Midterm 2 Questions

• Note: Proofs and theory are MUCH MORE a part of chapters 4-7 than they were chaps 1-3.

• Expect proofs! and expect T/F and/or give an example of... type questions to test your grasp of theory.

• Once again the majority of the test will be ‘long answer’ type questions that look similar to HW questions.

• The remainder will be short answer questions. Here is a guide of some of the types of short answer question(s) you may be asked.

• Things not on this list ARE fair game for the test.

Chapter 4

1. Most important HW type questions

   (a) Do part of the proof that \((L, \mathcal{M}_{m\times n}(\mathbb{R}), \mathcal{P}_n)\) is a vector space.

   (b) Prove or disprove \(W\) is a subspace of “blah blah blah.”

2. True/False

   (a) True/False: Every vector space has a zero vector.

   (b) True/False: For \(V\) a vector space, the zero vector of \(V\) is unique.

   (c) True/False: For \(u, v, w\) any vectors in a vector space \(V\), if \(u + v = w + v\), then \(u = w\).

   (d) True/False: For \(a, b\) any scalars and \(u\) any vector in a vector space \(V\), \((ab)u = a(bu)\)

   (e) True/False: For \(a\) any scalar and \(u, v\) any vector in a vector space \(V\), \((au)v = a(uv)\)

   (f) True/False: Every vector space is closed under vector addition.

   (g) True/False: Every subspace \(W\) of a vector space \(V\) contains the zero vector of \(V\).

3. Give an example of the following:

   (a) A proper subspace of \(\mathbb{R}^2\).

   (b) A proper subspace of \(\mathbb{R}^4\).

   (c) A proper subspace of \(\mathcal{M}_{3\times 2}(\mathbb{R})\).

   (d) A proper subspace of \(\mathcal{M}_{4\times 4}(\mathbb{R})\).

   (e) A proper subspace of \(\mathcal{P}_3\).

   (f) A proper subspace of \(L\).

   (g) A proper subspace of \(C[a, b]\).

   (h) A subset of \(\mathbb{R}^2\) that is NOT a subspace.

   (i) A subset of \(\mathbb{R}^4\) that is NOT a subspace.

   (j) A subset of \(\mathcal{M}_{3\times 2}(\mathbb{R})\) that is NOT a subspace.

   (k) A subset of \(\mathcal{P}_3\) that is NOT a subspace.
Chapter 5

4. Most important HW type questions

(a) Is (this set of vectors) linearly dependent or independent?
   i. If it’s linearly dependent find an explicit non-trivial linear relationship between them.

(b) Is \( w \) in the span of \( v_1, \ldots, v_k \)? if yes, write \( w \) as a linear combination of \( v_1, \ldots, v_k \).

(c) Give a basis for.... (like 9-13)

(d) Is (this set of vectors) a basis for (this vector space)?

5. True/False

(a) True/False: For any vectors \( v_1, \ldots, v_k \) in a vector space \( V \), \( \{v_1, \ldots, v_k\} \) is a subspace of \( V \).

(b) True/False: For any vectors \( v_1, \ldots, v_k \) in a vector space \( V \), \( \text{span}(\{v_1, \ldots, v_k\}) \) is a subspace of \( V \).

(c) True/False: For any vectors \( v_1, \ldots, v_k \) in a vector space \( V \), the dimension of \( \text{span}(\{v_1, \ldots, v_k\}) \) is \( k \).

(d) True/False: For any vectors \( v_1, \ldots, v_k \) in a vector space \( V \), if \( \text{span}(\{v_1, \ldots, v_k\}) \) is a subspace, then \( \{v_1, \ldots, v_k\} \) are linearly independent.

(e) True/False: For any vectors \( v_1, \ldots, v_k \) in a vector space \( V \), if \( \{v_1, \ldots, v_k\} \) form a basis for \( \text{span}(\{v_1, \ldots, v_k\}) \), then \( \{v_1, \ldots, v_k\} \) are linearly independent.

(f) True/False: For a fixed vector space \( V \), there is only one basis for \( V \).

(g) True/False: For a fixed finite dimensional vector space \( V \), every basis of \( V \) contains the same number of elements.

(h) True/False: Every set of 1 vector in \( \mathbb{R}^2 \) are linearly independent.

(i) True/False: Every set of 2 vectors in \( \mathbb{R}^2 \) are linearly independent.

(j) True/False: Every set of 2 vectors in \( \mathbb{R}^2 \) forms a basis for \( \mathbb{R}^2 \).

(k) True/False: Every set of 3 vectors in \( \mathbb{R}^2 \) spans \( \mathbb{R}^2 \).

(l) True/False: Every set of 2 linearly independent vectors in \( \mathbb{R}^2 \) forms a basis for \( \mathbb{R}^2 \).

