Sample Questions for

TEST 1

Here are sample questions picked by the course coordinator.

* Most, but not all, questions on Test 1 will look similar to questions in this packet.

* Test Questions do not come directly out of HW (and do not come directly from this packet).

* You can start your studying with questions from here, but your studying should include all topics we covered in class and in the suggested HW.
1. (8 points) Write the sets using interval notation:
   (a) \( \{x | x \neq 0, \pm 4\} \quad (1.1.4 \#13). \)
   
   (b) \( \{x | x \leq -3 \text{ or } x > 0\} \quad (1.1.4 \#16). \)

2. (10 points) Let \( P(-\frac{5}{3}, \frac{3}{2}) \) and \( Q(\frac{5}{2}, 2) \) be two points on the coordinate plane. Find the distance between \( P \) and \( Q \) and the midpoint \( M \) of the line segment connecting \( P \) and \( Q \). **Simplify your answer.** \( (1.1.4 \#25). \)

   Distance: __________________________

   Coordinates of \( M \): __________________________

3. (8 points) Find all points on the \( y \)-axis that are 5 units from the point \((-5, 3)\). \( (1.1.4 \#31). \)
4. (18 points) Find the coordinates of the x- and y-intercepts of the following graphs (if any exist).  

(a) \( y = x^2 - 2x - 8 \)  \( \text{ (1.2.2 #42)} \)

(b) \( y = \frac{x^2}{4} - 3x \)  \( \text{ (1.2.2 #44)} \)

(c) \( y = 2\sqrt{x+4} - 2 \)  \( \text{ (1.2.2 #46)} \)

5. (6 points) Determine if the following equations represent \( y \) as a function of \( x \). \textbf{Justify your answer.}

6. \( x^2 + y^2 = 4 \)  \( \text{ (1.3.1 #41)} \)

7. \( x^3 + y^3 = 4 \)  \( \text{ (1.3.1 #44)} \)
1. (9 points) Let \( f(x) = x^2 - 3x + 2 \). Find \( f(-x) \), \( f(x-4) \) and \( f(x^2) \). \textbf{Simplify your answer.} (1.4.2 #14).

4. (6 points)
   (a) Let \( f(x) = \frac{x}{x-1} \). Find \( f\left(\frac{3}{2}\right) \). \textbf{Simplify your answer.} (1.4.2 #15).

   (b) Let \( f(x) = \frac{2}{x} \). Find \( f\left(\frac{4}{3}\right) \). \textbf{Simplify your answer.} (1.4.2 #26).

8. (3 points) Let \( f(x) = x^2 - x - 12 \). Find \( f(0) \) and solve \( f(x) = 0 \). \textbf{Simplify your answer.} (1.4.2 #30).
4. (6 points) Let \( f(x) = \begin{cases} 
2x + 5 & \text{if } x \leq -3 \\
\sqrt{9-x^2} & \text{if } -3 < x \leq 3 \\
-x + 5 & \text{if } x > 3 
\end{cases} \) (1.4.2 #35).

Compute \( f(3), f(-3.001) \) and \( f(2) \).

5. (10 points) Find the domain of the following functions. Write your answers using interval notation.

(a) \( f(x) = \frac{3x}{x^2 + x - 2} \) (1.4.2 #40).

(b) \( f(x) = \frac{2x}{x^2 + 3} \) (1.4.2 #41).

(c) \( f(x) = \frac{2x}{x^2 - 3} \) (1.4.2 #42).

(d) \( f(x) = \frac{6}{4 - \sqrt{6x - 2}} \) (1.4.2 #52).

(e) \( f(x) = \frac{\sqrt{6x - 2}}{x^2 - 36} \) (1.4.2 #53).
12. (4 points) Let \( f(x) = -x^2 + x + 6, \ g(x) = x^2 - 9 \) and \( h(x) = \left( \frac{f}{g} \right)(x). \)

Find a simple expression for \( h(x) \) and find the domain of \( h(x) \). **Write your answers using interval notation.** (1.5.1 #16).

14. (12 points) Find a simple expression for the difference quotient \( \frac{f(x+h)-f(x)}{h} \) for the following functions.

(a) \( f(x) = -x^2 + 2x - 1 \). (1.5.1 #25).

(b) \( f(x) = \frac{x^2}{2x+1} \). (1.5.1 #38).

