• Steps for Constructing a Sign Diagram for a Rational Function:
Suppose \( r \) is a rational function.

1. Place any values excluded from the domain of \( r \) on the number line with an “?” above them
   (\( ? \) is called the interrobang, it is meant to “convey a sense of surprise, caution and wonderment”)
2. Find the zeros of \( r \) and place them on the number line with the number 0 above them.
3. Choose a test value in each of the intervals determined in steps 1 and 2.
4. Determine the sign of \( r(x) \) for each test value in step 3, and write that sign above the corresponding interval.

• Example: Make a sign Diagram for the function \( f(x) = \frac{(1 - x)(x - 2)}{(x - 3)(x + 2)^2} \)
• **Example:** How does the sign chart for \( f(x) = \frac{(1 - x)(x - 2)}{(x - 3)(x + 2)^2} \) relate to the graph of \( f(x) \)?

• **Steps for Graphing a Rational Function:**

  Suppose \( r \) is a rational function.

1. Find the domain of \( r \).
2. Reduce \( r(x) \) to lowest terms, (if applicable).
3. Find the \( x \)- and \( y \)-intercepts of the graph of \( y = r(x) \), if they exist.
4. Determine the location of any vertical asymptotes or holes in the graph, if they exist.
   Analyze the behavior of \( r \) on either side of the vertical asymptotes, if applicable.
5. (Analyze the end behavior of \( r \)).
   Find the horizontal asymptote, if one exists.
6. Use a sign diagram and plot additional points, as needed, to sketch the graph of \( y = r(x) \).
• **Examples:** Use the six-step procedure to graph the rational function. Be sure to draw any asymptotes as dashed lines.

1. \( f(x) = \frac{x^2 - 4x - 5}{x^3 - 2x^2 - 25x + 50} \)
2. \( f(x) = \frac{3x^2 + 3x - 6}{x^2 + 8x + 16} \)
3. (optional) \( f(x) = \frac{3x - 4}{x^2 + 9} \)
4. (optional) For \( f(x) = \frac{(x + 1)(x - 1)^4(2x - 10)(x + 8)}{(x^4 - 4x^2 - 5)(x + 1)^3} \)
   - Domain: \((-\infty, -\sqrt{5}) \cup (-\sqrt{5}, -1) \cup (-1, \sqrt{5}) \cup (\sqrt{5}, \infty)\)
   - VA: \( x = \sqrt{5}, x = -\sqrt{5}, x = -1 \)
   - Holes: none
   - HA: \( y = 2 \)

• **Note:** There is an incredibly high number of combinations for what can happen with rational functions:

- Are there holes or no holes?
- Where are the holes in relation to zeros and asymptotes?
- Is there a y-intercept or no?
- None, one, or several vertical asymptotes?
- At each vertical asymptote, does the function changes signs or not?
- Do you have enough points on the graph to know what to do at each vertical asymptote?
- Are there horizontal Asymptotes or not?
- If there is a horizontal Asymptote, is it \( y = 0 \) or \( y = \frac{a}{b} \)?
- Are there zeros to help you with the horizontal asymptotes?
- Are there points on the graph that help you know to approach each horizontal asymptote?
- Are there zeros of the function or not?
- Does the function change signs at each zeros or not?
- Are there 2 vertical asymptotes with no zeros between them?
- Are there 2 vertical asymptotes one or more zeros between them?
- etc....

It’s impossible that we cover all these examples in class. So it’s really important that you do your HW **without looking at the answers first** so you know how to prepare for every possibility.

Studying a few examples from class and not doing your HW is a death sentence in this section.