• **Steps for Constructing a Sign Diagram for a Rational Function:**
  Suppose \( r \) is a rational function.

  1. Place any values excluded from the domain of \( r \) on the number line with an “?!” above them
     (? is called the interrobang, it is meant to “convey a sense of surprise, caution and wonderment”)
  2. Find the zeros of \( r \) and place them on the number line with the number 0 above them.
  3. Choose a test value in each of the intervals determined in steps 1 and 2.
  4. Determine the sign of \( r(x) \) for each test value in step 3, and write that sign above the corresponding interval.

• **Example:** Make a sign Diagram for the function \( f(x) = \frac{(1 - x)(x - 2)}{(x - 3)(x + 2)^2} \)
• **Example:** How does the sign chart for \( f(x) = \frac{(1 - x)(x - 2)}{(x - 3)(x + 2)^2} \) relate to the graph of \( f(x) \)?

![Graph of a rational function]

• **Steps for Graphing a Rational Function:**

Suppose \( r \) is a rational function.

1. Find the domain of \( r \).
2. Reduce \( r(x) \) to lowest terms, (if applicable).
3. Find the \( x \)- and \( y \)-intercepts of the graph of \( y = r(x) \), if they exist.
4. Determine the location of any vertical asymptotes or holes in the graph, if they exist.
   - Analyze the behavior of \( r \) on either side of the vertical asymptotes, if applicable.
5. (Analyze the end behavior of \( r \)).
   - Find the horizontal asymptote, if one exists.
6. Use a sign diagram and plot additional points, as needed, to sketch the graph of \( y = r(x) \).
Two Important Rational Functions to Know the Graphs of

– You Should Know the General Shape.
– You should know all Asymptotes.
– You should know at least 2 points on them.
– (We’ll use these with transformations)

• $f(x) = \frac{1}{x}$

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{y=1/x.png}
\caption{Graph of $y = \frac{1}{x}$}
\end{figure}

• $f(x) = \frac{1}{x^2}$

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{y=1/(x^2).png}
\caption{Graph of $y = \frac{1}{x^2}$}
\end{figure}
• **Examples:** Use the six-step procedure to graph the rational function. Be sure to draw any asymptotes as dashed lines

1. \( f(x) = \frac{x^2 - 4x - 5}{x^3 - 2x^2 - 25x + 50} \)
2. \( f(x) = \frac{3x^2 + 3x - 6}{x^2 + 8x + 16} \)

• **Examples:** Use Transformations to sketch the graph of....

3. \( g(x) = \frac{2}{1 - x} \)
4. (optional) \( y = \frac{-1}{(x - 2)^2} + 3 \)

• **Note:** There is an incredibly high number of combinations for what can happen with rational functions:

  – Are there holes or no holes?
  – Where are the holes in relation to zeros and asymptotes?
  – Is there a y-intercept or no?
  – None, one, or several vertical asymptotes?
  – At each vertical asymptote, does the function changes signs or not?
  – Do you have enough points on the graph to know what to do at each vertical asymptote?
  – Are there horizontal Asymptotes or not?
  – If there is a horizontal Asymptote, is it \( y = 0 \) or \( y = \frac{a}{b} \) ?
  – Are there zeros to help you with the horizontal asymptotes?
  – Are there points on the graph that help you know to approach each horizontal asymptote?
  – Are there zeros of the function or not?
  – Does the function change signs at each zeros or not?
  – Are there 2 vertical asymptotes with no zeros between them?
  – Are there 2 vertical asymptotes one or more zeros between them?
  – etc....

  **It’s impossible that we cover all these examples in class. So it’s really important that you do your HW without looking at the answers first so you know how to prepare for every possibility.**

Studying a few examples from class and not doing your HW is a death sentence in this section.
• **Examples:** Use the six-step procedure to graph the rational function. Be sure to draw any asymptotes as dashed lines

5. (optional) \( f(x) = \frac{3x - 4}{x^2 + 9} \)

6. (optional) For \( f(x) = \frac{(x + 1)(x - 1)^4(2x - 10)(x + 8)}{(x^4 - 4x^2 - 5)(x + 1)^3} \)
   - Domain: \((-\infty, -\sqrt{5}) \cup (-\sqrt{5}, -1) \cup (-1, \sqrt{5}) \cup (\sqrt{5}, \infty)\)
   - VA: \( x = \sqrt{5}, x = -\sqrt{5}, x = -1 \)
   - Holes: none
   - HA: \( y = 2 \)