $S_n$ together.....

\[
S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + (a_1 + (n-2)d) + (a_1 + (n-1)d)
\]

\[
+ S_n = a_n + (a_n - d) + (a_n - 2d) + \cdots + (a_n - (n-2)d) + (a_n - (n-1)d)
\]

\[
\frac{2S_n}{2} = (a_1 + a_n) + (a_1 + a_n) + (a_1 + a_n) + \cdots + (a_1 + a_n) + (a_1 + a_n)
\]

We count the $(a_1 + a_n)$ terms on the right...
Add the two ways to write $S_n$ together.....

\[
S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + (a_1 + (n - 2)d) + (a_1 + (n - 1)d)
\]

\[
+S_n = a_n + (a_n - d) + (a_n - 2d) + \cdots + (a_n - (n - 2)d) + (a_n - (n - 1)d)
\]

\[
2S_n = \frac{1}{2} [(a_1 + a_n) + (a_1 + a_n) + (a_1 + a_n) + \cdots + (a_1 + a_n) + (a_1 + a_n)]
\]

We count the $(a_1 + a_n)$ terms on the right...
We see that $2S_n = n(a_1 + a_n)$ and...
Partial Sums of an Arithmetic Sequence (Cont.)

Add the two ways to write $S_n$ together.....

\[
S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + (a_1 + (n - 2)d) + (a_1 + (n - 1)d)
\]
\[
+ S_n = a_n + (a_n - d) + (a_n - 2d) + \cdots + (a_n - (n - 2)d) + (a_n - (n - 1)d)
\]
\[
\frac{2S_n}{2} = (a_1 + a_n) + (a_1 + a_n) + \cdots + (a_1 + a_n) + (a_1 + a_n)
\]

We count the $(a_1 + a_n)$ terms on the right...
We see that $2S_n = n(a_1 + a_n)$ and...

\[
S_n = \frac{n(a_1 + a_n)}{2} = \frac{n}{2}(a_1 + a_n)
\]
The $n^{th}$ partial sum of an Arithmetic Sequence $a_1, a_2, a_3, \ldots$ is given by

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Where $a_1$ is the first term of the Arithmetic Sequence and $a_n$ is the $n^{th}$ term of the Arithmetic Series.
Using the Formula the $n^{th}$ Partial Sum of an Arithmetic Sequence

For the Arithmetic Sequence 7, 10, 13, 16, 19, 21, ...

1. Find the 4$^{th}$ Partial Sum of the Sequence.

2. Find the 20$^{th}$ term of the Sequence

3. Find the 20$^{th}$ Partial Sum of the Sequence
Using the Formula the $n^{th}$ Partial Sum of an Arithmetic Sequence

For the Arithmetic Sequence 7, 10, 13, 16, 19, 21, …

1. Find the 4$^{th}$ Partial Sum of the Sequence.
   \[ S_4 = \frac{4}{2}(a_1 + a_4) = \frac{4}{2}(7 + 16) = 2(23) = 46 \]

2. Find the 20$^{th}$ term of the Sequence

3. Find the 20$^{th}$ Partial Sum of the Sequence
Using the Formula the $n^{th}$ Partial Sum of an Arithmetic Sequence

1. Find the $4^{th}$ Partial Sum of the Sequence.
   \[ S_4 = \frac{4}{2} (a_1 + a_4) = \frac{4}{2} (7 + 16) = 2(23) = 46 \]
   (We can double check that $7 + 10 + 13 + 16 = 46$)

2. Find the $20^{th}$ term of the Sequence

   \[ a_{20} = 7 + (20 - 1)3 = 7 + 19 \cdot 3 = 64 \]

3. Find the $20^{th}$ Partial Sum of the Sequence
   \[ S_{20} = \frac{20}{2} (7 + 64) = 10(71) = 710 \]
   Which is much faster than $7 + 10 + 13 + 16 + 19 + 22 + 25 + 28 + 31 + 34 + 37 + 40 + 43 + 46 + 49 + 52 + 55 + 58 + 61 + 64 = 710$
Using the Formula the $n^{th}$ Partial Sum of an Arithmetic Sequence

- For the Arithmetic Sequence 7, 10, 13, 16, 19, 21, ... 

1. Find the $4^{th}$ Partial Sum of the Sequence.
   
   $S_4 = \frac{4}{2}(a_1 + a_4) = \frac{4}{2}(7 + 16) = 2(23) = 46$
   
   (We can double check that $7 + 10 + 13 + 16 = 46$)

2. Find the $20^{th}$ term of the Sequence
   
   Our Arithmetic Sequence has $a_1 = 7$ and $d = 3$ so
   
   $a_n = 7 + (n - 1)3$, so ...

3. Find the $20^{th}$ Partial Sum of the Sequence
Using the Formula the $n^{th}$ Partial Sum of an Arithmetic Sequence

- For the Arithmetic Sequence 7, 10, 13, 16, 19, 21, …

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   Our Arithmetic Sequence has $a_1 = 7$ and $d = 3$ so
   \[ a_n = 7 + (n - 1)3, \text{ so } \]
   \[ a_{20} = 7 + (20 - 1)3 = 7 + 19 \cdot 3 = 64 \]

3. Find the $20^{th}$ Partial Sum of the Sequence
Using the Formula the $n^{th}$ Partial Sum of an Arithmetic Sequence

- For the Arithmetic Sequence 7, 10, 13, 16, 19, 21, …

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2. Find the 20<sup>th</sup> term of the Sequence.
   Our Arithmetic Sequence has $a_1 = 7$ and $d = 3$ so
   \[ a_n = 7 + (n - 1)3, \text{ so } \]
   \[ a_{20} = 7 + (20 - 1)3 = 7 + 19\cdot3 = 64 \]

3. Find the 20<sup>th</sup> Partial Sum of the Sequence.
   \[ S_{20} = \frac{20}{2}(7 + 64) = 10(71) = 710 \]
Using the Formula the $n^{th}$ Partial Sum of an Arithmetic Sequence

- For the Arithmetic Sequence 7, 10, 13, 16, 19, 21, ...

