THE COMPOSITE FERMION: A QUANTUM PARTICLE AND ITS QUANTUM FLUIDS

The fractional quantum-Hall effect and other exotic behaviors of electrons trapped in two dimensions can be understood in terms of composite particles—electrons sporting attached flux quanta.

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Discovery of new particles is not usually associated with condensed matter physics, because, at one level, we already know all the particles that go into the Hamiltonian—namely, electrons and ions. But it is a most profound fact of nature—indeed the very reason why physics can make progress at many different levels—that strongly interacting particles reorganize themselves to become more weakly coupled particles of a new kind. Often they are simple bound states of the old particles. But sometimes they are fantastically complicated collective objects (for example, solitons) that nonetheless behave as legitimate particles, with well-defined charge, spin, statistics, and other properties we attribute to particles.

These new particles are, in a sense, the true particles of the system in question, because it is reasonable to reserve the title "particle" for nearly independent objects. Once we have identified the true particles of a system, phenomena that were difficult or impossible to understand in terms of the old particles become simply comprehensible as properties of almost free particles. That is why condensed matter systems are often described in terms of phonons, magnons, Landau quasiparticles, or Cooper pairs, rather than electrons and ions.

This article concerns electrons confined to two dimensions. Such a system can be realized, for example, at the interface between two semiconductors. In strong transverse magnetic fields at sufficiently low temperature, such systems exhibit absolutely marvelous properties that are entirely unexpected and inexplicable when one thinks of them simply as a collection of weakly interacting electrons.

So, what are the true particles of this two-dimensional electron system? It happens that the electrons effectively "swallow" all or a substantial fraction of the external magnetic field, thus transforming themselves into particles that are called "composite fermions."[1,2] Numerous properties of these composite fermions and the quantum fluids they form have been established in the last decade:[3] Experimenters have observed their Fermi sea, their Shubnikov–de Haas oscillations, their cyclotron orbits, and their quantized Landau levels. They have measured the particles' charge, spin, statistics, mass, magnetic moment, and thermopower. In mesoscopic experiments, the composite fermions have been bounced around like billiard balls.

Not only has the composite fermion helped explain and predict remarkable phenomena. It has also provided the motivation for a microscopic theory that is practically exact without requiring new parameters. It is impossible, in this limited space, to do justice to the growing body of work in the field. So this article concentrates only on some of the most basic facts, pointing the interested reader to review articles or the most recent papers for further information.

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The quantum Hall effects

In the presence of a strong magnetic field B transverse to a two-dimensional system of electrons, the tiny cyclotron orbits of an electron are quantized to produce discrete kinetic energy levels, called "Landau levels." (See figure 1.) The degeneracy of each Landau level—that is to say, its maximum population per unit area—is B/ϕ_0, where $\phi_0 = h/e$ is the elementary quantum of magnetic flux. This degeneracy implies that the number of occupied Landau levels, called the filling factor, is $\nu = \rho\phi_0/B$, where ρ is the two-dimensional electron density. The integral quantum Hall effect, which is manifested by the development of spectacularly flat plateaus in the Hall conductance centered at integral values of ν, was discovered in 1980 by Klaus von Klitzing. (See PHYSICS TODAY, December 1985, page 17.)

In a sufficiently strong magnetic field, when ν is less than 1, all the electrons can be accommodated in the lowest Landau level and, to good approximation, one can neglect any mixing between Landau levels. The kinetic energy is then an irrelevant constant, and the Hamiltonian is simply given by the Coulomb potential of the electron assemblage:

$$H = \frac{1}{2}\sum_{j\neq k}\frac{1}{4\pi\epsilon_0}\frac{e^2}{|\mathbf{r}_j - \mathbf{r}_k|}. \tag{1}$$

The ultimate goal of theory is to solve the Schrödinger equation, $H\Psi = E\Psi$, as a function of ν in the Hilbert space of the lowest Landau-level states. Considering that we are dealing with a macroscopic system of interacting electrons, it should come as no surprise that we don't know the exact solution. To make matters worse, the standard approximate perturbative strategies are doomed by the absence of any small parameter, because the interaction energy is the only energy scale in the problem. Nonetheless, a trail of experimental clues has guided us to wavefunctions that are accurate and faithful representations of the exact eigenstates.

