

- ▶ JASON RUTE, *The computability of martingale convergence*.
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Martingales are an important tool in probability and analysis. This talk will be about the computability of martingale convergence.

A martingale, which can be thought of a fair betting strategy, is a sequence $(f_n)_{n \in \mathbb{N}}$ of integrable functions satisfying the property $\mathbb{E}[f_{n+1} \mid f_0, \dots, f_n] = f_n$. Doob showed that a martingale $(f_n)_{n \in \mathbb{N}}$ converges almost-surely if $\sup_n \|f_n\|_{L^1}$ is bounded.

Firstly, we may ask the usual questions from computable and constructive analysis. Given a martingale $(f_n)_{n \in \mathbb{N}}$: (1) Is the limit of $(f_n)_{n \in \mathbb{N}}$ computable? (2) If the limit is computable, is the rate of convergence computable? (3) If neither is computable, is there a different computable witness for convergence. I will address all of these questions, showing that the answers relate to the computability of various parameters.

Secondly, we may also use the field of algorithmic randomness to explore the almost-everywhere nature of martingale convergence. I will present a variety of theorems which characterize the exact points for which computable martingales converge under various assumptions. These classes of points are exactly the well-known notions of Schnorr randomness, computable randomness, and Martin-Löf randomness.

The connections between computable analysis and algorithmic randomness are very close. Further, these results have implications for differentiability, the law of large numbers, and de Finetti's theorem. They also parallel results about the ergodic theorem.

Studying topics such as this are important since they help us to understand how the field of probability theory can, on one hand, be incredibly applicable and computable, but on the other hand, be based heavily on non-constructive set-theoretic foundations.