6.2 Nullspace

• **Recall: Null Space**
  For $A$ be an $m \times n$ matrix. The nullspace of $A$, denoted by $\text{nullspace}(A)$ or $\mathcal{N}(A)$, is defined to be the set of all vectors $x$ in $\mathbb{R}^n$ such that $Ax = 0$.

  - The null space is also called the kernel of $A$.
  - In chapter 4 we proved this is a subspace of $\mathbb{R}^n$

• **Nullity**
  The dimension of the nullspace of a matrix $A$ is also known as the nullity of $A$.

  $$\text{nullity}(A) = \dim(\text{nullspace}(A)).$$

• **Left Nullspace**
  Let $A$ be a $m \times n$ matrix. The left nullspace of $A$ is defined to be the nullspace of the $n \times m$ matrix $A'$.

  - In other words the left nullspace of $A$ is the set of all vectors $x$ in $\mathbb{R}^m$ such that
    $$A'x = 0_{n \times 1}$$
  - (taking the transpose of the above equation)
    $$x^tA = 0_{1 \times n}$$
• Examples

1. For \( A = \begin{bmatrix} -1 & 1 & 1 & 1 & 0 \\ -1 & 2 & 0 & 1 & 2 \\ -3 & 4 & 2 & 3 & 2 \\ 1 & -1 & -1 & 0 & 2 \end{bmatrix} \).

(a) \( \text{nullspace}(A) \) is a subspace of ______.
(b) Find a basis for \( \text{nullspace}(A) \).
(c) What is \( \dim(\text{nullspace}(A)) \)?
(d) What is one vector in \( \text{nullspace}(A) \)?
(e) What is one vector not in \( \text{nullspace}(A) \)?
(f) What is the \( \text{nullity}(A) \)?

2. For \( A = \begin{bmatrix} -1 & 1 & 1 & 1 & 0 \\ -1 & 2 & 0 & 1 & 2 \\ -3 & 4 & 2 & 3 & 2 \\ 1 & -1 & -1 & 0 & 2 \end{bmatrix} \).

(a) The left nullspace of \( A \) is a subspace of ______.
(b) Find a basis for the left nullspace of \( A \).
(c) What is the dimension of the left nullspace of \( A \)?

• Examples

3. Let \( C = \begin{bmatrix} 1 & 2 & 3 & -1 & 0 \\ -1 & -2 & -2 & 1 & 1 \\ 1 & 2 & 2 & -1 & -1 \end{bmatrix} \).

(a) Find a basis for the nullspace of \( C \)
(b) Find a basis for the left nullspace of \( C \)
(c) What is the nullity of \( C \)?
(d) What is the dimension of the nullspace of \( C \)?
(e) What is the dimension of the left nullspace of \( C \)?