These wavefunctions reveal the simple physics of the problem, namely the formation of the composite fermion. The composite fermion was originally introduced to

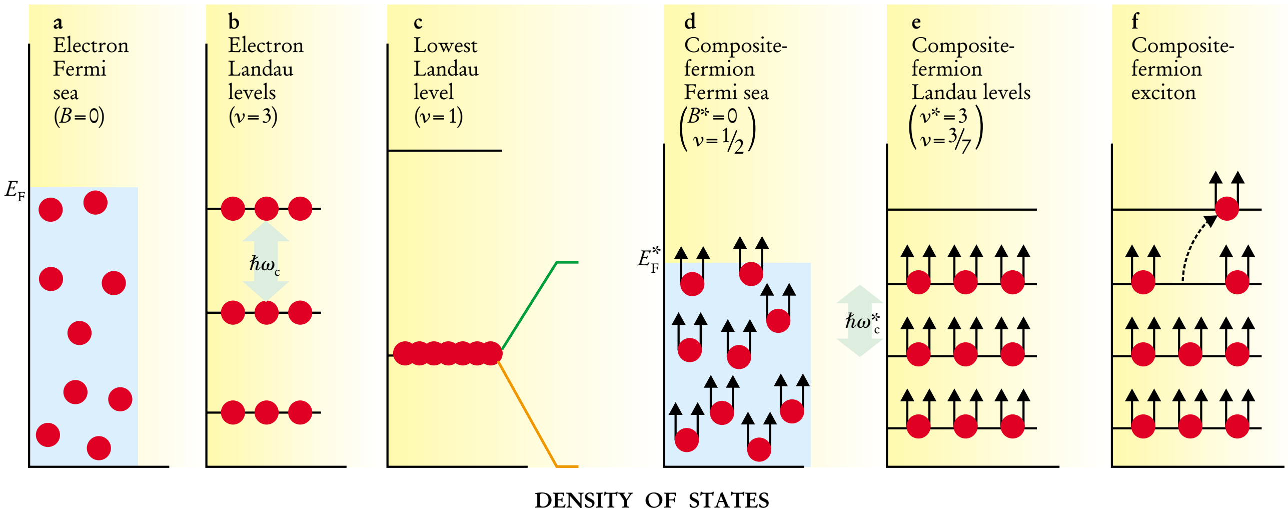

Figure 1. Evolution of a two-dimensional electron system as the transverse magnetic field B is increased. For independent electrons, the Fermi sea (**a**) (filled to Fermi energy E_F) at $B = 0$, splits into Landau levels (**b**) separated by the cyclotron energy. The lowest Landau level (**c**) is split by interactions into energy levels of composite fermions with attached flux quanta, which fill a composite-fermion Fermi sea (**d**) at $\nu = {}^1/_2$ and occupy composite-fermion Landau levels (**e**) at other filling factors. A jump out of such a level (**f**) creates an exciton, a neutral particle–hole excitation. At still higher fields, this scenario (**d—f**) repeats itself, but now with composite fermions carrying four or more flux quanta.

explain the fractional quantum Hall effect, discovered in 1982 by Daniel Tsui, Horst Stormer, and Arthur Gossard at simple fractional values of ν. (See Physics Today, December 1998, page 17.) But subsequent work has shown that it describes a superstructure that encompasses other phenomena as well.

The quickest way to introduce the composite fermion is through the following series of steps, which I call the "Bohr theory" of composite fermions because it obtains some of the essential results with the help of an oversimplified but useful picture. The outcome is that strongly interacting electrons in a strong magnetic field B transform into weakly interacting composite fermions in a weaker effective magnetic field B^*, given by

$$B^* = B - 2p\rho\phi_0\,, \qquad (2)$$

where $2p$ is an even integer. Equivalently, one can say that electrons at filling factor ν convert into composite fermions with filling factor $\nu^* = \rho\phi_0/|B^*|$, given by

$$\nu = \frac{\nu^*}{2p\nu^* \pm 1}\,. \qquad (3)$$

The minus sign corresponds to situations when B^* points antiparallel to B.

Start by considering interacting electrons in the transverse magnetic field B. Now attach to each electron an infinitely thin, massless magnetic solenoid carrying $2p$ flux quanta pointing antiparallel to B, turning it into a composite fermion. Such a conversion preserves the minus sign associated with an exchange of two fermions, because the bound state of an electron and an even number of flux quanta is itself a fermion. Hence the name. It also leaves the Aharonov–Bohm phase factors associated with all closed paths unchanged, because the additional phase factor due to a flux $\phi = 2p\phi_0$ is $\exp\{2\pi i\phi/\phi_0\} = 1$. In other words, the attached flux is unobservable, and the new problem, formulated in terms of composite fermions, is identical to the one with which we began.

So, what have we gained? Well, a "mean-field approximation" now suggests itself, in which the new attached magnetic field is smeared to produce an additional uniform magnetic field $-2p\rho\phi_0$. With that addition, we get the net magnetic field B^* of equation 2. The net effect, in a sense, is that each electron has absorbed $2p$ flux quanta from the external field to become a composite fermion that experiences only the residual magnetic field B^*. (See figure 2.)

The crucial point is that the many-particle ground state of electrons at $\nu < 1$ was highly degenerate in the absence of interaction, with *all* lowest Landau level configurations having the same energy. But now, the degeneracy of the composite-fermion ground state at the corresponding $\nu^* > 1$ is drastically smaller, even when the interaction between composite fermions is switched off. For integral values of ν^*, in fact, one gets a *non-degenerate* ground state.