(m) True/False: Every set of 2 linearly independent vectors in \( \mathcal{P}_2 \) forms a basis for \( \mathcal{P}_2 \).

(n) True/False: \( \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\} \) is a basis for \( \mathbb{R}^2 \).

(o) True/False: \( \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\} \) is a basis for \( \mathbb{R}^3 \).

(p) True/False: For \( v_1, v_2 \) vectors in a vector space \( V \), if \( v_1 \) and \( v_2 \) aren’t scalar multiples of each other, then \( \{v_1, v_2\} \) are linearly independent.

(q) True/False: For \( v_1, v_2, v_3 \) vectors in a vector space \( V \), if none of the vectors is a scalar multiple of any of the other vectors, then \( \{v_1, v_2, v_3\} \) are linearly independent.
(r) True/False: For $v_1, v_2, v_3$ vectors in a vector space $V$, $v_2$ is in span$\{v_1, v_2, v_3\}$.

(s) True/False: For $v_1, v_2, v_3$ vectors in a vector space $V$, $0$ is in span$\{v_1, v_2, v_3\}$.

(t) True/False: For $v_1, v_2, v_3$ vectors in a vector space $V$, $7v_1 + 13v_2 - \frac{1}{3}v_3$ is in span$\{v_1, v_2, v_3\}$.

(u) True/False: For $v_1, v_2, v_3$ vectors in a vector space $V$, $a v_1 + b v_2 + c v_3 = 1v_1 + 2v_2 + 3v_3$ then $a = 1, b = 2$ and $c = 3$.

6. Give an example of the following, (or state that it’s impossible)

(a) Three different bases for $\mathbb{R}^2$.

(b) Three different bases for $\mathcal{M}_{2\times2}(\mathbb{R})$.

(c) A basis for $\mathbb{R}^3$ containing the vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

(d) A set of 2 linearly independent vectors in $\mathbb{R}^3$

(e) A set of 3 linearly independent vectors in $\mathbb{R}^3$

(f) A set of 4 linearly independent vectors in $\mathbb{R}^3$

(g) A set of 2 linearly dependent vectors in $\mathbb{R}^3$

(h) A set of 3 linearly dependent vectors in $\mathbb{R}^3$

(i) A set of 4 linearly dependent vectors in $\mathbb{R}^3$

(j) A set of 2 vectors in $\mathcal{P}_2$ that does not form a basis for $\mathcal{P}_2$.

(k) A set of 3 vectors in $\mathcal{P}_2$ that does not form a basis for $\mathcal{P}_2$.

(l) A set of 4 vectors in $\mathcal{P}_2$ that does not form a basis for $\mathcal{P}_2$.

(m) A basis for $\mathbb{R}^3$ containing 2 vectors

(n) A basis for $\mathbb{R}^3$ containing 3 vectors

(o) A basis for $\mathbb{R}^3$ containing 4 vectors

(p) A vector space of dimension 5

(q) A vector space of dimension 100

(r) A subspace of $\mathbb{R}^3$ with dimensions 0

(s) A subspace of $\mathbb{R}^3$ with dimensions 1

(t) A subspace of $\mathbb{R}^3$ with dimensions 2

(u) A subspace of $\mathbb{R}^3$ with dimensions 3

(v) A subspace of $\mathbb{R}^3$ with dimensions 4

(w) 4 different vectors in span$\begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.

(x) A vector in $\mathbb{R}^4$ NOT in span$\begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.
Chapter 6

7. Most important HW type questions

(a) For matrix $A = ....$
   i. Find a basis for each of the 4 fundamental subspaces
   ii. Find the dimension of each of the 4 fundamental subspaces
   iii. Find $\text{rank}(A)$, $\text{nullity}(A)$

(b) 17, 18, 19, 23 are good theory/short answer questions
(c) 16 is a good question.
(d) You could be asked things about $\text{rowspace}(A^t)$, $\text{colspace}(A^t)$, $\text{nullspace}(A^t)$, $\text{leftnullspace}(A^t)$

8. Here is an $m \times n$ matrix $A$, and an REF form of $[A|I_m]$,

(a) Find a basis for each of the 4 fundamental subspaces
(b) Find the dimension of each of the 4 fundamental subspaces
(c) Find $\text{rank}(A)$, $\text{nullity}(A)$

9. Here is an $m \times n$ matrix $A$, and an REF form of $A$, along with an REF form of $A^t$,

(a) Find a basis for each of the 4 fundamental subspaces
(b) Find the dimension of each of the 4 fundamental subspaces
(c) Find $\text{rank}(A)$, $\text{nullity}(A)$

