Directions: Please answer the following questions and make sure your answers are legible. You must show your work to receive credit for your answers. You may not use a calculator (or any other technology) on this quiz. Good Luck.

1. (11 points) Solve the following equations.
   (a) \( \log_5(2x + 1) + \log_5(x + 2) = 1 \)  
   (b) \( \ln(\ln(x)) = 3 \)

   **Solution:**
   
   (a) Using the property of logarithms, we can combine the logarithms:
   \[
   \log_5((2x+1)(x+2)) = 1
   \]
   
   \[
   \frac{1}{5} = (2x+1)(x+2)
   \]
   
   Expanding:
   \[
   5 = 2x^2 + 5x + 2
   \]
   
   Rearranging:
   \[
   2x^2 + 5x - 3 = 0
   \]
   
   Factoring:
   \[
   (2x - 1)(x + 3) = 0
   \]
   
   Solutions:
   \( x = \frac{1}{2} \) or \( x = -3 \)

   (b) For \( \ln(\ln(x)) = 3 \), we isolate the logarithm:
   \[
   e^3 = \ln(x)
   \]
   
   **Solve for x:**
   
   \[
   e = x
   \]
2. (7 points) Solve the inequality \( x \ln(x) - x > 0 \), give your answer in interval notation.

\[
x \ln(x) - x > 0
\]

\[
\begin{align*}
\text{Domain} & : x > 0 \\
0 & \text{ is not in Domain}
\end{align*}
\]

\[
\begin{align*}
x \ln(x) - x & = 0 \\
x (\ln(x) - 1) & = 0 \\
x = 0 & \text{ or } \ln(x) = 1 \\
x & = e^1 \\
x & = e
\end{align*}
\]

\[
\begin{align*}
\text{Sign Diagram} & \\
0 & \text{ on 1 side}
\end{align*}
\]

\[
\begin{align*}
f(x) & = x [\ln(x) - 1] \\
f(e^1) & = (e^1) [\ln(e^1) - 1] \\
& = (e) (2 - 1) = e
\end{align*}
\]

Solve: \((e, \infty)\)

3. (7 points) For \( f(x) = 10^{2x-7} - 5 \) (Extra Questions # 8)
   (a) Find \( f^{-1}(x) \).
   (b) Find the domain of \( f^{-1} \).
   (c) Find the range of \( f^{-1} \)

\[
\begin{align*}
a) \quad & \text{Find } f^{-1} \\
y & = 10^{2x-7} - 5 \\
x & = 10^y - 2 \\
x + 5 & = 10^y - 2 \\
\log_{10} (x+5) & = \log_{10} (10^y - 2) \\
\log (x+5) & = 2 y - 7 \\
\log (x+5) + 7 &= 2 y \\
\frac{\log (x+5) + 7}{2} &= y \\
f^{-1}(x) & = \frac{\log (x+5) + 7}{2}
\end{align*}
\]

\[
\begin{align*}
b) & \text{Domain } f^{-1} : (-5, \infty) \\
c) & \text{Domain } f^{-1} : (-\infty, \infty)
\end{align*}
\]