Suppose $f$ and $g$ are functions and $x$ is in both the domain of $f$ and the domain of $g$

- The **sum** of $f$ and $g$, denoted $f + g$, is the function defined by the formula

$$ (f + g)(x) = f(x) + g(x) $$

- The **difference** of $f$ and $g$, denoted $f - g$, is the function defined by the formula

$$ (f - g)(x) = f(x) - g(x) $$

- The **product** of $f$ and $g$, denoted $fg$, is the function defined by the formula

$$ (fg)(x) = f(x)g(x) $$

- The **quotient** of $f$ and $g$, denoted $\frac{f}{g}$, is the function defined by the formula

$$ \left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)} $$

provided $g(x) \neq 0$

**Difference Quotient:**

- Given the function $f$, the *difference quotient of $f$* is the expression

$$ \frac{f(x + h) - f(x)}{h} $$
Summary of Common Economic Functions:

Suppose $x$ represents the quantity of items produced and sold.

- The price-demand function $p(x)$ calculates the price per item.
- The revenue function $R(x)$ calculates the total money collected by selling $x$ items at a price $p(x)$,

$$R(x) = xp(x)$$

- The cost function $C(x)$ calculates the cost to produce $x$ items. The value $C(0)$ is called the fixed cost or start-up cost.
- The average cost function $\overline{C}(x) = \frac{C(x)}{x}$ calculates the cost per item when making $x$ items. Here, we necessarily assume $x > 0$.
- The profit function $P(x)$ calculates the money earned after costs are paid when $x$ items are produced and sold,

$$P(x) = (R - C)(x) = R(x) - C(x).$$

Examples:

1. For $f(x) = 2 - \frac{1}{x}$ and $g(x) = 2x^2 + 5x$:
   (a) Find $(f + g)(-2)$
   (b) Find $(fg)(1)$
   (c) Find the domain of $f - g$ and find a formula for $(f - g)(x)$
   (d) Find the domain of $\frac{f}{g}$ and find a formula for $(\frac{f}{g})(x)$

2. Find and simplify the difference quotient of the following functions:
   (a) $f(x) = \frac{1}{1+3x}$
   (b) $g(x) = \sqrt{x}$

3. Let $x$ represent the number of pies baked and sold at a small bakery in a typical week. Suppose the cost (in dollars) to produce $x$ pies is $C(x) = 5x + 60$ (for $x \geq 0$) and the price, in dollars per pie, is given by $p(x) = 25 - \frac{x}{2}$ for $0 \leq x \leq 50$
   (a) Interpret $C(0)$
   (b) Find and interpret $\overline{C}(20)$.
   (c) Find and simplify expressions for the revenue function $R(x)$ and the profit function $P(x)$.
   (d) Find and interpret $R(0)$ and $P(0)$.
   (e) Find and interpret $R(20)$ and $P(20)$.
   (f) Solve $P(x) = 0$ and interpret the results.