3.1 Matrix Arithmetic

• **Matrix** is a rectangular array of real (or complex) numbers.

• **Matrix Notation**
  
  - The size of a matrix is \( m \times n \) when it has \( m \) rows and \( n \) columns.

  - The set of all real \( m \times n \) matrices is denoted \( M_{m \times n}(\mathbb{R}) \).

  - A **square matrix** is an \( n \times n \) matrix.

  - For \( A \) a matrix \((A)_{ij}\) denotes the element in the \( i \)-th row and \( j \)-th column.

  - Two matrices \( A = [a_{ij}] \) and \( B = [b_{ik}] \) are **equal** if they are the same size and \( a_{ij} = b_{ij} \) for all \( i \) and \( j \).

  - The \( m \times n \) **Zero Matrix** is the \( m \times n \) matrix where every entry is 0, denoted \( 0_{m \times n} \).

• **Matrix Addition**

  If \( A = [a_{ij}] \) and \( B = [b_{ij}] \) are both \( m \times n \) matrices, then their sum \( A + B \) is also an \( m \times n \) matrix defended by

  \[
  (A + B)_{ij} = [a_{ij} + b_{ij}]
  \]

  - In other words: we add matrices component wise
  - 2 matrices *must* be the same size to be added
  - If 2 matrices are different sizes, their sum is undefined (DNE)
• **Scalar Multiplication** (of a matrix)

   If $A = [a_{ij}]$ is an $m \times n$ matrix and $c$ is any scalar, then $cA$ is an $m \times n$ matrix defined by

   $$[cA]_{ij} = ca_{ij}$$

   – In other words: when multiplying a matrix $A$ by scalar $c$, multiply each component of $A$ by $c$

• **Examples:** Do the indicated operation(s)

   1. $\begin{bmatrix} 1 & 2 \\ 0 & -3 \\ 5 & 10 \\ -1 & 6 \end{bmatrix} + \begin{bmatrix} 0 & 9 \\ 1 & 1 \\ -3 & -4 \\ 10 & 0 \end{bmatrix}$

   2. $\begin{bmatrix} 1 & 2 \\ 0 & -3 \\ 5 & 10 \\ -1 & 6 \end{bmatrix} + \begin{bmatrix} 0 & 9 & 1 \\ 1 & 1 & 1 \\ -3 & -4 & -5 \\ 10 & 0 & 3 \end{bmatrix}$

   3. $-2 \begin{bmatrix} 3 & -1 & -1 \\ 0 & 5 & 7 \end{bmatrix}$

   4. $3 \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 5 & 0 & -2 \\ 1 & 8 & \frac{1}{2} \end{bmatrix}$
• Properties of Matrix Addition and Scalar Multiplication

Let $A, B, C$ be $m \times n$ matrices, and $b, c$ be scalars. Then

- $A + (B + C) = (A + B) + C$  (Matrix addition is associative).
- $A + B = B + A$  (Matrix addition is commutative).
- $A + 0 = 0 + A = A$  (Existence of zero matrix).
- $A + (-A) = A - A = 0$  (Existence of additive inverse).

In the language of abstract algebra, the above properties make the set of $m \times b$ matrices with matrix addition a commutative group.

- $1A = A$  (Identity element for scalar multiplication).
- $(bc)A = b(cA)$  (Associative property of scalar multiplication).
- $c(A + B) = cA + cB$  (Distributive property of scalar mult. with respect to matrix addition).
- $(b + c)A = bA + cA$  (Distributive property of scalar multi. with respect to scalar addition).

• Example  Do the indicated operations(s)

For $A = \begin{bmatrix} 1 & 2 & 3 \\ -3 & 1 & 0 \\ 7 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -2 & -2 \\ 5 & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

5. $-2(3A)$
6. $(2 - 5)(A + B) + 7(A + B) - 5(A - A)$