Homework 5: Part B

Due date: Wednesday February 24, 2016

- Complete this assignment on your own paper.
- Follow the rules for homework given as a handout on the first day of class.
- Start early and don’t be afraid to ask the instructor questions.
- Turn in on time. (However, if it must be late, I prefer correct and complete late homework over incorrect and incomplete timely homework.)

1 Some properties of $|=\$

#6. Let $\Sigma$ be a set of wffs, and let $\alpha$, $\beta$ and $\gamma$ be wffs. Then prove that the following hold:

(a) $\Sigma$ is unsatisfiable (not satisfiable) iff $\Sigma |= \bot$.
(b) If $\Sigma |= \alpha \lor \beta$, $\Sigma;\alpha |= \gamma$ and $\Sigma;\beta |= \gamma$, then $\Sigma |= \gamma$

Remark. Part (a) was something I mentioned in class as an exercise. Many of you used it on the exam. You should prove it. Recall $\bar{v}(\bot) = F$. Part (b) is part of the lemma used to prove the soundness theorem.

2 Induction on proofs

Prove the following by induction on proofs.

#7. (Finite character) Let $\Sigma$ be a set of wffs, and let $\varphi$ be a wff. Prove the following by induction on proofs: If $\Sigma \vdash \varphi$ then there is a finite subset $\Sigma_0 \subseteq \Sigma$ such that $\Sigma_0 \vdash \varphi$.

Remark: For induction on proofs there are 15 induction steps. I will not make you do all 15. Instead only do the base case (assumption rule) and the induction steps corresponding to the following seven rules: $\land I$, $\land E_1$, $\lor I$, $\lor E$, $\lor I_1$, $\lor E$, RAA. Then say “The rest follow similarly.” In the future I will let you get by with even less steps.

Hint: All the big work in shrinking down $\Sigma$ to a finite set $\Sigma_0$ is done at the base step. The induction steps just carry through the changes recursively. (You will also need to use weakening in your induction steps.)