R4 Polynomials

1. Add/subtract/multiply the polynomials below:
   (a) \((5x^4 - \pi x^3 + 17x - 4) + 4(x^3 - \frac{1}{3}x^2 + 6x)\)
   (b) \(x(2x^8 - 6x + 1) - (x^6 - 5x^2 + \frac{3}{7}x)\)
   (c) \((2x - 1)(x^2 + 3x + 6)\)

2. Use the special product formulas to help you express the following as a single polynomial in standard form:
   (a) \((3x + 4)^2\)
   (b) \((3x + 4)^3\)
   (c) \((7x - 4)(7x + 4)\)

3. Find the quotient and remainder when \(6x^3 - 5x^2 + 3\) is divided by \(2x - 1\)

Special Product Formulas

- **Difference of Two Squares**: \((x - a)(x + a) = x^2 - a^2\)
- **Perfect Squares**: \((x + a)^2 = x^2 + 2ax + a^2\) \((x - a)^2 = x^2 - 2ax + a^2\)
- **Perfect Cubes**: \((x + a)^3 = x^3 + 3ax^2 + 3a^2x + a^3\) \((x - a)^3 = x^3 - 3ax^2 + 3a^2x - a^3\)
- **Difference of Two Cubes**: \((x - a)(x^2 + ax + a^2) = x^3 - a^3\)
- **Sum of Two Cubes**: \((x + a)(x^2 - ax + a^2) = x^3 + a^3\)
R5: Factoring Polynomials

4. Factor the following polynomials completely:

(a) \((6x - 4)\)
(b) \(5x^4 - 10x^2\)
(c) \(x^2 - 9\)
(d) \(x^3 + 27\)
(e) \(y^3 - 1\)
(f) \(x^2 - 20x + 100\)
(g) \(16x^2 + 24x + 9\)
(h) \(x^2 + 2x - 24\)
(i) \(27x^3 + 8\)
(j) \(x^3 - 3x^2 + 4x - 12\)
(k) \((x - 2)^2(x + 1)^2 + (x - 2)(x + 1)^2(x + 3)\)
(l) \(x^4 + 2x^2 - 3\)

5. Determine the number that must be added to \(y^2 + 8y\) to complete the square. Then factor the result.

6. Determine the number that must be added to \(x^2 - 10x\) to complete the square. Then factor the result.

Common Factoring Approaches

- *Any Polynomial:* Look for common factors first (common factors may be monomial like \(3x\) or polynomials like \(x + 3\))
- *Binomials of Degree 2 of more:* Look for specials products (Difference of Two Squares, Difference of Two cubes, Sum of Two Cubes)
- *Trinomials of Degree 2:* Check for Perfect Square or factor by hand
- *Four or More Terms:* Factor By Grouping
- *Don't forget:* Sometimes substations can make things with higher degrees fit lower degree patterns.
R7: Rational Expressions

7. Reduce the following to lowest terms:
   (a) \( \frac{x^2 + 6x + 9}{x^2 - x - 12} \)
   (b) \( \frac{(x - 3)(x^2 + 3x + 9)}{x^3(3 - x)} \)

8. Perform the indicated operation and simplify the result:
   (a) \( \frac{(x + 3)^2}{(x^4 - 16)} \cdot \frac{(x + 2)}{(2x + 6)} \)
      \( \frac{(x+1)(x-4)}{x^2+7x+12} \)
   (b) \( \frac{x^2 - x - 12}{x^2 - x - 12} \)

9. Perform the indicated operation and simplify the result, leave your answers in factored from
   (a) \( \frac{x - 1}{3x - 4} + \frac{x^2 + 4x - 13}{3x - 4} \)
   (b) \( \frac{x^2 - x}{x - 4} + \frac{4x - 11}{4 - x} \)
   (c) \( \frac{x}{x + 3} - \frac{x - 10}{x + 6} \) for \( x \neq -3, -6 \)
   (d) \( \frac{x + 1}{(x^2 - 9)} + \frac{x^4}{x^2 + 3x} \)
   (e) \( \frac{x}{x^2 + 3x + 2} + \frac{2x - 3}{x^2 - 1} \) for \( x \neq -2, -1, 1 \)

10. Simplify
    (a) \( \frac{1}{3} + \frac{2}{x} + \frac{1}{11} \)
    (b) \( \frac{x^2}{x - 2} + \frac{1}{x + 1} + \frac{1}{x - 3} + \frac{1}{x} \)