Asymmetric Learning from Price and Post-Earnings Announcement Drift*

Jaewon Choi† Jared Williams‡

October 14, 2014

Abstract

Motivated by research in psychology and experimental economics, we assume that investors who observe positive (negative) private information learn from price, but only if the equilibrium price is low (high). We examine the implications of this form of learning from price in the context of trade around earnings announcements. We show that asymmetric learning from price causes post-earnings announcement drift, but only among firms whose earnings are expected (ex ante) to be persistent.

Keywords: **Post-Earnings Announcement Drift, False Consensus Effect, Market Underreaction, Trading Volume**

JEL Classification: D80, G02, G14, M41

*We thank Zhi Da, Yelena Larkin, Hong Qu, and seminar participants at Penn State. All errors are our own.

†Department of Finance, College of Business, University of Illinois. 340 Wohlers Hall, 1206 South Sixth Street, Champaign, IL, 61820. E-mail: jaewchoi@illinois.edu

‡Department of Finance, Smeal College of Business, Penn State University. 344 Business Building, University Park, PA. E-mail: jmw52@psu.edu

1 Introduction

The tendency for stocks' abnormal returns to drift in the direction of earnings surprises following earnings announcements is one of the most puzzling anomalies in capital markets research. To date, the most widely accepted explanations for this "post-earnings announcement drift" (PEAD) are that investors underestimate the persistence of earnings surprises (Bernard and Thomas, 1990) and that investors have limited attention (Hirshleifer, Lim, and Teoh (2009) and DellaVigna and Pollet (2009)). In this paper, we provide an alternative explanation for PEAD; we show that if investors learn from price—but only when the price clearly reveals that other investors observed different private information—then PEAD should be expected as long as the firm's earnings are likely to be persistent.

We demonstrate our PEAD prediction by modeling trade around the earnings announcement of a firm whose earnings are composed of two different components. These components can represent items such as revenue and cost as well as different lines of a company's business. In contrast to the limited attention explanation for PEAD, when the firm announces its earnings, the realizations of each of the components is publicly observed by all market participants. Although all investors observe the earnings realizations, they receive different pieces of private information: some investors observe whether one component is likely to be persistent, while others observe whether the other component is likely to persist. In practice, this corresponds to the idea that some investors are skilled at analyzing certain components a firm's earnings (e.g., a specific segments), whereas others are skilled at analyzing other components of an earnings report. Unlike the Bernard and Thomas (1990) explanation for PEAD, all the investors in our model correctly assess the unconditional autocorrelation of the firm's earnings. Compared to the two predominant explanations for PEAD, the investors in our model are quite sophisticated—they are attentive and they have correct beliefs about the time series properties of the firm's earnings.

Our only departure from traditional models concerns the way that investors learn from the asset's price. It is common for researchers to assume that investors learn nothing from price (DO models) or that investors believe the researcher's model and update their beliefs about the model's other variables upon observing the equilibrium price (RE models). In practice, investors' task of learning from price is made difficult by the fact that different investors infer different pieces of private information. The equilibrium price provides investors with information about whether others' private information is qualitatively similar (positive or negative) to their own private information, but not necessarily whether others' private information is *the same* as their own.

Consider an investor who observes an earnings report, and is skilled at interpreting the value-relevance of the disclosure of one of the company's segments but not its other segment. Without loss of generality, suppose the investor infers good private information from the firm's disclosure about

the segment he is skilled at analyzing. If the market price following the earnings announcement is below what should be expected given the firm's overall earnings, e.g., if the stock is trading at a lower than expected price-earnings ratio, then the investor can likely infer that other investors were able to infer negative private information from the disclosure about the firm's other segments. Hence, the investor should revise his beliefs about the firm's valuation upon observing the equilibrium price. Conversely, if the market price following the earnings announcement is greater than what should be expected given the firm's overall earnings, then it is unclear whether, and how, the investor should use the market price to revise his beliefs—if the equilibrium price is unusually high because others inferred the same private information that he inferred, then the equilibrium price is incrementally uninformative. However, if the equilibrium price is high because others were able to infer different pieces of positive private information (e.g., by analyzing the disclosure of the other segment's performance), then the investor should revise his valuation of the company upwards upon observing the market price. In DO models, investors do not learn from price in either scenario, whereas in RE models, they learn from price in both scenarios. In contrast, our key assumption is that investors are able to properly update their beliefs in the former scenario, but that they refrain from revising their beliefs in the latter, more difficult scenario. More formally, we examine what would happen if investors learn from price, but only when they observe positive (negative) private information and the equilibrium price is below (above) the price that is justified based on the public information (e.g., the reported earnings).

Our paper is fundamentally different than most of the existing literature in behavioral finance on biased beliefs. Most of this literature can be divided into three partially overlapping categories. One strand focuses on people's assessments of their abilities relative to their true ability. Psychologists have documented that most people think they are better than average, and that people's confidence intervals tend to be too narrow. This has been interpreted as evidence of overconfidence, and there are several prominent models that incorporate overconfidence. This literature includes, but is not limited to, Odean (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), Daniel, Hirshleifer, and Subrahmanyam (2001), Gervais and Odean (2001), Scheinkman and Xiong (2003), Malmendier and Tate (2005), and Gervais, Heaton, and Odean (2011).

A second strand focuses on mistakes people make in updating their beliefs upon observing random sequences. Examples in this literature include (but are not limited to) Barberis, Shleifer, and Vishny (1998), Rabin (2002), Rabin and Vayanos (2010), and Benjamin, Rabin, and Raymond (2013).

The third strand of literature focuses on the limits of people's ability (or willingness) to process information. In addition to the limits of attention literature (e.g., Hirshleifer and Teoh (2003) and Hirshleifer, Lim, and Teoh (2011)), this literature also includes models with boundedly rational agents (e.g., Hong and Stein (1999)).

Unlike these previous studies, we focus on people's perceptions of the *similarity* between their private information and others'. In this regard, our paper is closely related to Williams (2013), who models sequential forecasting of firms' earnings when analysts overestimate their similarity to other analysts. People's perceptions of their similarity to others is an important question in social psychology. As Hirshleifer (2001) notes, social psychology can potentially be an important factor in financial markets, with the tendency for people to overestimate their similarity to others (the "false consensus effect") being one example.

More generally, our paper is related to the recent literature in finance on higher order beliefs. Banerjee (2011) develops a model that nests RE and DO within it—each of these equilibrium concepts corresponds to a different parameter value. He finds that on average, empirical evidence is more supportive of RE than DO, particularly among larger firms. Banerjee, Kaniel, and Kremer (2009) provide a possible explanation for price momentum by showing that returns can exhibit positive autocorrelation when there are higher-order differences of opinion. Banerjee and Green (2013) develop a model where some investors are uninformed almost surely, while others are informed with some positive probability. The uninformed traders update their beliefs about whether others are informed based on price and dividend realizations. Our model differs from all of these in that in our model, future price changes are positively associated with current earnings surprises, i.e., our model predicts PEAD.

Our paper is organized as follows. We describe the evidence supporting our key assumption in Section 2. In Section 3, we develop two models of trade around earnings announcement that differ in their realism and tractability. Our baseline model is highly stylized and tractable, whereas our extended model is more realistic but less tractable. We show that both models predict PEAD, but only when earnings are ex ante expected to be persistent. Section 4 concludes.

2 Learning from Price

Like researchers who invoke RE or DO assumptions, we are not certain that investors learn from price in the manner that we assume. However, research in psychology and experimental economics supports our assumption.

Social psychologists have long been interested in people's perception of their similarity to others. People's perception of their similarity to others should affect how they learn from price. Financial markets consist of interactions among large groups of people with different sets of knowledge, skills, insights, and reasons for trading (e.g., liquidity versus speculative trade). A trader's beliefs about others' motives and information is important because it affects how he updates his beliefs upon observing others' actions. If others are trading for liquidity reasons, there is little information content in their trades. If others are trading a stock at high prices because they are optimistic about

the company for the same reason as the trader, then there is little reason for the trader to update his valuation upon observing the high prices, as it provides little to no incremental information content. If, however, others are trading a stock at high prices because they have private information that the trader lacks, then the trader should treat the price as incrementally informative.

The social psychology literature suggests that people *overestimate* their similarity to others. Early evidence of this tendency dates all the way back to Katz and Allport (1931), who document that cheaters' estimate of the prevalence of cheating is higher than non-cheaters' estimates. There was not widespread interest in the topic until several decades later, when Ross, Greene, and House (1977) asked students a series of two different types of questions. In one type of question, students were asked whether they engage in a certain behavior (e.g. watch television at least 30 hours a month), have a certain preference (e.g. prefer wheat or white bread), hold a particular belief (e.g. that nuclear weapons will be used in warfare in the next 20 years), etc. In the other type of question, the same students were asked to estimate the percentage of other students who engage in the type of behavior, have a particular preferences, struggle with a particular problem, hold a certain belief, etc. In the case of television watching, people who watched television at least 30 hours a month provided an average estimate of 49.2% for the percentage of students who watch television at least 30 hours a month, whereas the average estimate provided by those who did not watch television at least 30 hours a month was only 40.9%. Similar patterns arose for the other questions: the average estimates provided for the percent of students who believe (participate in, prefer, etc.) X were higher among those students who believe (participate in, prefer, etc.) X than among those who do not believe (participate in, prefer, etc.) X, supporting the hypothesis that people overestimate the degree to which they are similar to others.[1] Welch (2000) documents that financial economists who believe in a high equity premium believe that others do too, consistent with the false consensus effect. See Krueger (1998) or Krueger (2000) for more comprehensive reviews of the literature on the false consensus effect.

If investors overestimate their similarity to other investors, they will likely overestimate the likelihood that other investors infer the same pieces of private information from a firm's earnings

[1]The evidence described above does not necessarily imply people *overestimate* their similarity to others—an individual's own type is a data point, so the evidence described above is perfectly consistent with rational Bayesian behavior (Dawes and Mulford, 1996). Later researchers modified the criterion by defining a "truly false consensus effect" (TFCE) to occur if respondents weight their own responses more heavily than the response of a randomly chosen individual (Engelmann and Strobel, 2000) or if the correlation of a dummy for whether an individual engages in a certain behavior (i.e., his "endorsement") is positively correlated with his forecast error for the percent of people who actually engage in the behavior (Krueger and Zeiger, 1993). Both types of TFCE are generally supported by the literature. Of more specific interest to financial economists, Egan, Merkle, and Weber (2010) document a TFCE when analyzing investors' survey responses to questions about others' beliefs about future stock returns. Engelmann and Strobel (2000) are an exception in that they argue that monetary incentives cause the TFCE to disappear, but in a later study (Engelmann and Strobel, 2012) they document that a TFCE arises when there are monetary incentives as long as mental effort is required to determine the responses of other subjects.

report as them. For example, if they infer that an earnings component is high quality (i.e., likely to persist), they will likely overestimate the likelihood that others inferred the same thing. If the equilibrium price is consistent with their private information, there is little reason for investors to update their beliefs that others might have observed different private information. Conversely, if the equilibrium price is inconsistent with the investor's own private information, then it should be obvious to him that it is possible that others observed different private information than him, and that he should therefore view the price as incrementally informative.

Even though the false consensus effect is well-documented, it has received very little attention from theorists. Since people's beliefs about others is undoubtedly important in many applications, the false consensus should affect behavior in many economically interesting environments. To our knowledge, the only other study to examine the effects of the false consensus effect is Williams (2013), who shows that financial analysts' earnings forecasts are consistent with analysts overestimating the correlation of their private signal errors.

