Section 3.2 (Compounding Frequencies)

• Example:
  Suppose Olaf invests $5,000 in an investment that pays 6% interest (per year), compounded monthly. How much does he have at the end of one year? How much does he have at the end of 5 years?

  Compound Interest Formula \( FV = PV(1 + i)^n \)

  \( FV \) = future value (dollar amount)
  \( PV \) = present value (dollar amount)
  \( i \) = interest rate \( \text{per compounding period} \) (as a decimal)
  \( n \) = number of compounding periods

Rather than being able to read these numbers directly in the question (like we often could in earlier sections) we’ll have to do a little work.

1. Identify how often the interests compounds: \( \text{e.g. monthly, daily, etc} \)

2. Find \( i \) by....
   (a) Identify how many time the interest compounds in 1 year, \( \text{call this } k \)
   (b) Identify the interest rate (as a decimal) \( \text{call this } R \)
   (c) Find \( i = \frac{R}{k} \)

3. Find \( n \) by EITHER
   • Identify the Term (always in years) \( \text{call this } T \)
     Find \( n = T \cdot k \).
   (OR)
   • Reading the question (paying careful attention to the units of time) and asking \( \text{how many compounding periods does this loan/investment/etc last for?} \)

• Relationship between PV, FV, and Interest
  – Future Value = Present Value + Interest \( \text{Or } FV - \text{Interest} = PV \)
  – This is just like the relationship with Simple Interest:
    (Total Amount to Be Repaid) = Principal + Interest
  – And just like the relationship with Simple Discount:
    Maturity Value - Discount = Proceeds
• Example:

Suppose Olaf invests $5,000 in an investment that pays 6% interest (per year), compounded monthly. How much does he have at the end of one year?

- The interest is compounded...
- The number of compounding periods in one year is $k = 12$.
- The (annual) interest rate (as a decimal is) $R = 0.06$.
- So we work out $i = 0.005$.
- number of compounding periods: $n = 12$.

\[
FV = PV(1 + i)^n
\]
\[
FV = 5000\left(1 + \frac{0.06}{12}\right)^{12}
\]
\[
FMV = 5000(1 + 0.005)^{12}
\]
\[
FV = 5000(1.005)^{12}
\]
\[
FV = 5000 \times 1.06167781186
\]
\[
FV = 5308.3890593
\]

So Olaf will have $5308.39 in his account at the end of one year.

OR you can save time and enter

\[
5000(1+.06/12)^{12} = \]

\[
5000(1+.005)^{12} =
\]

into your calculator (if you have the TI-30X)

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• Example

Suppose Olaf invests $5,000 in an investment that pays 6% interest (per year), compounded monthly. How much does he have at the end of 5 years?

- The interest is compounded...
- The number of compounding periods in one year is $k = 12$.
- The (annual) interest rate (as a decimal is) $R = 0.06$.
- So we work out $i = 0.005$.
- number of compounding periods: $n = 60$.

\[
FV = 5000(1 + \frac{0.06}{12})^{60}
\]
\[
FV = 5000\left(1 + \frac{0.06}{12}\right)^{60}
\]
\[
FV = 5000(1.005)^{60} = 5000 \times 1.34885015255 = 6744.25076275
\]

So Olaf will have $6744.25 in his account at the end of five years.

OR you can save time and enter

\[
5000(1+.06/12)^{60} = \]

\[
5000(1+.005)^{60} =
\]

into your calculator (if you have the TI-30X)
Note! You need to get used to entering longer expressions (correctly) into your calculator, because otherwise you will be copying and typing decimals like 1.34885015255, which is WAY TOO SLOW and leaves you prone to errors.

1. Find the future value of $700 at 2.75% interest compounded daily for 3 years.

2. You found a CD that pays 4.6% interest with a term of 5 years, and you plan to invest $8000 in it, but you forgot to write down how often the interest is compounded. Find the future value of the CD if the interest is compounded....
   (a) Annually
   (b) Weekly
   (c) Every Minute

3. For Question 2, how much interest did you earn in each case?

4. How much do you need to deposit today in an account that earns 2.94% interests compounded daily so that you will have $20,000 in 30 years?

Vocabulary for interest compounding:

<table>
<thead>
<tr>
<th>Interest compounds with frequency</th>
<th>times per year (k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annually</td>
<td>1</td>
</tr>
<tr>
<td>Semianually</td>
<td>2</td>
</tr>
<tr>
<td>Quarterly</td>
<td>4</td>
</tr>
<tr>
<td>Monthly</td>
<td>12</td>
</tr>
<tr>
<td>Every other week biweekly*</td>
<td>26</td>
</tr>
<tr>
<td>fortnightly</td>
<td></td>
</tr>
<tr>
<td>Weekly</td>
<td>52</td>
</tr>
<tr>
<td>Daily (bankers rule)</td>
<td>360</td>
</tr>
<tr>
<td>Daily</td>
<td>356</td>
</tr>
<tr>
<td>Every Minute</td>
<td>525,600</td>
</tr>
</tbody>
</table>

*Unfortunately in everyday speech people some use biweekly to mean 'every 2 weeks' and sometimes use it to mean 'twice a week'.
**Continuously Compounding Interest**

- Is when interest is compounded so frequently it’s as though you’re compounding ‘infinitely many times a year’.
- It’s compounding faster than daily... faster than every minute,... faster than every second,... faster than 1000 times a second, ... etc.

- **Continuous Compounding** \( FV = PV e^{rt} \)
  
  - \( FV \) = future value
  - \( PV \) = present value
  - \( e \) = a mathematical constant (approx. 2.71828)
  - \( r \) = annual interest rate
  - \( t \) = number of years

- Note, there is a button for \( e \) on your calculator: (assuming you have TI-30X)
  - hit 2nd (upper left hand corner)
  - then hit the LN button
  (which has the label \( e^x \) above it in the same color as your 2nd button)
  this should add \( e^{(} \) to your screen

5. Find the future value of $1,150, invested at 1.27% interest for 3 years, assuming the interest compounds continuously.

\[
1150e^{(0.0127*3)} = 1194.66037787
\]

6. Find the future value of $4321, invested at 3.18% interests for 1000 days if the interest is compounded:

(a) Yearly  
(b) Daily  
(c) Continuously

7. (Optional) Gerda invests $3,500 in an investment that earns 4.3% compounded quarterly for 4.5 years. What is the future value of this investment? How much will Gerda earn with this investment? If instead of compounding quarterly the interest compounded continuously, how much would Gerda earn?