6.4 Logarithmic Equations and Inequalities

• (Book’s Directions)

Steps for Solving an Equation involving Logarithmic Functions

1. Isolate the logarithmic function.
2. (a) If convenient, express both sides as logs with the same base and equate
   the arguments of the log functions.
   (b) Otherwise, rewrite the log equation as an exponential equation.

• (My Directions)

Steps for Solving an Equation involving Logarithmic Functions

Use algebra (including properties of Exponentials/Logs) to get the equation into
one of the following forms, and then do the recommended thing.

(u, v represent any expression with variables. eg. \( u = x - 1 \) or \( v = \frac{17x^2 + 1}{5} \))

- \( \log_b(u) = \log_b(v) \):
  * Equate the insides of the logs, and solve \( u = v \)
  * Remember you need to add ‘as long as \( u > 0 \)’ and ‘as long as \( v > 0 \)’
  * Or check your answer(s)

- \( \log_b(u) = \text{some} \# \):
  * Rewrite the log statement as an exponential statement.
  * Remember you need to add ‘as long as \( u > 0 \)’
  * Or check your answer(s).

• Examples: Solve the following equations:

1. \( \log_{19}(x^2 - 16) = \log_{19}(8 - 2x) \)
2. \( 1 - \ln(x - 4) = 3 \)
3. \( \log_5(x + 3) + \log_5(x - 1) = 1 \)
4. \( \log_2(x^2 + 3x + 14) = \log_2(x + 1) + 3 \)
5. \( \log_3(x) = \frac{1}{2} + \log_9(2x - 3) \)
• Solving Inequalities with Logarithms
  – Get 0 on one side of the inequality, call the other side $f(x)$.
  – Make a sign chart for $f(x)$
    * You’ll need the Domain of $f(x)$
    * You’ll need the Zeros of $f(x)$
  – Pick the appropriate Interval(s) and points(s)

• Examples: Solve the following inequalities:

6. \( \frac{2}{1 - \ln(x + 1)} < 3 \)

7. \( x \log_6(2x - 8) \geq 2x \)

8. \( (\log_3(x))^2 + 6 \log_3(x) \geq -8 \)

• Optional Questions

9. Solve the following equations
   (a) \( \log_7(1 - 2x) = 1 - \log_7(3 - x) \)
   (b) \( \log(\ln(x)) = 2 \)

10. Solve the following inequality: \( 2 < -\ln(x) \leq 5 \)