Kwasnica, Velthuis, and Williams (2013) provide more direct evidence in support of our assumption. In their experiments, subjects issue forecasts for a hypothetical firm's earnings, equal to revenues minus costs. Subjects first choose whether to observe revenues or costs, each of which takes one of two values with equal probability, and the high and low values are equally far apart (so that the two components are equally informative). Subjects are not allowed to observe both earnings components. Next, subjects issue initial forecasts for the firm's earnings. After issuing their initial forecasts, subjects are informed whether a randomly chosen other subject (the subject's "partner") observed "good news" or "bad news," where good news can represent either high revenues or low costs, and bad news can represent either low revenues or high costs.[2] Finally, after observing this information, subjects issue revised forecasts for the firm's earnings. Kwasnica, Velthuis, and Williams (2013) find that subjects typically issue revised forecasts as though they naïvely assume others observe the same information as them *unless* it is clear that their partner observed different information, in which case the subject's revised forecast is optimal. More specifically, most subjects' revised forecasts are exactly equal to their initial forecasts when their partner's information is qualitatively similar to theirs (e.g., both good news or both bad news), which is optimal only if they are certain their partner chose to observe the same information as them. On the other hand, when their partner's information is qualitatively dissimilar from theirs, their revised forecast differs from their initial forecast and it is consistent with them properly aggregating their private information and their partner's information.

While the psychology and experimental evidence described above support our assumption, we acknowledge that to our knowledge, no one knows how investors actually learn from price. Given that DO and RE models make polar opposite assumptions about how investors learn from price,

[2]Subjects are not informed *which* earnings component their partner observed.

at least one of them must be flawed in describing how people update their beliefs in practice. Nevertheless, both are considered mainstream, and researchers continue working with both types of models. As Banerjee (2011) notes, "The true behavior of investors is likely to be neither as efficient as in an RE equilibrium nor as inefficient as in a pure DO equilibrium, but somewhere in between" (page 3058). We agree, and the behavior of the investors in our model is less efficient than behavior in RE and more efficient than behavior in DO models.

3 Models of Trade around Earnings Announcements when Investors Asymmetrically Learn from Price

In this section, we develop two models of trade around earnings announcements. Our benchmark model is simplified in that earnings take discrete realizations, there are no arbitrageurs, and private information is evenly split across the investor population every period. These simplifications keep the model relatively tractable. Our extended model relaxes these simplifying assumptions: earnings realizations are continuous, there is a random proportion of arbitrageurs each period, and the proportion of investors who observe a given piece of private information each period is random. Both models predict abnormal returns to drift in the direction of earnings surprises, but only if the firm's earnings are expected to be persistent.

3.1 Assumptions Common to Both Models

There are two assets: a risk-free asset with interest rate normalized to 0, and a risky asset (firm) with per capita supply Q that pays no dividends until a random liquidation time T, at which point it pays a liquidating dividend equal to its cumulative earnings:

$$D_T = D_0 + \sum_{t=1}^{T} e_t. \tag{1}$$

The liquidation probability, ℓ, is constant across time. We assume ℓ and Q are arbitrarily close to 0.

Each period, a continuum of investors with mean-variance utility enter the market. The utility function of each investor j is given by

$$U_j = \hat{\mathbf{E}}_j[W_T^j] - \left(\frac{\alpha}{2}\right) \widehat{\mathrm{Var}}_j(W_T^j), \tag{2}$$

where $\hat{\mathbf{E}}_j[\cdot]$ and $\widehat{\mathrm{Var}}_j[\cdot]$ are the expectation and variance taken with respect to j's beliefs, W_T^j is j's terminal wealth, and α is the coefficient of absolute risk aversion. We assume that each period,

investors form their demands as though they plan to hold their investment that period until the asset liquidates.

Each period's earnings consist of two components: $e_t = e_t^A + e_t^B$. If an earnings component is "high quality" in a given period, then it is expected to persist into the next period; in contrast, if an earnings component is "low quality" at a given time, it is not expected to persist. More formally,

$$\mathbf{E}[e_{t+1}^i | e_t^i \text{ is high quality}] > 0, \text{ and} \tag{3}$$

$$\mathbf{E}[e_{t+1}^i | e_t^i \text{ is low quality}] \leq 0. \tag{4}$$

The likelihood that earnings quality is high is equal across components and is denoted by p. Earnings quality realizations are independent across components (A and B) and across time. All investors correctly understand the time series properties of each earnings component, including the parameter p.[3]

At time t, the firm publicly announces its time t earnings, e_t, along with the individual components, e_t^A and e_t^B. When the firm announces its time t earnings, $\{e_t^A, e_t^B\}$ is common knowledge—all investors pay attention to the earnings announcement.[4] In addition, investors infer the quality (high or low) of at least one of the two earnings components. Since investors have to use their expertise in evaluating earnings announcements (including analyzing soft information from conference calls between management and analysts) in order to infer earnings quality, we consider earnings quality to be private information, as it is not publicly revealed during the earnings announcement. We refer to the "rational" investors who infer the quality of both earnings components as "type R" investors, or "arbitrageurs." We refer to the investors who only observe the quality of earnings component A (B) as "type A" ("type B") investors.

It is standard for economists to invoke one of two polar assumptions about how investors learn from price. In difference of opinion (DO) models, investors never learn from price, whereas in rational expectations (RE) models, they always learn from price. Let I_{it} denote the information that investors of type i use to form their beliefs about D_T at time t. In a DO framework, investors incorporate all non-price information that they have observed; in our setting, this corresponds to

$$I_{it}^{\text{DO}} \equiv \{e_1, \ldots, e_t, \mathbf{1}_{\{e_t^i \text{ is high quality}\}}\}, \qquad i = A, B. \tag{5}$$

In an RE framework, investors incorporate all information that they have observed (including the

[3] An obvious corollary of this is that all investors correctly estimate the autocorrelation of the firm's earnings, which is in contrast to the Bernard and Thomas (1990) explanation for PEAD. Hence, the investors in our model are relatively sophisticated.

[4] This is in contrast to the other prominent explanation for PEAD, i.e., limited attention (Hirshleifer, Lim, and Teoh, 2011). Again, relative to the alternative explanation, the investors in our model are sophisticated.

equilibrium price); in our setting, this corresponds to

$$I_{it}^{\mathrm{RE}} \equiv \{e_1, \ldots, e_t, \mathbf{1}_{\{e_t^i \text{ is high quality}\}}, P_1^*, \ldots, P_t^*\}, \qquad i = A, B. \tag{6}$$

Finally, let I_t^e denote the the earnings information that is public at time t:

$$I_t^e \equiv \{e_1, \ldots, e_t\}. \tag{7}$$

Clearly, for all i and t, $I_t^e \subseteq I_{it}^{\mathrm{DO}} \subseteq I_{it}^{\mathrm{RE}}$.

Let P_t^e denote the equilibrium price that would arise if no investor observed private information; P_t^e is the price that is justified based on public information. It trivially follows from (2) that if investors form their beliefs about D_T based only on non-price public information, then each investor's demand is given by

$$x_t^e \quad = \quad \frac{\mathbf{E}[D_T|I_t^e] - P_t^e}{\alpha \mathrm{Var}[D_T|I_t^e]}. \tag{8}$$

Market clearing implies

$$P_t^e \quad = \quad \mathbf{E}[D_T|I_t^e] - \alpha Q \mathrm{Var}[D_T|I_t^e]. \tag{9}$$

We say that investors observe good (bad) private information if $\mathrm{Priv}_{it} > 0$ ($\mathrm{Priv}_{it} < 0$), where Priv_{it} is defined as the difference between type i investors' expectation for the liquidating dividend and the expectation of the liquidating dividend conditional on all public information:

$$\mathrm{Priv}_{it} \quad \equiv \quad \mathbf{E}[D_T|I_{it}^{\mathrm{DO}}] - \mathbf{E}[D_T|I_t^e]. \tag{10}$$

If $e_t^i > 0$, then investors of type i observe good (bad) private information if the quality of e_t^i is high (low). Conversely, if $e_t^i < 0$, then investors of type i observe good (bad) private information if the quality of e_t^i is low (high).

Let P_t^* denote the actual equilibrium price for the risky asset. Suppose that at time t, investors of type i observe good private information. If $P_t^* < P_t^e$, then investors of type i can reasonably infer that other investors observed *bad* private information—otherwise, the equilibrium price would have been no less than P_t^e. Since they inferred positive private information about e_t^i, it must be the case that other investors inferred negative private information about the other earnings component. As discussed in Section 2, we assume that investors learn from price in this scenario, i.e., $I_{it} = I_{it}^{\mathrm{RE}}$.

Now consider the case where investors of type i observe positive private information about e_t^i and the equilibrium price is greater than what is implied by public information, $P_t^* > P_t^e$. In this scenario, it is less clear how i should update his beliefs upon observing P_t^*—since the investors'

private information and the equilibrium price are directionally consistent vis-a-vis the price justified by public information, investors cannot know whether other investors inferred the same private as them. As discussed in Section 2, we assume that investors treat the equilibrium price as being incrementally uninformative in this scenario, i.e., $I_{it} = I_{it}^{\mathrm{DO}}$.

More generally, we assume that the information that investors of type i ($i = A, B$) use to update their beliefs about D_T at time t is given by

$$I_{it} = \begin{cases} I_{it}^{\mathrm{RE}} & \text{if } \mathrm{sgn}(\mathrm{Priv}_{it}) \neq \mathrm{sgn}(P_t^* - P_t^e) \\ I_{it}^{\mathrm{DO}} & \text{if } \mathrm{sgn}(\mathrm{Priv}_{it}) = \mathrm{sgn}(P_t^* - P_t^e) \end{cases} \tag{11}$$

where $\mathrm{sgn}(\cdot)$ is the sign function, and I_{it}^{DO} and I_{it}^{RE} are given by (5) and (6).[5] Equation (11) formalizes the notion that investors learn from the equilibrium price (as in RE models), but only when it is clear that others have observed private information that differs from their own.

P_t^* is an equilibrium pricing function if (11) holds and markets clear, in which case we let I_{it}^* denote the information set satisfying (11).

3.2 Baseline Model

In our baseline model, earnings are discrete. Each of the earnings components ($\{e^A\}$ and $\{e^B\}$) are independent and identically distributed Markov chains, taking values $-\sigma$ and σ. If an earnings component is high quality at time t, then its time $t+1$ realization equals its time t realization: $e_t^i = e_{t+1}^i$. Conversely, if it is low quality at time t, then it reverses at time $t+1$: $e_t^i = -e_{t+1}^i$. Since p is the likelihood that an earnings component is high quality, it follows that each of the earnings components follow a Markov chain with switching probability $1-p$. It clearly follows that total earnings ($e = e^A + e^B$) also follows a Markov chain. Let s_1, s_2, and s_3 be the states $\{e = -2\sigma\}$, $\{e = 0\}$, and $\{e = 2\sigma\}$, respectively, so that the transition matrix, P, is given by

$$P = \begin{pmatrix} p^2 & 2p(1-p) & (1-p)^2 \\ p(1-p) & p^2 + (1-p)^2 & p(1-p) \\ (1-p)^2 & 2p(1-p) & p^2 \end{pmatrix}, \tag{12}$$

where P_{ij} denotes the probability of moving to state j next period conditional on being in state i this period.

Our assumption that investors infer the earnings quality of an earnings component is equivalent to assuming that at time t, each trader is able to infer *one* (but not both) of the firm's time $t+1$ earnings component (e_{t+1}^A or e_{t+1}^B) based on his analysis of the time t earnings report. For example, some traders may be skilled at determining whether a certain earnings components is likely to be

[5] The sign function maps positive numbers to 1, negative numbers to -1, and 0 to 0.

transitory or persistent, while others may be able to infer whether a different component is likely to be transitory or persistent. In our baseline model, we assume that at each time t, 50% of the incoming investors observe e_t^A's earnings quality, and the other 50% of investors observe e_t^B; there are no arbitrageurs in the baseline model.

