Math 22: Fall 2015
5.3 Other Algebraic Functions

- Let $x$ be a real number and $n$ a natural number. If $n$ is odd, the **principal $n$th root of** $x$, denoted $\sqrt[n]{x}$ is the unique real number satisfying $(\sqrt[n]{x})^n = x$. If $n$ is even, $\sqrt[n]{x}$ is defined similarly provided $x \geq 0$ and $\sqrt[n]{x} \geq 0$.

  The **index** is the number $n$ and the **radicand** is the number $x$. For $n = 2$, we write $\sqrt{x}$ instead of $\sqrt[2]{x}$.

- **Properties of Radicals:** Let $x$ and $y$ be real numbers and $m$ and $n$ be natural numbers. If $\sqrt[n]{x}$, $\sqrt[n]{y}$ are real numbers, then
  - **Product Rule:** $\sqrt[n]{xy} = \sqrt[n]{x} \sqrt[n]{y}$
  - **Powers of Radicals** $\sqrt[n]{x^m} = (\sqrt[n]{x})^m$
  - **Quotient Rule:** $\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$, as long as $y \neq 0$
  - If $n$ is odd $\sqrt[n]{x^n} = x$; if $n$ is even, $\sqrt[n]{x^n} = |x|$.

- **Fractional Exponents**
  - $x^{1/n} = \sqrt[n]{x}$ whenever $\sqrt[n]{x}$ is defined.
  - $x^{m/n} = (\sqrt[n]{x})^m$ whenever $(\sqrt[n]{x})^m$ is defined.

- **Steps for Constructing A Sign Diagram for Algebraic Functions**
  Suppose $f$ is an algebraic function.

  1. Place any values excluded from the domain of $f$ on the number line with an ↑ above them.
  2. Find the zeros of $f$ and place them on the number line with the number 0 above them.
  3. Choose a test value in each of the intervals determined in steps 1 and 2.
  4. Determine the sign of $f(x)$ for each test value in step 3, and write that sign above the corresponding interval.
• **Examples:** For the following functions, state their domains and create sign diagrams.

1. \( f(x) = 2x\sqrt{4-x} \)
2. \( f(x) = \sqrt{3 - \sqrt{x+1}} \)

• **Examples:** Solve the following inequalities:

3. (Refer to the previous exercises) \( 2x\sqrt{4-x} \leq 0 \)
4. \( x^{4/3} + x^{2/3} \leq 12 \)
Graphs of Even Roots

All graphs contain the points (0, 0) and (1, 1) then it’s easy to find a point (?, 2) if you need a 3rd point.
Graphs of Odd Roots

All graphs contain the points (0, 0) and (1, 1) and (−1, −1)
• **Examples:** Graph the following functions as transformations

5. \( f(x) = 2\sqrt[3]{x + 3} - 1 \)

• **Examples:** More Examples:

6. Solve the inequality \( 4(3 - x)^{1/3} \geq x(3 - x)^{-2/3} \)

7. Solve the equation \( \sqrt{6 - x} + 4 = x \)

8. Graph \( f(x) = \sqrt{-x} + 1 \) as a transformation

9. State the domain of \( f(x) = \frac{\sqrt{x^2 - 3x + 7}}{\sqrt{x^2 - 2x - 24}} \)

10. Solve the inequality \( \frac{1}{2}x^{-1/2}(x - 4)^{2/3} + \frac{2}{3}x^{1/2}(x - 4)^{-1/3} < 0 \)

11. Solve the inequality \( \frac{x(x - 1)}{\sqrt{x^2 - 1}} > 0. \)