10. (a) For $A$ an $m \times n$ matrix...
   i. True/False: $\text{colspace}(A) = \text{rowspace}(A)$
   ii. True/False: $\text{colspace}(A) = \text{rowspace}(A^t)$
   iii. True/False: $\text{colspace}(A) = \text{colspace}(A^t)$
   iv. True/False: $\text{colspace}(A) = \text{nullspace}(A)$
   v. True/False: $\text{colspace}(A) = \text{nullspace}(A^t)$
   vi. True/False: $\text{colspace}(A) = \text{nullity}(A)$
   vii. True/False: $\text{colspace}(A) = \text{rank}(A)$
   viii. True/False: $\dim(\text{colspace}(A)) = \text{nullity}(A)$
   ix. True/False: $\dim(\text{colspace}(A)) = \text{rank}(A)$
   x. True/False: $\dim(\text{nullspace}(A)) = \text{nullity}(A)$
   xi. True/False: $\dim(\text{nullspace}(A)) = \text{rank}(A)$
   xii. True/False: $\text{rowspace}(A)$ is closed under vector addition.
   xiii. True/False: $\text{rowspace}(A)$ is a subspace of $\mathbb{R}^m$
   xiv. True/False: $\text{rowspace}(A)$ is a subspace of $\mathbb{R}^n$
   xv. True/False: $\text{colspace}(A)$ is a subspace of $\mathbb{R}^m$
   xvi. True/False: $\text{colspace}(A)$ is a subspace of $\mathbb{R}^n$
   xvii. True/False: $\text{colspace}(A^t)$ is a subspace of $\mathbb{R}^m$
   xviii. True/False: $\text{colspace}(A^t)$ is a subspace of $\mathbb{R}^n$
   xix. True/False: $\text{colspace}(A) + \text{rowspace}(A) = n$
xx. True/False: \( \dim(\text{colspan}(A)) + \dim(\text{rowspan}(A)) = n \)

xxi. True/False: \( \dim(\text{colspan}(A)) + \dim(\text{nullspace}(A)) = m \)

xxii. True/False: \( \dim(\text{colspan}(A)) + \dim(\text{nullspace}(A)) = n \)

xxiii. True/False: \( \dim(\text{rowspan}(A)) + \dim(\text{nullspace}(A)) = m \)

xxiv. True/False: \( \dim(\text{rowspan}(A)) + \dim(\text{nullspace}(A)) = n \)

xxv. True/False: \( \dim(\text{left nullspace}(A)) = m - \text{rank}(A) \)

xxvi. True/False: \( \dim(\text{left nullspace}(A)) = n - \text{rank}(A) \)

xxvii. True/False: \( \text{rank}(A) + \text{nullity}(A) = m \)

xxviii. True/False: \( \text{rank}(A) + \text{nullity}(A) = n \)

(b) True/False: Every subspace of \( \mathbb{R}^4 \) is the column space of some matrix \( A \).

11. Give an example of the following (or explain why it’s impossible)

(a) A matrix \( A \) where the rowspace(\( A \)) and colspace(\( A \)) have the same dimension

(b) A matrix \( A \) where the rowspace(\( A \)) and colspace(\( A \)) are the same subspace of \( \mathbb{R}^n \)

(c) A matrix \( A \) where the rowspace(\( A \)) and colspace(\( A \)) have different dimensions

(d) A matrix \( A \) where the rowspace(\( A \)) and colspace(\( A \)) are different subspaces.

(e) An \( n \times n \) matrix \( A \) where nullspace(\( A \)) is the trivial subspace.

(f) An \( m \times n \) matrix \( A \) where nullspace(\( A \)) is the trivial subspace.

(g) A square matrix \( A \) where nullspace(\( A \)) and left nullspace(\( A \)) have the same dimension.

(h) A square matrix \( A \) where nullspace(\( A \)) and left nullspace(\( A \)) have different dimensions.

(i) A non-square matrix \( A \) where nullspace(\( A \)) and left nullspace(\( A \)) have the same dimension.

(j) A non-square matrix \( A \) where nullspace(\( A \)) and left nullspace(\( A \)) have different dimensions.

(k) A matrix \( A \) where rowspace(\( A \)) and nullspace(\( A \)) have the same dimension.

(l) A matrix \( A \) where rowspace(\( A \)) and nullspace(\( A \)) have different dimension.

(m) A matrix \( A \) where rank(\( A \)) = nullity(\( A \))

(n) A matrix \( A \) where rank(\( A \)) < nullity(\( A \))

(o) A matrix \( A \) where rank(\( A \)) > nullity(\( A \))

(p) A matrix \( A \) where nullity(\( A \)) = 0

(q) A matrix \( A \) where dim(left nullspace(\( A \)) = 0

(r) A matrix \( A \) where rank(\( A \)) = 0

Chapter 7

12. Most important HW type questions

(a) Here’s a function \( T \), determine if \( T \) is a linear transformation. if it is, find its matrix relative to (some basis)
   These are like 1-8 and 14-21

(b) Something like 9, 10, 11

(c) Something like 22-25