3.2.1 Equilibrium

The following lemma establishes that in any equilibrium, the price is (approximately) the average of traders' ex post expectations for the liquidating value.

Lemma 1. *For any $\xi > 0$, there exist $\overline{Q} > 0$ and $\overline{\ell} > 0$ such that if $Q < \overline{Q}$ and $\ell < \overline{\ell}$, then $\left|P_t^* - \overline{E[D_T|I_{it}]}\right| < \xi$ for any equilibrium pricing function P_t^*, where $\overline{E[D_T|I_{it}]}$ denotes investors' average expectation for the liquidating dividend.*

See Appendix A for proofs of all propositions.

The proof for Lemma 1 is also valid if agents condition on I_{it}^{RE} or I_{it}^{DO}. Since we assume Q and ℓ to be arbitrarily small, we will use the proposition to express all equilibria prices as the average of investors' ex post beliefs.[6] For example, the time t DO equilibrium price can be expressed as

$$\begin{aligned} P_t^{\text{DO}} &= \frac{\mathbf{E}[D_T|e_1,\ldots,e_t,e_{t+1}^A] + \mathbf{E}[D_T|e_1,\ldots,e_t,e_{t+1}^B]}{2} \\ &= D_t + \frac{1}{2}\left(\frac{q}{1-q}\right)e_t + \frac{1}{2}\left(\frac{1}{1-q}\right)e_{t+1}, \end{aligned} \tag{13}$$

where $D_t \equiv \sum_{j=1}^t e_j$ and

$$q \equiv 2p - 1. \tag{14}$$

(To compute (13), we use the fact that ℓ is arbitrarily close to 0.)

Let P_t' and P_t^* be defined as

$$P_t' = \begin{cases} D_t - \frac{e_t}{4} & \text{if } p > 0.5 \\ D_t + \left(\frac{1+q}{4(1-q)}\right)e_t & \text{if } p < 0.5 \end{cases} \tag{15}$$

$$P_t^* = \begin{cases} P_t^{\text{DO}} & \text{if } (e_t, e_{t+1}) \neq (\pm 2\sigma, 0) \text{ or } p = 0.5 \\ P_t' & \text{if } (e_t, e_{t+1}) = (\pm 2\sigma, 0) \text{ and } p \neq 0.5 \end{cases} \tag{16}$$

[6] By "ex post," we mean after the equilibrium price is revealed. Note, however, that not all investors will use the equilibrium price in their conditioning set. For example, in DO equilibria, none of the investors include the equilibrium price in their conditioning set.

The following proposition establishes that P_t^* is an equilibrium pricing function.

Proposition 1. *P_t^* is an equilibrium pricing function.*

To see how P_t^* is constructed, suppose first that the time $t+1$ earnings are extreme ($e_{t+1} = \pm 2\sigma$). All investors receive qualitatively similar private information (all good or all bad), P_t^* will naturally be above (below) P_t^e if all investors observe good (bad) private information, and no investors learn from price since it is consistent with their private information. P_t^* is simply the DO equilibrium price in this scenario.

Now consider the case where the time $t+1$ earnings are normal ($e_{t+1} = 0$). If the time t earnings are also normal ($(e_t, e_{t+1}) = (0, 0)$), then the strength of the good news observed by half of the investor population is equal to the strength of the bad news observed by the other half. The equilibrium price is therefore equal to the price implied by the public information, which is also equal to the difference of opinion equilibrium price: $P_t^* = P_t^e = P_t^{\mathrm{DO}}$. If the time t earnings are extreme ($(e_t, e_{t+1}) = (\pm 2\sigma, 0)$) and $p \neq 0.5$, then half of the population observes a reversal in an earnings component, while the other half of the population observes a continuation. Which of these events is more informative (i.e., surprising) depends on whether $p > 0.5$ or $p < 0.5$. Without loss of generality, suppose that the A component reverses. If $p > 0.5$, A's reversal is more surprising than B's continuation, $\mathrm{sgn}(P_t^* - P_t^e) = \mathrm{sgn}(e_{t+1}^A)$, and investors of type A do not learn from price, whereas investors of type B do learn from the price. $P_t^* = P_t'$ is simply the average of investors' posterior means for D_T given that one group learns from price while the other does not. In the $p < 0.5$ case, the continuation is more informative than the reversal, so the group of investors who observe the reversal learns from price, whereas those who observe the continuation do not learn from price.

Although P_t^* is not the unique equilibrium pricing function, it is unique in most scenarios and it is the most natural equilibrium when multiple equilibria exist. It can be shown that P_t^* is the unique equilibrium *unless* (i) $(e_t, e_{t+1}) = (0, 0)$ or (ii) $p = 0.5$. In the former case, an equilibrium can arise in which the price differs from $P_t^e (= P_t^{\mathrm{DO}})$, the group observing the private information that conflicts with the equilibrium price learns from the price and has a rational posterior belief, while the group observing the private information that is consistent with the equilibrium price does not learn from the price. Allowing for these alternative equilibria have no effect on our PEAD predictions as long as we assume that when $(e_t, e_{t+1}) = (0, 0)$, the $P_t > P_t^e$ alternative equilibrium is as likely to occur as the $P_t < P_t^e$ alternative equilibrium. Similar alternative equilibria can exist in the knife-edge case where p is exactly equal to 0.5.

Specifically, investors' only "mistake" is that they do not consider the equilibrium price to be

incrementally informative when it is qualitatively consistent with their private information. E.g., when they observe positive private information and see that the equilibrium price is above what is implied by the (non-price) public information, they assume the price is incrementally uninformative given their other information. The equilibrium price is simply the price that equates the supply of the risky asset with investors' demand for it given their beliefs, where their beliefs can be updated based on the equilibrium price, as in RE models.

3.2.2 Post-Earnings Announcement Drift

We begin by examining the relationship between earnings surprises and subsequent price drift. Let PEAD_j denote the average profit (per round trip transaction) from the strategy that buys (sells) at time $t+j-1$ if the time t earnings are greater (less) than expected and unwinds the position at time $t+j$. Formally,

$$\text{PEAD}_j = \begin{cases} \mathbf{E}[\Delta P^*_{t+j}|e_1,\ldots,e_t] & \text{if } e_t > \mathbf{E}[e_t|e_1,\ldots,e_{t-1}] \\ -\mathbf{E}[\Delta P^*_{t+j}|e_1,\ldots,e_t] & \text{if } e_t < \mathbf{E}[e_t|e_1,\ldots,e_{t-1}] \end{cases} \tag{17}$$

We plot PEAD_j by p, the earnings persistence parameter, in Figure 1.

[INSERT FIGURE 1 HERE]

For more details on the construction of this figure, see Appendix B.

Empirically, earnings are persistent. Bernard and Thomas (1990) show that a \$1 earnings surprise (relative to a seasonal random walk model) in quarter t is associated with average earnings surprise of \$0.34, \$0.19, and \$0.06 in quarters $t+1$, $t+2$, and $t+3$, respectively. In our model, if $p = 0.7$, then a \$1 earnings realization in quarter t is associated with average earnings of \$0.4, \$0.16, and \$0.064 in quarters $t+1$, $t+2$, and $t+3$, respectively; if $p = 0.65$, then a \$1 earnings realization in quarter t is associated with average earnings of \$0.3, \$0.09, and \$0.027 in quarters $t+1$, $t+2$, and $t+3$, respectively. Hence, it is reasonable to assume the typical firm has a p near the $(0.65, 0.7)$ range, and the PEAD_1 plot in Figure 1 implies the following prediction.

Prediction 1. *Cumulative abnormal returns will drift in the direction of earnings surprises. That is, firms' returns will exhibit post-earnings announcement drift.*

Notice the relationship between the PEAD_1, PEAD_2, and PEAD_3 plots in the figure—in the $p \in [0.65, 0.7]$ range, positive earnings surprises at quarter t are associated with average returns of

approximately 0.15σ, 0.05σ, and 0.02σ at times $t+1$, $t+2$, $t+3$, respectively. That is, announcement returns around future earnings announcements are predictable based on current earnings realizations, and the average size (in absolute value) of these returns diminishes the further into the future the earnings announcement. This is consistent with the evidence documented by Bernard and Thomas (1990).

Prediction 2. *Positive (negative) earnings surprises in quarter t are associated with declining (in absolute value) positive (negative) earnings announcement returns in quarters $t+1$, $t+2$, and $t+3$.*

Despite being documented over 40 years ago by Ball and Brown (1968), post-earnings announcement drift is not well-understood by accounting researchers and financial economists.[7] Our model provides an explanation for the anomaly that is consistent with the earnings announcement return patterns documented by Bernard and Thomas (1990).

In addition, it offers a new testable prediction. For example, it is clear from the figure that our model only predicts PEAD if the firm's earnings are expected (ex ante) to be persistent, which (to our knowledge) has never been tested.

Prediction 3. *PEAD strategies will earn negative profits among firms whose earnings are (ex ante) expected to reverse.*

It is worth noting that our PEAD predictions are *not* a simple consequence of introducing earnings persistence into a standard model; it is easy to verify that future RE and DO price changes *cannot* be predicted based on current (or historical) earnings: for all $j \geq 1$,

$$\mathbf{E}[\Delta P^{\mathrm{RE}}_{t+j} | e_1, \ldots, e_t] = 0 \tag{18}$$

$$\mathbf{E}[\Delta P^{\mathrm{DO}}_{t+j} | e_1, \ldots, e_t] = 0. \tag{19}$$

The intuition behind (18) is obvious—in RE models, the agents properly learn from all the public information (including price), so they would exploit any mispricing until it disappears. Equation (19) follows from the fact that each investor i rationally updates his beliefs upon observing $\{e_1, \ldots, e_t, e^i_{t+1}\}$; the equilibrium DO price is simply the average of expectations that are conditioned on finer information sets than the information on which the PEAD strategy trades. Hence, future DO price changes cannot be predicted based on current earnings, because those current earnings have already been (correctly) incorporated in all investors' beliefs.

[7] For a review of the empirical research on PEAD, see Livnat and Mendenhall (2006).

To understand why there is PEAD in our model, consider the case in which earnings are ex ante expected to persist ($p > 0.5$) and current earnings are high ($e_t = 2\sigma$). If the next period's earnings are either extremely high or low, i.e., $e_{t+1} = \pm 2\sigma$, then all investors receive qualitatively similar private information—all observe good private information or all observe bad private information. The equilibrium price will be consistent with everyone's private information, so investors will believe the equilibrium price is uninformative, and $P_t^* = P_t^{\mathrm{DO}}$. Now suppose next period's earnings are 0—without loss of generality, $e_{t+1}^A = \sigma$ and $e_{t+1}^B = -\sigma$. Investors who infer that $e_{t+1}^A = \sigma$ observe relatively weak positive information (recall that e_t^A is ex ante expected to persist), whereas investors who infer that $e_{t+1}^B = -\sigma$ observe relatively strong negative information. It can be verified that $P_t^* < P_t^{\mathrm{DO}}$, the investors who privately inferred that $e_{t+1}^A = \sigma$ learn from the price that $e_{t+1}^B = -\sigma$, whereas those who privately inferred that $e_{t+1}^B = -\sigma$ *do not* learn from the equilibrium price that $e_{t+1}^A = \sigma$, and P_t^* is simply the average of the investors' posterior beliefs. Since $P_t^* = P_t^{\mathrm{DO}}$ when $e_{t+1} = \pm 2\sigma$ and $P_t^* < P_t^{\mathrm{DO}}$ when $e_{t+1} = 0$, it trivially follows that $\mathbf{E}[P_t^* | e_t = 2\sigma] < \mathbf{E}[P_t^{\mathrm{DO}} | e_t = 2\sigma]$. By (19), there is no PEAD in the DO equilibrium prices, and $\mathbf{E}[P_{t+j}^{\mathrm{DO}} | e_t = 2\sigma] = \mathbf{E}[P_t^{\mathrm{DO}} | e_t = 2\sigma]$ for all $j > 0$. It can be verified that as j increases, $\mathbf{E}[P_{t+j}^{\mathrm{DO}} - P_{t+j}^* | e_t = 2\sigma]$ converges monotonically to 0. It follows that $\mathbf{E}[P_{t+j}^* | e_t = 2\sigma] > \mathbf{E}[P_t^* | e_t = 2\sigma]$, i.e., high earnings are followed by positive subsequent returns.