(c) \( f(x) = \sqrt{-4x+5} \). (Simplify by rationalizing the numerator.) (1.5.1 #41).
15. (5 points) Sketch the graph of the function \( f(x) = 3 - 2\sqrt{x + 2} \). Find the domain of the function, and the coordinates of the \( x \)- and \( y \)-intercepts (if any). (1.6.2 #10)

16. (5 points) Sketch the graph of the function

\[
f(x) = \begin{cases} 
  x^2 & \text{if } x \leq -2 \\
  3 - x & \text{if } -2 < x < 2 \\
  4 & \text{if } x \geq 2.
\end{cases}
\] (1.6.2 #19)

17. (5 points) Let \( f(x) = \frac{\sqrt[3]{x^3} + x}{5x} \). Determine analytically if \( f(x) \) is even, odd or neither. (1.6.2 #41)
(20 points)

Use the graph of $y = f(x)$ to answer the following:

(a) Find the domain of $f$. Write your answer using interval notation. (1.6.2 #42)

(b) Find the range of $f$. Write your answer using interval notation. (1.6.2 #43)

(c) Find the zeros of $f$. (1.6.2 #48)

(d) Solve $f(x) \geq 0$. Write your answer using interval notation. (1.6.2 #49)

(e) List the intervals where $f$ is increasing. (1.6.2 #52)

(f) List the intervals where $f$ is decreasing. (1.6.2 #53)

(g) List the local maxima of $f$ (if any exist). (1.6.2 #54)

(h) List the local minima of $f$ (if any exist). (1.6.2 #55)

(i) Find the maximum of $f$ (if it exists). (1.6.2 #56)

(j) Find the minimum of $f$ (if it exists). (1.6.2 #57)
19. (5 points) Suppose \((2, -3)\) is a point on the graph of \(y = f(x)\). Find a point on the graph of \(y = \frac{4 - f(3x - 1)}{7}\). \((1.7.1 \#18)\)

20.

Use the graph of \(y = f(x)\) given above to graph the transformed functions:

(a) (5 points) \(n(x) = 4f(x - 3) - 6\). \((1.7.1 \#47)\)

(b) (5 points) \(p(x) = 4 + f(1 - 2x)\). \((1.7.1 \#48)\)
27. (10 points) Let \( f(x) = \sqrt{x} \). Find a formula for a function \( g \) whose graph is obtained from \( f \) from the given sequence of transformations.

(a) (1) shift up 1 unit; (2) reflect across the x-axis. (1.7.1 #57)

(b) (1) reflect across the y-axis; (2) shift left 1 unit; (3) shift up 2 units. (1.7.1 #59)

(c) (1) shift left 3 units; (2) shift down 4 units; (3) vertical stretch by a factor of 2. (1.7.1 #61)

28. (5 points) Find the equation of the line that passes through the points \( P(-\sqrt{3}, -1) \), \( Q(\sqrt{3}, 1) \). (2.1.1 #20)

29. (5 points) Sketch the graph of the function \( f(x) = \frac{1-x}{2} \). Find the slope, and the coordinates of the x-intercepts and y-intercepts (if any). (2.1.1 #26)
26. (5 points) Find all points on the line \( y = 2x + 1 \) that are 4 units from the point \((-1, 3)\).  

27. (5 points) A salesperson is paid $200 per week plus 5% commission on her weekly sales of \( x \) dollars. Find a linear function that represents her total weekly pay, \( W \) (in dollars) in terms of \( x \). What must her weekly sales be in order for her to earn $475.00 for the week?  

28. (5 points) Let \( f(x) = \frac{x + 4}{x - 3} \). Find the average rate of change of \( f(x) \) over the interval \([5, 7]\). Simplify your answer.  

(2.1.1 #27)  

(2.1.1 #32)  

(2.1.1 #48)
18. (5 points) Let \( f(x) = 3x^2 + 2x - 7 \). Find the average rate of change of \( f(x) \) over the interval \([x, x + h]\).
Simplify your answer. (2.1.1 #48)

14. (5 points) Find the equation of a line that passes through the point \( P(1, -1) \), and is parallel to the line 
\[ y = \frac{4 - x}{3}. \] (2.1.1 #62)

15. (5 points) Find the equation of a line that passes through the point \( P(1, -1) \), and is perpendicular to the 
line \( y = \frac{4 - x}{3} \). (2.1.1 #68)