Review for Final

Final: 8:00 am - 9:50 am 106 Osmond
Don't oversleep Our classroom

Material on Both Midterms
See previous notes (2/15 and 4/4)
Will use similar A-B-C breakdown

Definability (B-level)

- $A \subseteq \mathcal{O}_2 \times \ldots \times \mathcal{O}_2$ is a definable relation in $\mathcal{O}_2$ iff there is a formula $\phi(s,t)$.
For $\phi \equiv [a_1, \ldots, a_n] \leftrightarrow \langle a_1, \ldots, a_n \rangle \in A$

- Be able to write formula defining a relation

- Be able to write set of formulas defining a class of structures

- A point $a \in \mathcal{O}_2$ is definable in $\mathcal{O}_2$ iff $\langle a \rangle$ is definable.

- A function $f : \mathcal{O}_2^n \rightarrow \mathcal{O}_2$ is definable in $\mathcal{O}_2$ iff \( \{ \langle a_1, \ldots, a_n, b \rangle \mid s(a_1, \ldots, a_n) = b \} \) is definable.
Homomorphisms (B-level)

- Homomorphisms
- Isomorphisms (into and onto)
- Isomorphic $\equiv$
- Homomorphism Thm (A-level)
  - Statement (4 parts)
  - Proof
- Elementary equivalence $\equiv$
  - Know difference between $\equiv$ and $\equiv$

- Automorphisms
- Showing a relation is not definable by finding an automorphism which does not preserve it

Proof trees for first-order logic (B-level)

- Be able to use proof trees to prove a formula is valid
- Use proof trees to prove results of form $\varphi, \psi \vdash \theta$

- Proofs must follow the rules exactly, have each step labeled, and cancel hypotheses (labelling where hypothesis came from).
- I will prove a handout with the allowable rules.
Proof theory (B-level)

- Understand what $\Sigma \vdash \varphi$ means
- Know basic facts about $\Sigma \vdash \varphi$
  - Weakening: $\Sigma \vdash \varphi$ and $\Delta \vdash \Sigma \Rightarrow \Delta \vdash \varphi$
  - Finite character: $\Sigma \vdash \varphi \Rightarrow \exists \varphi' \text{ for some finite } \Sigma' \subseteq \Sigma$

- Deduction Theorem: $\Sigma, \varphi \vdash \psi \iff \Sigma \vdash \varphi \rightarrow \psi$
  (Know proof)

- $\Sigma \vdash \varphi \iff \Sigma \cup \varphi^2$ is inconsistent
  (Know proof)

Induction on proofs (A-level)

Substitution and substitutability (B-level)

- $\varphi^x_t$ know definition
- $t$ is substitutable for $x$ in $\varphi$ (Know def)
Soundness / Completeness / Compactness (B-level)

- **Soundness:**
  1. $\Sigma \vdash \varphi \implies \Sigma \vdash \varphi$
  2. $\Sigma$ satisfiable $\implies \Sigma$ consistent

- **Completeness:**
  1. $\Sigma \models \varphi \implies \Sigma \vdash \varphi$
  2. $\Sigma$ consistent $\implies \Sigma$ satisfiable

- **Compactness:**
  1. $\Sigma \models \varphi \implies$ there is some finite $\Sigma' \subseteq \Sigma$
     s.t. $\Sigma' \models \varphi$
  2. $\Sigma$ is finitely satisfiable $\implies \Sigma$ satisfiable

  Prove 1 $\iff$ 2

Proof of compactness using soundness/completeness/finite character

- Consistent / Inconsistent
- Satisfiable / Unsatisfiable
- Finitely Satisfiable

- $\Sigma$ is satisfiable $\iff$ $\Sigma \not\models \bot$
- $\Sigma \vdash \varphi$ $\iff$ $\Sigma \cup \neg \varphi$ is unsatisfiable

Given $\varphi$: either give proof tree (if valid)
  or countermodel (if not valid)
Soundness/Completeness/Compactness (A-level)

- Proof of soundness (Main ideas)
- Proof of completeness (Main ideas)

- Applications of compactness theorem (Similar to class and homework)