Note that as in Hong and Stein (1999), there is gradual information diffusion in our baseline model: at time t, each investor is able to infer some, but not all, of the time $t+1$ earnings components. When the firm publicly announces its time $t+1$ earnings, the information fully diffuses across the entire population of investors. Hong and Stein (1999) assume investors do not learn from price (as in DO models), and their focus is on price momentum rather than price drift following earnings announcements—they show that gradual information diffusion leads to positive autocorrelation in DO equilibrium price changes, i.e., their model generates price momentum. Although their focus is on momentum rather than the profitability of strategies that sort on publicly observed (non-price) signals such as earnings, they write (page 2165):

> It is easy to embellish our model so that it also generates short-run underreaction to public news. For example, one might argue that although the news announcement itself (e.g., "earnings are up by 10 percent") is public, it requires some other, private, information (e.g., knowledge of the stochastic process governing earnings) to convert this news into a judgment about value. If this is true, the market's response to public news involves the aggregation of private signals, and our previous underreaction results continue to apply.

Their model does predict the market to "underreact" to public news in the sense that announcement returns are positively correlated with future returns, controlling for the publicly observed earnings

surprise. However, their model cannot explain why portfolios that sort on publicly observed earnings (as opposed to announcement returns) earn differential returns. To get PEAD from their model, one would have to assume that when a firm announces its earnings, investors only observe one component of the firm's current earnings, and that investors only observe the firm's net income at a later date. Since net income is the most salient and arguably the most informative piece of information in a firm's earnings announcement, this interpretation of their model—which the authors do not advance—is not compelling. Indeed, equation (19) shows that regardless of whether the firm's underlying earnings are persistent, it is not possible to predict future price changes based on current earnings; the PEAD that our model predicts is driven by the fact that sometimes investors learn from price, but sometimes they do not.

3.3 Extended Model

Our baseline model in the previous section is highly stylized. There are no arbitrageurs, earnings components have two possible realizations, and exactly 50% of investors receive private information about each component each period. In this section, we develop a more realistic model that allows for continuous earnings realizations, the presence of arbitrageurs, and for the proportion of investors observing private information about the quality of earnings components to be random and time-varying. We maintain our key assumption about investor learning from price: each investor learns from price, but only when the price and the investor's private information are directionally opposite relative to the price justified by public information. That is, the investor learns from price only if his private information is positive (negative) and the price is below (above) the price that is justified by the public information. For more discussion of this assumption, see Section 2. This model is less tractable than the baseline model, but like the model in the previous section, it still predicts PEAD if and only if earnings are ex ante persistent. This suggests that the baseline model's prediction is a robust consequence of asymmetric learning from price, and not a consequence of the previous model's more simplified assumptions (e.g., the discrete nature of earnings realizations).

Each of the risky asset's earnings components ($i = A, B$) are continuous and can be decomposed as follows:

$$e^i_{t+1} = \begin{cases} \lambda e^i_t + \varepsilon^i_{t+1} & \text{if } e^i_t \text{ is high quality} \\ \varepsilon^i_{t+1} & \text{if } e^i_t \text{ is low quality,} \end{cases} \tag{20}$$

for some $\lambda \in (0, 1)$. The ε^i_t's are distributed

$$\varepsilon^i_{t+1} \sim N\left(0, \frac{1}{2\tau_e}\right). \tag{21}$$

As in the baseline model, the likelihood of an earnings component being high is p. ε_t^A and ε_t^B are independent of one another they are independent across time, and they are independent of whether the earnings are high quality or low quality.

It trivially follows that

$$\mathbf{E}[e_{t+1}|e_t] \quad = \quad p\lambda. \tag{22}$$

Let q be defined as

$$q \equiv p\lambda, \tag{23}$$

so that

$$\mathbf{E}[e_{t+1}|e_t] \quad = \quad qe_t. \tag{24}$$

As in the baseline model, q denotes the rate at which earnings are expected to decay.[8]

At time t, the firm publicly announces its time t earnings component realizations, which is observed by all investors. Unlike the baseline model, we allow for the existence of arbitrageurs who can observe the earnings quality of both components: a proportion ρ_t of the investor population observes the quality of both earnings components, where $\rho_t \sim U(0, \overline{\rho})$ for some $\overline{\rho} \in (0, 1]$. The remaining $1 - \rho_t$ of the investor population at time t only observes the quality of one of the two earnings components: a proportion $1 - \beta_t$ of these investors infer the quality of e_t^A (high or low), while a proportion β_t of these investors infer the quality of e_t^B, where $\beta_t \sim U(0, 1)$. The random variables ε_t^A, ε_t^B, ρ_t, β_t are all distributed independently of one another.

We let μ_{it} denote the size of the investor population of type i at time t:

$$\mu_{it} \quad \equiv \quad \begin{cases} \rho_t & \text{if } i = R \\ (1-\rho_t)(1-\beta_t) & \text{if } i = A \\ (1-\rho_t)\beta_t & \text{if } i = B, \end{cases}$$

and we let I_{Rt} denote the arbitrageurs' information set at time t:

$$I_{Rt} \equiv \{e_1, \ldots, e_t, \mathbf{1}_{\{e_t^A \text{ is high quality}\}}, \mathbf{1}_{\{e_t^B \text{ is high quality}\}}, P_1^*, \ldots, P_t^*\} \tag{25}$$

[8]The link between q and p, expressed in (14) and (23), differ because of the differences in the structure of the baseline and extended model. It is worth reiterating, however, that in both models, q denotes the rate at which expected earnings decay.

3.3.1 Equilibrium

The following lemma verifies that the equilibrium price will be (roughly) equal to the average expectation of the asset's liquidating value, where the average is taken across the investor population:

Lemma 2. *For any $\xi > 0$, there exist $\overline{Q} > 0$ and $\overline{\ell} > 0$ such that if $Q < \overline{Q}$ and $\ell < \overline{\ell}$, then*

$$\left| P_t^* - \sum_{j \in \{A,B,R\}} \mu_{jt} \mathbf{E}[D_T | I_{jt}] \right| < \xi \tag{26}$$

for any equilibrium pricing function P_t^*.

The following proposition defines the equilibrium whose price drifts following earnings announcements we will later analyze.

Proposition 2. *Let $I_{Rt}^* \equiv I_{Rt}$, and for $i = A, B$, let I_{it}^* be defined by*

$$I_{it}^* \equiv \begin{cases} I_{it}^{DO} & \text{if } sgn\left(Priv_{it}\right) = sgn\left(\sum_{j \in \{A,B,R\}} \mu_{jt} Priv_{jt}\right) \\ I_{it}^{RE} & \text{if } sgn\left(Priv_{it}\right) \neq sgn\left(\sum_{j \in \{A,B,R\}} \mu_{jt} Priv_{jt}\right), \end{cases} \tag{27}$$

The following pricing function is an equilibrium:

$$P_t^* = \sum_{j \in \{A,B,R\}} \mu_{jt} \mathbf{E}[D_T | I_{jt}^*]. \tag{28}$$

The equilibrium described above is the most natural equilibrium to analyze in that it is the only equilibrium satisfying the condition that, relative to the price justified by public information, the equilibrium price lies in the direction of the stronger private information and/or the private information of the largest groups of investors. More formally, the equilibrium is the only one that satisfies the following condition:

Condition 1.

$$sgn\left(\sum_{j \in \{A,B,R\}} \mu_{jt} Priv_{jt} \right) = sgn\left(P_t^* - P_t^{\varepsilon}\right). \tag{29}$$

To analyze price drifts following earnings announcements, we need to compute the following conditional expectations for the firm's liquidating dividend. Letting $e_t^{-A} \equiv e_t^B$ and $e_t^{-B} \equiv e_t^A$, it can easily be verified that

$$\begin{aligned}
P_t^e &= \mathbf{E}[D_T|I_t^e] \\
&= D_t + e_t\left(\frac{q}{1-q}\right) && (30)\\
\mathbf{E}[D_T|I_{it}^{\mathrm{DO}}] &= D_t + e_t^{-i}\left(\frac{q}{1-q}\right) + \lambda e_t^i\left(\frac{1}{1-q}\right)\mathbf{1}_{\{e_t^i \text{ is high quality}\}}, \qquad i = A, B, && (31)\\
\mathbf{E}[D_T|I_{it}^{\mathrm{RE}}] &= \mathbf{E}[D_T|I_{Rt}] \\
&= D_t + \lambda\left(\frac{1}{1-q}\right)\left(e_t^A\mathbf{1}_{\{e_t^A \text{ is high quality}\}} + e_t^B\mathbf{1}_{\{e_t^B \text{ is high quality}\}}\right), \quad i = A, B. && (32)
\end{aligned}$$

Proposition 2 and equations (30)-(32) suffice for computing the equilibrium prices as a function of cumulative earnings, D_t, current earnings realizations, e_t^A and e_t^B, and the quality of current earnings components, $\mathbf{1}_{\{e_t^A \text{ is high quality}\}}$ and $\mathbf{1}_{\{e_t^B \text{ is high quality}\}}$.

3.3.2 Simulations

Compared to the baseline model, the extended model is not very tractable, so we rely on simulations to analyze price drifts following earnings announcements. Unlike the baseline model, earnings surprises relative to public information (i.e., $e_t - \mathbf{E}[e_t|I_{t-1}^e]$) can take infinitely many values in the extended model. To make our analysis comparable to the empirical PEAD literature, we divide earnings surprises into deciles, and examine the difference in subsequent price changes between deciles 1 and 10.

To simulate the long run distribution of earnings, note that the length of consecutive high quality earnings components preceeding an arbitrary period t is distributed geometrically with parameter p. Let n_t denote the realization of that random variable. If $n_t = 0$, $e_t^i = \varepsilon_t^i$. If $n_t = 1$, $e_t^i = \lambda\varepsilon_{t-1}^i + \varepsilon_t^i$. More generally,

$$e_t^i = \lambda^{n_t}\varepsilon_{t-n_t}^i + \lambda^{n_t-1}\varepsilon_{t-(n_t-1)}^i + \ldots + \lambda\varepsilon_{t-1}^i + \varepsilon_t^i. \tag{33}$$

Since $\sum_{i=0}^{n_t}(\lambda^2)^i = \frac{1-(\lambda^2)^{n_t+1}}{1-\lambda^2}$, and $\mathrm{Var}(\varepsilon_t^i) = \frac{1}{2\tau_e}$, it follows that the distribution of e_t^i conditional on n_t is

$$e_t^i|_{n_t} \sim N\left(0, \frac{1-(\lambda^2)^{n_t+1}}{2\tau_e(1-\lambda^2)}\right). \tag{34}$$

Hence, to simulate the long run distribution of earnings, we can first generate a geometric random

variable (n_t), and then use (34) to generate the earnings at an arbitrary starting date ($t = 0$). For subsequent time periods, earnings can easily be simulated using (20) and (21). By taking this approach, the distribution of our simulated earnings is the same at every time t. Once the earnings realizations and the sizes of the investor populations are simulated (ρ_t and β_t), equilibrium prices can be computed using Proposition 2 and equations (30)-(32).