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   (We can double check that $7 + 10 + 13 + 16 = 46$)

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   Our Arithmetic Sequence has $a_1 = 7$ and $d = 3$ so
   \[ a_n = 7 + (n - 1)3, \text{ so } \]
   \[ a_{20} = 7 + (20 - 1)3 = 7 + 19 \cdot 3 = 64 \]

3. Find the 20$^{th}$ Partial Sum of the Sequence
   \[ S_{20} = \frac{20}{2}(7 + 64) = 10(71) = 710 \]
   Which is much faster than
Using the Formula the \( n^{th} \) Partial Sum of an Arithmetic Sequence

- **For the Arithmetic Sequence** \( 7, 10, 13, 16, 19, 21, \ldots \)
  1. Find the \( 4^{th} \) Partial Sum of the Sequence.
     \[
     S_4 = \frac{4}{2} (a_1 + a_4) = \frac{4}{2} (7 + 16) = 2(23) = 46
     \]
     (We can double check that \( 7 + 10 + 13 + 16 = 46 \))
  2. Find the \( 20^{th} \) term of the Sequence
     Our Arithmetic Sequence has \( a_1 = 7 \) and \( d = 3 \) so
     \[
     a_n = 7 + (n - 1)3, \text{ so } \]
     \[
     a_{20} = 7 + (20 - 1)3 = 7 + 19 \cdot 3 = 64
     \]
  3. Find the \( 20^{th} \) Partial Sum of the Sequence
     \[
     S_{20} = \frac{20}{2} (7 + 64) = 10(71) = 710
     \]

*Which is much faster than*

\[
7 + 10 + 13 + 16 + 19 + 22 + 25 + 28 + 31 + 34 + 37 + 40 + 43 + 46 + 49 + 52 + 55 + 58 + 61 + 64 = 710
\]
Examples, Real World Arithmetic Sequences (Number 3)

A new company has a loss of $2,500 in its first month, but they expect their monthly profit to increase by $400 each month. What is their profit in the 12\textsuperscript{th} month? What is their total profit/loss of the year?
A new company has a loss of $2,500 in its first month, but they expect their monthly profit to increase by $400 each month. What is their profit in the 12th month? What is their total profit/loss of the year?

Fill in the Table:

<table>
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<tr>
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<tr>
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</tr>
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Examples, Real World Arithmetic Sequences (Number 3)

A new company has a loss of $2,500 in its first month, but they expect their monthly profit to increase by $400 each month. What is their profit in the 12th month? What is their total profit/loss of the year?

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This is an Arithmetic Sequence with \( a_1 = −$2500 \) and \( d = 400 \)

So \( a_n \) represents the monthly profit/loss in month \( n \) and

\[
 a_n = −$2500 + (n − 1)\times$400
\]
Examples, Real World Arithmetic Sequences (Number 3) Cont.

- The profit in the 12th month:

- The total profit/loss for the year:
Examples, Real World Arithmetic Sequences (Number 3) Cont.

- The profit in the $12^{th}$ month: is represented by $a_{12}$

- The total profit/loss for the year:
Examples, Real World Arithmetic Sequences (Number 3) Cont.

- The profit in the 12th month:
  is represented by \( a_{12} \)
  \[ a_{12} = -2500 + (12 - 1)400 = 1900 \]

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Examples, Real World Arithmetic Sequences (Number 3) Cont.

- The profit in the 12th month:
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- The total profit/loss for the year:

  \((\text{profit/loss for Jan}) + (\text{profit/loss for Feb}) + \cdots + (\text{profit/loss for Dec})\)
Examples, Real World Arithmetic Sequences
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- The profit in the 12th month:
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which can be represented in symbols as \( a_1 + a_2 + \cdots + a_{12} = S_{12} \)
The profit in the 12th month:
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Total Profits for the Year are $S_{12}$
Examples, Real World Arithmetic Sequences (Number 3) Cont.

- The profit in the 12th month:
  
  is represented by \( a_{12} \)
  
  \[
  a_{12} = -$2500 + (12 - 1) \times $400 = $1900
  \]

- The total profit/loss for the year:

  \( \text{(profit/loss for Jan)} + \text{(profit/loss for Feb)} + \cdots + \text{(profit/loss for Dec)} \)

  which can be represented in symbols as \( a_1 + a_2 + \cdots + a_{12} = S_{12} \)

  Total Profits for the Year are \( S_{12} \)

  \[
  S_{12} = \frac{12}{2} (a_1 + a_{12}) = \frac{12}{2} (-2500 + 1900) = 6(-600) = -$3600
  \]
Examples, Real World Arithmetic Sequences (Number 3) Cont.

- The profit in the 12th month:
  is represented by $a_{12}$
  $a_{12} = -$2500 + (12 - 1)$400 = $1900

- The total profit/loss for the year:

$$(\text{profit/loss for Jan}) + (\text{profit/loss for Feb}) + \cdots + (\text{profit/loss for Dec})$$

which can be represented in symbols as $a_1 + a_2 + \cdots + a_{12} = S_{12}$

Total Profits for the Year are $S_{12}$
$S_{12} = \frac{12}{2} (a_1 + a_{12}) = \frac{12}{2} (-2500 + 1900) = 6(-600) = -$3600

They lost a total of $3600 for the year.
Homework

It is NOT in your book.

It IS at the end of the printout on the course website.