For each value of $p \in \{0, 0.01, \ldots, 0.99\}$, we create one million simulations of earnings and price realizations at times $t = 0, 1, 2, 3$ and 4, where D_{-1} is set to 0.[9] For each value of p, we examine the relationship between earnings surprises and subsequent price drift by dividing the time 1 simulations into deciles based on the unexpected earnings at time 1, defined as $\text{UE}_1 \equiv e_1 - qe_0$. We report the average price change from time 1 to time 2 for the highest (lowest) decile of UE_1 in Figure 2a (Figure 2b). In these simulations, we arbitrarily set λ, τ_e, and $\overline{\rho}$ equal to 0.9, 5, and 0.3, respectively.[10]

[INSERT FIGURE 2 HERE]

From Figure 2a, it is clear that if $p < 0.5$, then prices are expected to fall in the period following high earnings surprises, and from Figure 2b, it is clear that prices are expected to rise following low earnings surprises. That is, Figure 2 shows that prices drift in the *opposite* direction of earnings surprises if $p < 0.5$. Conversely, if $p > 0.5$, it is clear from Figure 2 that prices drift in *the same* direction as earnings surprises, i.e., there is post-earnings announcement drift.

In Figure 3, we repeat the simulations from Figure 2, except we also consider the average price changes from times 2 to 3 and 3 to 4.

[INSERT FIGURE 3 HERE]

From the figure, we can see that our model predicts the finding of Bernard and Thomas (1990) that returns around future earnings announcements are positively correlated with current earnings surprises, and that the relationship diminishes as the length of time between earnings announcements increases.

To ensure that our results are robust to other parameter regions, we repeat the simulation procedure above for each quadruple $(p, \lambda, \overline{\rho}, \tau_e)$ such that $p \in \{0.1, 0.2, \ldots, 0.9\}$ (excluding $p = 0.5$), $\lambda \in \{0.2, 0.4, 0.6, 0.8, 1\}$, $\overline{\rho} \in \{0.2, 0.4, 0.6, 0.8, 1\}$, and $\tau_e \in \{1, 2, 3, 4, 5\}$. There are 1,000 quadruples

[9] Since we analyze price changes, the initial level of cumulative earnings is irrelevant.

[10] The qualitative results that we discuss below are not affected by our choice of parameters.

in the cartesian product of the above sets, and since we run simulations separately for the low earnings surprise deciles and high earnings surprise deciles, we conduct a total of 2,000 simulations, each simulation consisting of 1 million observations. For each of these 2,000 simulations, we examined whether the PEAD result was consistent with our baseline model, i.e., we tested whether we observed PEAD if, and only if, $p > 0.5$. This prediction was confirmed for all but 10 of the 2,000 parameter values.[11] We re-ran the simulations for these 10 parameter values, and each time, the output was consistent with the baseline model's prediction of PEAD, if and only if, $p > 0.5$. Hence, our simulations suggest that our prediction is consistent for all 1,000 parameter values that we tested.

Summarizing, Predictions 1-3 from our baseline model continue to hold in the extended model. This suggests that the model's predictions are not due to the baseline model's simplified assumptions, but rather are robust predictions that follow from our assumption that investors learn from price in an asymmetric manner around earnings announcements.

4 Discussion and Conclusions

Research in psychology and experimental economics suggests that people might learn from price in an asymmetric manner—when the price is consistent with their private information, they will view the price as incrementally uninformative, but when the price is not consistent with their private information, they will consider the possibility that others have observed private information that differs from their own private information. We examined the pricing implications of this asymmetric learning from price in the context of earnings announcements, and we showed that it predicts prices to drift in the direction of earnings surprises, i.e., post-earnings announcement drift.

While our models are likely to be considered "behavioral" because our investors are less than fully rational, it is worth noting that our investors are far more sophisticated than investors in competing theories of PEAD. Two of the most prominent explanations for PEAD in the accounting and finance literatures are: (i) investors underestimate the persistence of firms' earnings (Bernard and Thomas (1990)), and (ii) a subset of investors have limited attention (DellaVigna and Pollet (2009), Hirshleifer, Lim, and Teoh (2009), and Hirshleifer, Lim, and Teoh (2011)). In our opinion, the first explanation is unappealing because PEAD was first documented over 45 years ago (Ball and Brown, 1968), and autocorrelations are easy to estimate; hence, for this to explain PEAD, investors must have very low cognitive ability. Regarding the limited attention explanation for PEAD, Hong and Stein (2007) note that these models assume that a sizable class of investors are unsophisticated in two logically distinct ways: (i) they do not pay attention to the earnings announcement, and (ii) they are completely unaware of the fact that the firm announced earnings.[12] Compared to these

[11] These 10 parameter combinations were: $(p, \lambda, \overline{\rho}, \tau_e) = \{(0.1, 0.2, 0.8, 1), (0.4, 0.2, 0.6, 5), (0.4, 0.2, 1, 1), (0.4, 0.2, 1, 5), (0.4, 0.8, 1, 3), (0.4, 0.2, 0.8, 1), (0.4, 0.2, 1, 2), (0.4, 0.2, 1, 4), (0.6, 0.2, 0.2, 3),$ and $(0.6, 0.2, 0.8, 5)\}$.

[12] Alternatively, limited attention models are applicable if investors are aware of the earnings announcement but

explanations, the investors in our model are quite sophisticated—they correctly understand the time series properties of firms' earnings, and they all pay attention to and fully understand (non-price) information as soon as it is made public. The only sense in which our investors are "biased" relative to rational expectations models is that our investors view the equilibrium price as incrementally uninformative when it is consistent with the private information they have observed. Hence, the investors in our model are also significantly more sophisticated than investors in DO models, and DO models have become standard within the finance and economics literature.

Regarding testable predictions, the "investors underestimate the persistence of earnings" explanation of PEAD is ad hoc and offers few additional testable predictions, so it is difficult to run an empirical horse race to judge our model versus it. However, limited attention theory is broad and provides additional testable predictions, as shown by Hirshleifer, Lim, and Teoh (2011). Hence, rather than relying on personal priors and tastes regarding which set of assumptions one prefers, researchers can compare the models by testing the new empirical predictions. Prediction 3 can be used to test our theory against limited attention, since limited attention models predict PEAD regardless of whether firms' earnings are (ex ante) expected to be persistent or not (Hirshleifer, Lim, and Teoh (2011)).[13]

There are at least two avenues for future research. First, researchers can develop a method for determining (ex ante) whether a firm's earnings are likely to be persistent or not. Prediction 3 says that for such earnings announcements, we should observe post-earnings announcement reversals. Sorting firms on their past earnings persistence will not suffice; in unreported empirical analysis, we found that firms earnings are likely to be persistent even among firms whose past earnings tended to reverse. Firms' past use of accruals may be useful in identifying firms whose future earnings are likely to reverse. Second, more research can be done on determining how investors actually learn from price. Currently, RE and DO models are both standard in the literature despite the fact that they make polar opposite assumptions about investors' learning from price. Our models show that a better understanding of how investors learn from price might lead to a better understanding of why prices drift in the direction of earnings surprises.

they dismiss the possibility that other investors analyzed the information, i.e., if investors do not observe the earnings announcement and they treat the price as uninformative.

[13] Prediction 3 also distinguishes our theory from earlier behavioral theories of price drift following public announcements, e.g., Daniel, Hirshleifer, and Subrahmanyam (1998).

A Proofs

Proof. (**Lemma 1**)

A complication with our framework is that at certain times, some investors fully learn next period's earnings realizations (if they learn from the equilibrium price), whereas other investors only learn one of next period's earnings components (if they do not learn from the equilibrium price). Nevertheless, with Q and ℓ arbitrarily small, the equilibrium price must be arbitrarily close to the average of all traders' subjective expectations for the liquidating dividend.

Let U_t (I_t) denote the group of relatively uninformed (informed) investors at time t.[14] Let $\mathbf{E}_{U,t}[D_T]$ and $\sigma_{U,t}$ denote the U_t investors' time t conditional expectation of D_T. Let $\mathbf{E}_{I,t}[D_T]$ and $\sigma_{I,t}$ be defined analogously, and let κ_t be defined as

$$\kappa_t = 1 - \frac{\sigma_{I,t}}{\sigma_{U,t}}$$

(so that $\sigma_{I,t} = (1-\kappa_t)\sigma_{U,t}$).

Letting Q denote the per capita supply of the asset and α denote investors' risk aversion coefficient, market clearing implies

$$\frac{\mathbf{E}_{U,t}[D_T] - P_t}{\alpha\sigma_{U,t}} + \frac{\mathbf{E}_{I,t}[D_T] - P_t}{\alpha\sigma_{I,t}} = Q, \tag{35}$$

i.e.,

$$P_t = \frac{(1-\kappa_t)\mathbf{E}_{U,t}[D_T] + \mathbf{E}_{I,t}[D_T]}{2-\kappa_t} - \frac{\alpha(1-\kappa_t)\sigma_{U,t}Q}{2-\kappa_t}. \tag{36}$$

Since the $\frac{\alpha(1-\kappa_t)\sigma_{U,t}Q}{2-\kappa_t}$ term obviously vanishes as $Q \to 0$, it suffices to show that as $\ell \to 0$, $\kappa_t \to 0$.

Consider the transition matrix for one of the earnings components, e^A or e^B. The two cases are symmetric, and for notational convenience, we will drop the A/B terminology and simply refer to the component as e. The transition matrix for the earnings component can be expressed:

$$P = \begin{pmatrix} p & 1-p \\ 1-p & p \end{pmatrix}, \tag{37}$$

P^n can be expressed as $P^n = A + q^n B$, where $A = (1/2, 1/2; 1/2, 1/2)$ and $B = (1/2, -1/2; -1/2, 1/2)$.

[14] If all investors are equally informed at time t, randomly assign investors to U_t and I_t.

(See, e.g., Dekking and Kong (2011).) It follows that

$$P^n = \begin{pmatrix} \frac{1+q^n}{2} & \frac{1-q^n}{2} \\ \frac{1-q^n}{2} & \frac{1+q^n}{2} \end{pmatrix} \tag{38}$$

It is easily verified that

$$\begin{aligned} E[e_{t+n}|e_t] &= q^n e_t & (39) \\ var[e_{t+n}|e_t] &= \left(\frac{1+q^n}{2}e_t^2 + \frac{1-q^n}{2}e_t^2\right) - (q^n e_t)^2 & (40) \\ &= (1-q^{2n})e_t^2 & (41) \end{aligned}$$

It can be verified that for $m \geq n$,

$$Cov(e_{t+n}, e_{t+m}|e_t) = e_t^2 q^{m-n}(1-q^{2n}) \tag{42}$$

For some scalar $\tau > t$, let $V_\tau = var[e_{t+1} + ... + e_\tau|e_t]$. Note that V_τ represents the conditional variance of the unrealized earnings from $t+1$ to τ *assuming the investors cannot infer next period's earnings given this period's earnings.* It can be shown that V_τ can be expressed

$$V_\tau = V_{\tau-1} + (1-q^{2(\tau-t)})e_t^2 + C\left(q - q^\tau - q^{\tau+1} + q^{2\tau}\right) \tag{43}$$

where $C = 2e_t^2 \frac{q^{-t}}{1-q}$.

Iterating from $t+2$ to τ,

$$\begin{aligned} V_\tau &= V_{t+1} + e_t^2 \left\{(\tau-t-1) - q^2 \frac{1-q^{2(\tau-t-1)}}{1-q^2}\right\} \\ &+ C\left\{(\tau-t-1)q - q^{t+2}\frac{1-q^{\tau-t-1}}{1-q} - q^{t+3}\frac{1-q^{\tau-t-1}}{1-q} + (q^2)^{t+2}\frac{1-(q^2)^{\tau-t-1}}{1-(q^2)}\right\} \end{aligned} \tag{44}$$

where $V_{t+1} = (1-q^2)e_t^2$ and $C = 2e_t^2 \frac{q^{-t}}{1-q}$.

Now let $V_\tau^I = var[e_{t+1} + ... + e_\tau|e_{t+1}]$. Note that V_τ^I represents the conditional variance of the earnings from $t+1$ to τ *assuming the investors are able to infer next period's earnings given this*

period's earnings. It clearly follows that

$$\begin{aligned} V_\tau^I &= V_{\tau-1} && (45)\\ &= V_{t+1} + e_t^2\left\{(\tau-t-2) - q^2\frac{1-q^{2(\tau-t-2)}}{1-q^2}\right\} \\ &+ C\left\{(\tau-t-2)q - q^{t+2}\frac{1-q^{\tau-t-2}}{1-q} - q^{t+3}\frac{1-q^{\tau-t-2}}{1-q} + (q^2)^{t+2}\frac{1-(q^2)^{\tau-t-2}}{1-(q^2)}\right\} && (46)\end{aligned}$$

where $V_{t+1} = (1-q^2)e_t^2$ and $C = 2e_t^2\frac{q^{-t}}{1-q}$.

As $\tau \to \infty$, V_τ and V_τ^I both increase at a rate of $e_t^2 + Cq = e_t^2\left(1 + 2\frac{q^{-t+1}}{1-q}\right)$. Moreover,

$$\begin{aligned} V_\tau - V_\tau^I &= e_t^2 + \frac{q^2}{1-q^2}\left(q^{2(\tau-t-1)} - q^{2(\tau-t-2)}\right) && (47)\\ &+ C\left\{q + \frac{q^{t+2}}{1-q}(q^{\tau-t-1} - q^{\tau-t-2}) + \frac{q^{t+3}}{1-q}(q^{\tau-t-1} - q^{\tau-t-2}) + \frac{q^{2(t+2)}}{1-q^2}\left(q^{2(\tau-t-2)} - q^{2(\tau-t-1)}\right)\right\}.\end{aligned}$$

Note that as $\tau \to \infty$, $V_\tau - V_\tau^I \to e_t^2 + Cq$. In particular, *it is a bounded sequence.*

Now suppose the asset liquidates with probability ℓ each period, and let T denote the liquidation time. Let $S_T = e_{t+1} + \ldots + e_T$, and let $V_T = var[S_T|e_t]$. Applying the law of total expectation,

$$\begin{aligned} V_T &= \mathbf{E}[(S_T - \mathbf{E}[S_T])^2] && (48)\\ &= \mathbf{E}\left\{(S_T - \mathbf{E}[S_T])^2|T = t+1\right\}\mathbf{P}(T = t+1) + \mathbf{E}\left\{(S_T - \mathbf{E}[S_T])^2|T = t+2\right\}\mathbf{P}(T = t+2) + \ldots \\ &= \sum_{k=1}^{\infty}(1-\ell)^{k-1}\ell V_{t+k}. && (49)\end{aligned}$$

Similarly,

$$V_T^I = \sum_{k=1}^{\infty}(1-\ell)^{k-1}\ell V_{t+k}^I. \qquad (50)$$

Hence,

$$V_T - V_T^I = \sum_{k=1}^{\infty}(1-\ell)^{k-1}\ell(V_{t+k} - V_{t+k}^I) \qquad (51)$$

It suffices to show that as $\ell \to 0$, $\frac{V_T^I}{V_T} \to 1$, or equivalently, $\frac{V_T - V_T^I}{V_T} \to 0$. But this follows from the fact that as $\tau \to \infty$, V_τ and V_τ^I increase without bound, whereas $V_\tau - V_\tau^I$ converges to $e_t^2 + Cq$, completing the proof.

□

Proof. (**Proposition 1**)

The following lemma states that if agents incorporate the equilibrium price in their information set, they can infer the following period's earnings, e_{t+1}.

Lemma 3. *For every agent i and time t,* $\mathbf{E}[D_T|e_1,\ldots,e_t,e^i_{t+1},P^*_1,\ldots,P^*_t] = \mathbf{E}[D_T|e_1,\ldots,e_{t+1}]$.

To complete the proof, we must verify that P^*_t clears the market given traders' demands. From (36), it suffices to show that P^*_t is the average of traders' posterior expectations for D_T given the information they use in their information set, i.e., that

$$P^*_t = \frac{\mathbf{E}[D_T|I_{At}] + \mathbf{E}[D_T|I_{Bt}]}{2}. \tag{52}$$

Using Lemma 3 to justify (54), we express investors' posterior subjective expectations for D_T based on the information they condition on:

$$\begin{aligned}
\mathbf{E}[D_T|I^{\mathrm{DO}}_{it}] &= \mathbf{E}[D_T|e_1,\ldots,e_t,e^i_{t+1}] \\
&= D_t + \frac{q}{1-q}\left(e_t - e^i_t\right) + \left(\frac{1}{1-q}\right)e^i_{t+1}. \qquad (53)\\
\mathbf{E}[D_T|I^{\mathrm{RE}}_{it}] &= \mathbf{E}[D_T|e_1,\ldots,e_{t+1}] \\
&= D_t + \left(\frac{1}{1-q}\right)e_{t+1}. \qquad (54)
\end{aligned}$$

<u>*Case 1:*</u> $e_{t+1} = \pm 2\sigma$.

Then $P^*_t = P^{\mathrm{DO}}_t$, all traders observe directionally similar private information at time t: $e^i_{t+1} = e^j_{t+1}$ for all investors i and j, and $\mathrm{sgn}(e^i_{t+1}) = \mathrm{sgn}(P^{\mathrm{DO}}_t - P^e_t)$. It follows that $I_{it} = I^{\mathrm{DO}}_{it}$, i.e., investors do not update their beliefs upon observing the equilibrium price, and it can easily be verified using (53) that P^*_t is the average of investors' ex post expectations of D_T.

<u>*Case 2:*</u> $(e_t, e_{t+1}) = (2\sigma, 0)$ and $p > 0.5$.

Without loss of generality, $e^A_{t+1} = \sigma$ and $e^B_{t+1} = -\sigma$. It is easily verified that $P^*_t < P^e_t$. Hence, investors who infer e^A_{t+1} incorporate price into their information set, whereas investors who infer e^B_{t+1} do not. Recalling (53) and (54), investors' average posterior beliefs about the expected liquidating

dividend is given by

$$\begin{aligned}
\overline{\mathbf{E}[D_T|I_{it}]} &= \frac{\mathbf{E}[D_T|I_{At}^{\mathrm{RE}}] + \mathbf{E}[D_T|I_{Bt}^{\mathrm{DO}}]}{2} \\
&= D_t - \frac{\sigma}{2} \\
&= D_t - \frac{e_t}{4} \\
&= P_t^*, \qquad (55)
\end{aligned}$$

where the last equality follows from (15) and (16).

The other cases are easily verified by analogous reasoning.

□

Proof. (**Lemma 3**)

It trivially follows from (13), (15), and (16) that the time t price, P_t^*, is completely determined by the cumulative earnings through time t (D_t) and the time t and time $t+1$ earnings, e_t and e_{t+1}. It suffices to show that for any (D_t, e_t, e_{t+1}^A), the equilibrium price P_t^* is injective (or "one-to-one") when expressed as a function of e_{t+1}^B. Letting $P_t^*|_{e_{t+1}^B=-\sigma}$ and $P_t^*|_{e_{t+1}^B=\sigma}$ denote the equilibrium prices as a function of e_{t+1}^B, we must show that for all (D_t, e_t, e_{t+1}^A), $P_t^*|_{e_{t+1}^B=-\sigma} \neq P_t^*|_{e_{t+1}^B=\sigma}$. Without loss of generality, $e_{t+1}^A = \sigma$.

<u>*Case 1:*</u> $p = 0.5$.

If $p = 0.5$, then $P_t^* = P_t^{\mathrm{DO}}$, and it trivially follows from (13) that $P_t^{\mathrm{DO}}|_{e_{t+1}^B=-\sigma} \neq P_t^{\mathrm{DO}}|_{e_{t+1}^B=\sigma}$.

<u>*Case 2:*</u> $e_t \in \{-2\sigma, 2\sigma\}$ and $p \in \left(\frac{1}{2}, 1\right)$.

Recalling (16) and our assumption that $e_{t+1}^A = \sigma$,

$$\begin{aligned}
P_t^*|_{e_{t+1}^B=-\sigma} &= P_t'|_{e_{t+1}^B=-\sigma} \\
&= D_t - \frac{e_t}{4}. \qquad (56) \\
P_t^*|_{e_{t+1}^B=\sigma} &= P_t^{\mathrm{DO}}|_{e_{t+1}^B=\sigma} \\
&= D_t + \frac{1}{2}\left(\frac{q}{1-q}\right) e_t + \frac{1}{2}\left(\frac{1}{1-q}\right)(2\sigma). \qquad (57)
\end{aligned}$$

It is easily verified that $\left|\frac{1}{2}\left(\frac{q}{1-q}\right) e_t + \frac{1}{2}\left(\frac{1}{1-q}\right)(2\sigma)\right| > \left|\frac{e_t}{4}\right|$, so $P_t^*|_{e_{t+1}^B=-\sigma} \neq P_t^*|_{e_{t+1}^B=\sigma}$.

<u>*Case 3:*</u> $e_t \in \{-2\sigma, 2\sigma\}$ and $p \in \left(0, \frac{1}{2}\right)$.

Recalling (16) and our assumption that $e^A_{t+1} = \sigma$,

$$\begin{aligned} P^*_t|_{e^B_{t+1}=-\sigma} &= P'_t|_{e^B_{t+1}=-\sigma} \\ &= D_t + \left(\frac{1+q}{4(1-q)}\right) e_t. \qquad (58) \\ P^*_t|_{e^B_{t+1}=\sigma} &= P^{\mathrm{DO}}_t|_{e^B_{t+1}=\sigma} \\ &= D_t + \frac{1}{2}\left(\frac{q}{1-q}\right) e_t + \frac{1}{2}\left(\frac{1}{1-q}\right)(2\sigma). \qquad (59) \end{aligned}$$

Hence,

$$\begin{aligned} P^*_t|_{e^B_{t+1}=\sigma} - P^*_t|_{e^B_{t+1}=-\sigma} &= -\frac{1}{4}e_t + \frac{1}{2}\left(\frac{1}{1-q}\right)(2\sigma) \\ &> 0. \qquad (60) \end{aligned}$$

<u>*Case 4:*</u> $e_t = 0$ and $p \neq 0.5$.

In this scenario,

$$\begin{aligned} P^*_t|_{e^B_{t+1}=\sigma} - P^*_t|_{e^B_{t+1}=-\sigma} &= P^{\mathrm{DO}}_t|_{e^B_{t+1}=\sigma} - P^{\mathrm{DO}}_t|_{e^B_{t+1}=-\sigma} \\ &= \frac{1}{2}\left(\frac{1}{1-q}\right)(2\sigma) \\ &> 0. \qquad (61) \end{aligned}$$

□

Proof. (**Lemma 2**) Note that an earnings component at any time $t+j$ can be expressed

$$\begin{aligned} e^i_{t+j} &= \varepsilon^i_{t+j} + \lambda\varepsilon^i_{t+j-1}\mathbf{1}_{\{e^i_{t+j-1} \text{ is high quality}\}} + \lambda^2\varepsilon^i_{t+j-2}\mathbf{1}_{\{e^i_{t+j-2} \text{ and } e^i_{t+j-1} \text{ are high quality}\}} + \ldots + \\ &\quad \lambda^{j-1}\varepsilon^i_{t+1}\mathbf{1}_{\{e^i_{t+1},\ldots,e^i_{t+j-1} \text{ are all high quality}\}} + \lambda^j e^i_t \mathbf{1}_{\{e^i_t,\ldots,e^i_{t+j-1} \text{ are all high quality}\}}. \qquad (62) \end{aligned}$$

Since ε^i_j and ε^i_k are independent for $j \neq k$, it follows from (62) that

$$\begin{aligned} \mathrm{Var}(e^i_{t+j}|e^i_t) &= \frac{1}{2\tau_e}\sum_{k=0}^{j-1}(\lambda^2 p)^k + \lambda^{2j}e^{i2}_t p^j(1-p^j) \\ &= \left(\frac{1}{2\tau_e}\right)\left(\frac{1-(\lambda^2 p)^j}{1-\lambda^2 p}\right) + \lambda^{2j}e^{i2}_t p^j(1-p^j) \qquad (63) \end{aligned}$$

and

$$\mathrm{Var}(e^i_{t+j}|e^i_t, \mathbf{1}_{\{e^i_t \text{ is high quality}\}}) = \left(\frac{1-(\lambda^2 p)^j}{2\tau_e(1-\lambda^2 p)}\right) + \lambda^{2j} e^{i2}_t p^{j-1}(1-p^{j-1}) * \mathbf{1}_{\{e^i_t \text{ is high quality}\}} \quad (64)$$

For $n > m$, it follows from (62) and the fact that ε^i_j and ε^i_k are independent for $j \neq k$ that

$$\begin{aligned} \mathrm{Cov}(e^i_{t+m}, e^i_{t+n}|e^i_t) &= \mathrm{Cov}(e^i_{t+m}, \lambda^{n-m} e^i_{t+m} \mathbf{1}_{\{e^i_{t+m}, e^i_{t+m+1}, \ldots, e^i_{t+n} \text{ are all high quality}\}}) \\ &= (\lambda p)^{n-m} \mathrm{Var}(e^i_{t+m}|e^i_t) \end{aligned} \quad (65)$$

and

$$\mathrm{Cov}(e^i_{t+m}, e^i_{t+n}|e^i_t, \mathbf{1}_{\{e^i_t \text{ is high quality}\}}) = \mathrm{Cov}(e^i_{t+m}, e^i_{t+n}|e^i_t). \quad (66)$$

Since

$$\mathrm{Var}(\sum_{j=1}^{n} e^i_{t+j}|e^i_t) = \sum_{j=1}^{n} \mathrm{Var}\left(e^i_{t+j}|e^i_t\right) + 2\sum_{k=1}^{n-1}\sum_{j=k+1}^{n} \mathrm{Cov}(e^i_{t+k}, e^i_{t+j}|e^i_t), \quad (67)$$

and the same equation holds when we condition everywhere on both e^i_t and $\mathbf{1}_{\{e^i_t \text{ is high quality}\}}$ instead of just e^i_t, it follows that

$$\begin{aligned} \mathrm{Var}(\sum_{j=1}^{n} e^i_{t+j}|e^i_t) - \mathrm{Var}(\sum_{j=1}^{n} e^i_{t+j}|e^i_t, \mathbf{1}_{\{e^i_t \text{ is high quality}\}}) &= \sum_{j=1}^{n} \lambda^{2j} e^{i2}_t p^{j-1} * \Big\{p(1-p^j) - \\ &\quad (1-p^{j-1}) * \mathbf{1}_{\{e^i_t \text{ is high quality}\}}\Big\} \\ &< \sum_{j=1}^{n} \lambda^{2j} e^{i2}_t, \end{aligned} \quad (68)$$

which is clearly a bounded series. Hence, as $n \to \infty$, the difference between the variance of the liquidating dividend conditional on the relatively informed investors (i.e., the arbitrageurs and the investors who learn earnings quality from price) and the variance of the liquidating dividend conditional on the relatively uninformed investors (i.e., the non-arbitrageurs who do not learn earnings quality from price) is bounded, whereas the two variances themselves increase without bound as $n \to \infty$. The lemma now follows analogously to the arguments used to prove Lemma 1 for the baseline model.

□

Proof. (**Proposition 2**) The existence of an equilibrium is guaranteed within the proof of Lemma 2. We need to verify that I_{it}^* (defined in (27)) is the information set that each group of investors conditions on when forming their beliefs about $\mathbf{E}[D_T]$ when the pricing function is given by P_t^* in (28), and when investors are assumed to learn from price as described in (11). It suffices to show that for $i = A, B$, the following two equalities are equivalent, i.e., one holds if and only if the other holds:

$$\begin{aligned} \operatorname{sgn}(\mathrm{Priv}_{it}) &= \operatorname{sgn}\left(\sum_{j\in\{A,B,R\}} \mu_{jt}\mathrm{Priv}_{jt}\right) && (69)\\ \operatorname{sgn}(\mathrm{Priv}_{it}) &= \operatorname{sgn}(P_t^* - P_t^e), && (70) \end{aligned}$$

i.e., it suffices to show that Condition 1 (on page 18) is satisfied. But this follows from the fact that

$$\begin{aligned} \operatorname{sgn}\left(\sum_{j\in\{A,B,R\}} \mu_{jt}\mathrm{Priv}_{jt}\right) &= \operatorname{sgn}\left(\sum_{j\in\{A,B,R\}} \mu_{jt}\left(\mathbf{E}[D_T|I_{jt}^{\mathrm{DO}}] - \mathbf{E}[D_T|I_t^e]\right)\right) && (71)\\ &= \operatorname{sgn}\left(\sum_{j\in\{A,B,R\}} \mu_{jt}\left(\mathbf{E}[D_T|I_{jt}^*] - \mathbf{E}[D_T|I_t^e]\right)\right) && (72)\\ &= \operatorname{sgn}\left(P_t^* - P_t^e\right), && (73) \end{aligned}$$

where (72) follows from the fact that $\mathbf{E}[D_T|I_{it}^*] = \mathbf{E}[D_T|I_{it}^{\mathrm{DO}}]$ *unless* $\operatorname{sgn}(\mathrm{Priv}_{it}) \neq \operatorname{sgn}\left(\sum_{j\in\{A,B,R\}} \mu_{jt}\mathrm{Priv}_{jt}\right)$, in which case

$$\operatorname{sgn}\left(\mathbf{E}[D_T|I_{it}^*] - \mathbf{E}[D_T|I_{it}^{\mathrm{DO}}]\right) = \operatorname{sgn}\left(\mathbf{E}[D_T|I_t^e] - \mathbf{E}[D_T|I_{it}^{\mathrm{DO}}]\right), \quad (74)$$

and (73) follows from the definition of P_t^* in (28) and the fact that $\sum_{j\in\{A,B,R\}} \mu_{jt} = 1$.

□

B Notes on Construction of Figures

To construct Figure 1, we need to compute the joint distribution trading volume and changes in the equilibrium price, ΔP_t^*.

Lemma 4. *Let E_t be defined as the triple (e_{t-1}, e_t, e_{t+1}). If $p > .5$, then*

$$\Delta P_t^* = \begin{cases} \frac{1}{2}\left\{\left(\frac{-q}{1-q}\right)e_{t-1} + e_t + \left(\frac{1}{1-q}\right)e_{t+1}\right\} & \text{if } E_t = (\pm 2\sigma, \pm 2\sigma, \pm 2\sigma), (0,0,0), (0,0,\pm 2\sigma), \text{ or } (0, \pm 2\sigma, \pm 2\sigma) \\ -\frac{1}{2}\left(\frac{q}{1-q}\right)e_{t-1} + \frac{1}{4}\left(\frac{1-3q}{1-q}\right)e_t & \text{if } E_t = (\pm 2\sigma, \pm 2\sigma, 0) \text{ or } (0, \pm 2\sigma, 0) \\ \left(\frac{1}{4}\right)e_{t-1} + \frac{1}{2}\left(\frac{1}{1-q}\right)e_{t+1} & \text{if } E_t = (\pm 2\sigma, 0, \pm 2\sigma) \text{ or } (\pm 2\sigma, 0, 0). \end{cases}$$

If $p < .5$, then

$$\Delta P_t^* = \begin{cases} \frac{1}{2}\left\{\left(\frac{-q}{1-q}\right)e_{t-1} + e_t + \left(\frac{1}{1-q}\right)e_{t+1}\right\} & \text{if } E_t = (\pm 2\sigma, \pm 2\sigma, \pm 2\sigma), (0,0,0), (0,0,\pm 2\sigma), \text{ or } (0, \pm 2\sigma, \pm 2\sigma) \\ -\frac{1}{2}\left(\frac{q}{1-q}\right)e_{t-1} + \left(\frac{3}{4}\right)e_t & \text{if } E_t = (\pm 2\sigma, \pm 2\sigma, 0) \text{ or } (0, \pm 2\sigma, 0) \\ -\frac{1}{4}\left(\frac{1+q}{1-q}\right)e_{t-1} + \frac{1}{2}\left(\frac{1}{1-q}\right)e_{t+1} & \text{if } E_t = (\pm 2\sigma, 0, \pm 2\sigma) \text{ or } (\pm 2\sigma, 0, 0). \end{cases}$$

All that remains is computing the probability distribution of (e_{t-1}, e_t, e_{t+1}). Recall that total earnings ($e = e^A + e^B$) follows a Markov chain with transition probabilities given by P in (37). Let s_1, s_2, and s_3 be the states $\{e = -2\sigma\}$, $\{e = 0\}$, and $\{e = 2\sigma\}$. If $\mathbf{u}$ represents an arbitrary starting probability distribution over the states s_1, s_2, and s_3, then the distribution after n steps is given by

$$\mathbf{u}^{(n)} = \mathbf{u}P^n. \tag{75}$$

It is straightforward to verify that the earnings process is ergodic and regular, so $P^n \to W$, where all the rows of W are the same (call them $\mathbf{w}$), and $\mathbf{w}$ is the steady state probability distribution: $\mathbf{w}P = \mathbf{w}$. (And $\mathbf{w}$ is the *unique* probability distribution satisfying $\mathbf{w}P = \mathbf{w}$.)

It is straightforward to verify that $\mathbf{w} = (0.25, 0.5, 0.25)$ satisfies $\mathbf{w}P = \mathbf{w}$, so that $\mathbf{w} \equiv (0.25, 0.5, 0.25)$ is the steady state probability distribution for e—regardless of where the process is at time t, the conditional probability distribution of where the chain will be at time $t + k$ is $\mathbf{w} = (0.25, 0.5, 0.25)$ for sufficiently large k. It follows that to compute the distribution of (e_{t-1}, e_t, e_{t+1}), we can simply use $(0.25, 0.5, 0.25)$ as the probability distribution for e_{t-1} over the states $(-2\sigma, 0, 2\sigma)$. From here, the distribution of e_t conditional on e_{t-1}, and then the distribution of e_{t+1} conditional on (e_t, e_{t-1}) can easily be computed from the transition matrix, P, defined in (37).

References

Ball, R., and P. Brown (1968): "An Empirical Evaluation of Accounting Income Numbers," *Journal of Accounting Research*, 6, 159–178.

Banerjee, S. (2011): "Learning from Prices and the Dispersion in Beliefs," *The Review of Financial Studies*, 24, 3025–3068.

Banerjee, S., and B. Green (2013): "Learning Whether Other Traders are Informed," *Working Paper*.

Banerjee, S., R. Kaniel, and I. Kremer (2009): "Price Drift as an Outcome of Differences in Higher-Order Beliefs," *The Review of Financial Studies*, 22, 3707–3734.

Barberis, N., A. Shleifer, and R. Vishny (1998): "A Model of Investor Sentiment," *Journal of Financial Economics*, 49, 307–343.

Benjamin, D., M. Rabin, and C. Raymond (2013): "A Model of Non-Belief in the Law of Large Numbers," *Working Paper*.

Bernard, V., and J. Thomas (1990): "Evidence that Stock Prices do not fully Reflect the Implications of Current Earnings for Future Earnings," *Journal of Accounting and Economics*, 13, 305–340.

Daniel, K., D. Hirshleifer, and A. Subrahmanyam (1998): "Investor Psychology and Security Market Under- and Overreactions," *The Journal of Finance*, 53, 1839–1885.

——— (2001): "Overconfidence, Arbitrage, and Equilibrium Asset Pricing," *The Journal of Finance*, 56, 921–965.

Dawes, R., and M. Mulford (1996): "The False Consensus Effect and Overconfidence: Flaws in Judgment or Flaws in How We Study Judgment?," *Organizational Behavior and Human Decision Processes*, 65, 202–211.

Dekking, M., and D. Kong (2011): "Multimodality of the Markov Binomial Distribution," *Journal of Applied Probability*, 48, 938–953.

DellaVigna, S., and J. Pollet (2009): "Investor Inattention and Friday Earnings Announcements," *The Journal of Finance*, 64, 709–749.

Egan, D., C. Merkle, and M. Weber (2010): "The Beliefs of Others - Naive Realism and Investment Decisions," *Working Paper*.

Engelmann, D., and M. Strobel (2000): "The False Consensus Effect Disappears if Representative and Monetary Incentives are Given," *Experimental Economics*, 3, 241–260.

——— (2012): "Deconstruction and Reconstruction of an Anomaly," *Games and Economic Behavior*, 76, 678–689.

Gervais, S., J. Heaton, and T. Odean (2011): "Overconfidence, Compensation Contracts, and Capital Budgeting," *The Journal of Finance*, 66, 1735–1777.

Gervais, S., and T. Odean (2001): "Learning to be Overconfident," *The Review of Financial Studies*, 14, 1–27.

Hirshleifer, D. (2001): "Investor Psychology and Asset Pricing," *The Journal of Finance*, 56, 1533–1597.

Hirshleifer, D., S. S. Lim, and S. H. Teoh (2009): "Driven to Distraction: Extraneous Events and Underreaction to Earnings News," *The Journal of Finance*, 64, 2289–2325.

——— (2011): "Limited Investor Attention and Stock Market Misreactions to Accounting Information," *Review of Asset Pricing Studies*, 1, 35–73.

Hirshleifer, D., and S. H. Teoh (2003): "Limited Attention, Information Disclosure, and Financial Reporting," *Journal of Accounting and Economics*, 36, 337–386.

Hong, H., and J. Stein (1999): "A Unified Theory of Underreaction, Momentum Trading, and Overreaction in Asset Markets," *The Journal of Finance*, 54, 2143–2184.

——— (2007): "Disagreement and the Stock Market," *Journal of Economic Perspectives*, 21, 109–128.

Katz, D., and F. Allport (1931): *Students' Attitudes*. Syracuse: Craftsman Press.

Krueger, J. (1998): *Advances in Experimental Social Psychology*vol. 30, pp. 163–240. Academic Press.

——— (2000): *Handbook of Social Comparison: Theory and Research*pp. 323–351. Plenum Publishers.

Krueger, J., and J. Zeiger (1993): "Social Categorization and the Truly False Consensus Effect," *Journal of Personality and Social Psychology*, 65, 670–680.

Kwasnica, A., R. Velthuis, and J. Williams (2013): "Did You See What I Saw? Interpreting Others' Forecasts When Their Information is Unknown," *Working Paper*.

Livnat, J., and R. Mendenhall (2006): "Comparing Post-Earnings Announcement Drift for Surprises Calculated from Analyst and Time-Series Forecasts," *Journal of Accounting Research*, 44, 177–205.

Malmendier, U., and G. Tate (2005): "CEO Overconfidence and Corporate Investment," *The Journal of Finance*, 60, 2661–2700.

Odean, T. (1998): "Volume, Volatility, Price, and Profit When All Traders Are Above Average," *The Journal of Finance*, 53, 1887–1934.

Rabin, M. (2002): "Inference by Believers in the Law of Small Numbers," *The Quarterly Journal of Economics*, 117, 775–816.

Rabin, M., and D. Vayanos (2010): "The Gambler's and Hot-Hand Fallacies: Theory and Applications," *The Review of Economic Studies*, 77, 730–778.

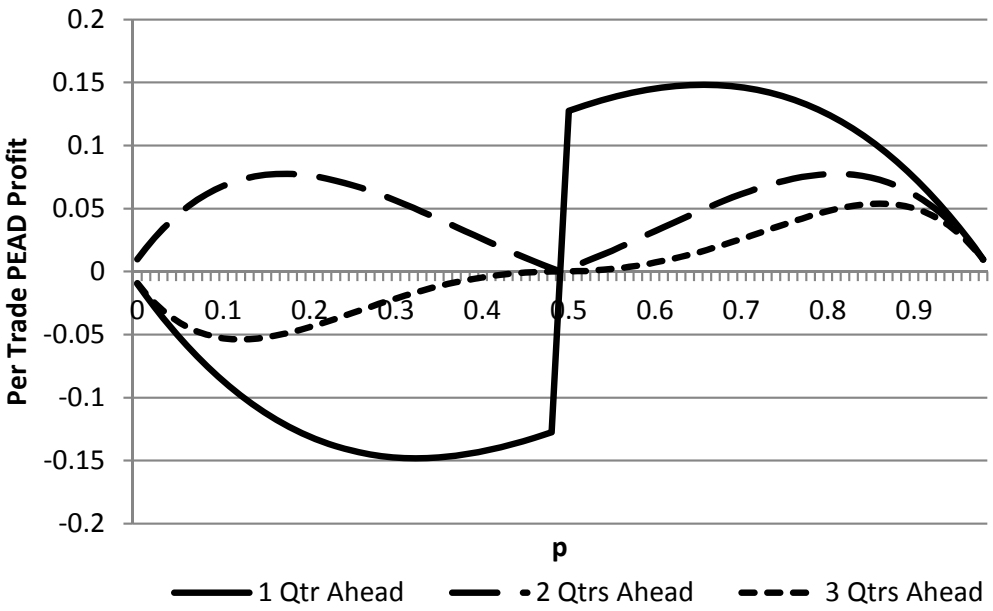
Ross, L., D. Greene, and P. House (1977): "The "False Consensus Effect": An Egocentric Bias in Social Perception and Attribution Processes," *Journal of Experimental Social Psychology*, 13, 279–301.

Scheinkman, J., and W. Xiong (2003): "Overconfidence and Speculative Bubbles," *Journal of Political Economy*, 111, 1183–1219.

Welch, I. (2000): "Views of Financial Economists on the Equity Premium and on Professional Controversies," *Journal of Business*, 73, 501–537.

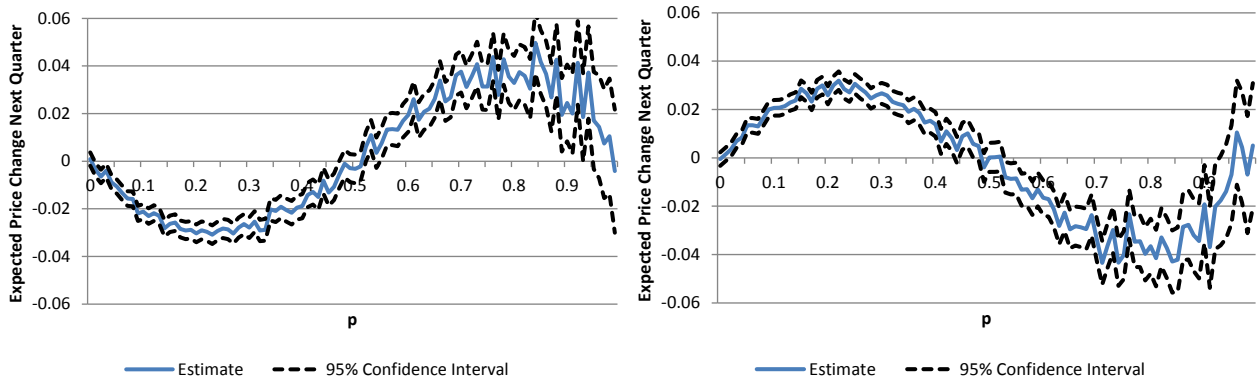
Williams, J. (2013): "Financial Analysts and the False Consensus Effect," *Journal of Accounting Research*, 51, 855–907.

Figure 1: PEAD Profits around Subsequent Earnings Announcements, by Earnings Persistence (Baseline Model)



We plot average (round-trip transaction) PEAD profits around earnings announcements 1 quarter, 2 quarters and 3 quarters ahead in our baseline model, by (ex ante) earnings persistence parameter, p. That is, we plot PEAD_1, PEAD_2, and PEAD_3 defined in (17) on page 13, by p. The parameter σ is normalized to 1 in this figure.

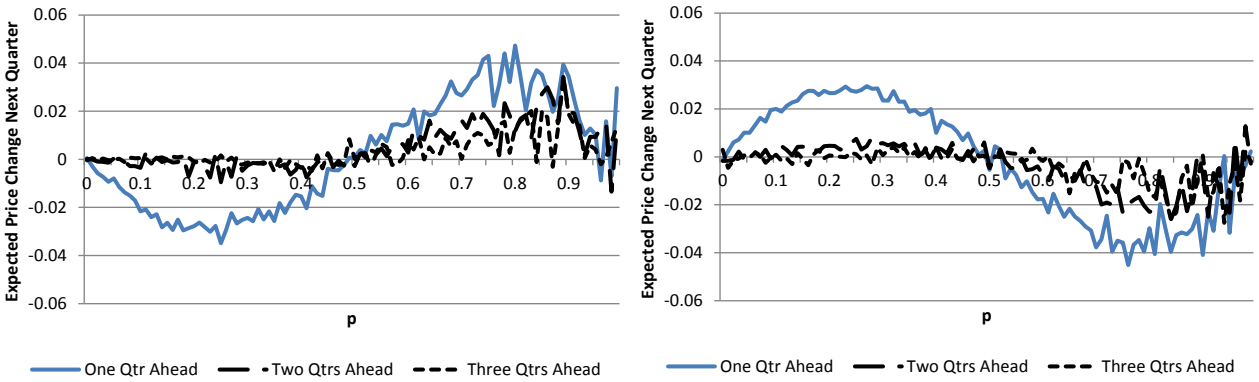
Figure 2: Price Changes in the Quarter following High and Low Earnings Surprises, by Earnings Persistence (Extended Model)



(a) High Earnings Surprises (b) Low Earnings Surprises

We run simulations to estimate the expected future price changes, $\mathbf{E}[P^*_{t+1} - P^*_t]$, in our extended model following high earnings surprises (Figure 2a) and low earnings surprises (Figure 2b). High (low) earnings surprises consist of the decile with the highest (lowest) earnings surprises. One million simulations were conducted for each value of $p \in \{0, 0.01, \ldots, 0.99\}$, where p measures the ex ante likelihood of an earnings component being high quality (i.e., likely to persist). The parameters λ, τ_e, and $\overline{\rho}$ were set to 0.9, 5, and 0.3, respectively. 95% confidence intervals are plotted in the dashed lines.

Figure 3: PEAD Profits around Subsequent Earnings Announcements, by Earnings Persistence (Extended Model)



(a) High Earnings Surprises (b) Low Earnings Surprises

We run simulations to estimate the expected future price changes, $\mathbf{E}[P^*_{t+j} - P^*_{t+j-1}]$ $(j = 1, 2, 3)$, in our extended model following high time t earnings surprises (Figure 3a) and low time t earnings surprises (Figure 3b). High (low) earnings surprises consist of the decile with the highest (lowest) earnings surprises. One million simulations were conducted for each value of $p \in \{0, 0.01, \ldots, 0.99\}$, where p measures the ex ante likelihood of an earnings component being high quality (i.e., likely to persist). The parameters λ, τ_e, and $\overline{\rho}$ were set to 0.9, 5, and 0.3, respectively.