The reduced degeneracy suggests that one might start by treating the composite fermions as independent. In that approximation, the composite fermions fill a Fermi sea of their own whenever B^* vanishes ($1/\nu = 2p$), and form composite-fermion Landau levels when it does not. All of this action, of course, takes place inside the lowest electronic Landau level, as shown in figure 1.

Having identified interacting electrons at filling factor ν with independent composite fermions at ν^*, we write the (unnormalized) microscopic wavefunctions for interacting electrons at a given ν as:

$$\Psi_\nu = \Phi_{\nu^*}\prod_{j<k}(z_j - z_k)^{2p}, \qquad (4)$$

where $z_j = x_j - iy_j$ denotes the position of the jth electron as a complex number, and Φ_{ν^*} are the known Slater-determinant wavefunctions for *non-interacting* electrons at the corresponding ν^*. For simplicity, we have assumed that the electron population is fully polarized and we have suppressed the spin part of the wavefunction.

The wavefunctions Ψ_ν, which turn out to be extremely accurate approximations of the actual electron eigenstates, give a precise meaning to the intuitive physics of our composite-fermion discussion. The factors in the product over all electrons in equation 4 tell us that every electron sees $2p$ vortices at every other electron. That is to say, as the jth electron executes a closed path around the

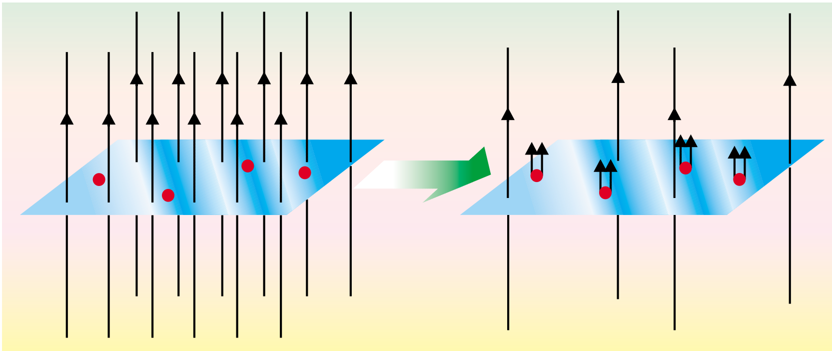

FIGURE 2. CAPTURING TWO FLUX quanta transforms each electron in the plane into a composite fermion that experiences, in effect, a reduced residual magnetic field.

kth, it generates a phase of magnitude $2p \times 2\pi$. By definition, a closed loop around a unit vortex generates a phase of 2π. Thus the product in equation 4 attaches $2p$ vortices to each electron in the noninteracting Slater-determinant wavefunction.

So we see that the flux quanta of our "Bohr theory" represent the microscopic vortices of the many-particle wavefunction, and the composite fermion is actually the bound state of an electron and $2p$ quantum vortices. A flux quantum is topologically similar to a vortex; it also produces an Aharonov–Bohm phase of 2π for a closed path around itself. Therefore it is often useful to model the vortices as flux quanta and envision the composite fermion as an electron carrying $2p$ flux quanta.

How do the vortices cancel part of the external B field? Consider a path in which one particle executes a counterclockwise loop enclosing an area A, with all the other particles held fixed. Equating the sum of the Aharonov–Bohm phase $2\pi BA/\phi_0$ and the phase $-2\pi 2p\rho A$ coming from the encircled vortices to an *effective* Aharonov–Bohm phase $2\pi AB^*/\phi_0$, we get the new field B^* of equation 2. Of course a magnetometer will still measure simply B. But, as far as a composite fermion is concerned, B^* is the real field, as we shall see.

The form of the wavefunction Ψ provides an insight into why the repulsive interaction between electrons might force vortices on them. The wavefunction is very effective in keeping the electrons apart. The probability that any two will come within a distance r of each other vanishes like $r^{2(2p+1)}$. Contrast that with the r^2 vanishing for a typical state satisfying the Pauli principle. In essence, then, electrons transmute into composite fermions by capturing $2p$ vortices because that is how they best screen the repulsive Coulomb interaction. The interaction between composite fermions is weak because most of the Coulomb interaction has been screened out—or used up—in making them.

Equations 2, 3, and 4 are the master equations describing the quantum fluid of composite fermions. Since the first two are the same and can be derived from the third, everything ultimately stems from a single equation. The quantum numbers of the composite fermion follow straightforwardly from the observation that each one is produced by a single electron. It has the same charge and spin as the electron, and it is also a fermion.

Seeing composite fermions

The crucial, non-perturbative respect in which composite fermions distinguish themselves from electrons is that they experience an effective magnetic field, B^*, that is drastically different from the external magnetic field. The effective magnetic field is so central, direct, and dramatic a consequence of the formation of composite fermions that its observation is tantamount to an observation of the composite fermion itself.

At filling factors ν less than 1, the experiments clearly show us composite fermions subject to the magnetic field B^* rather than electrons subject to B. The most compelling experimental evidence for the composite fermion comes simply from plotting the high-field magnetoresistance as a function of $1/\nu^*$, which is proportional to B^*.

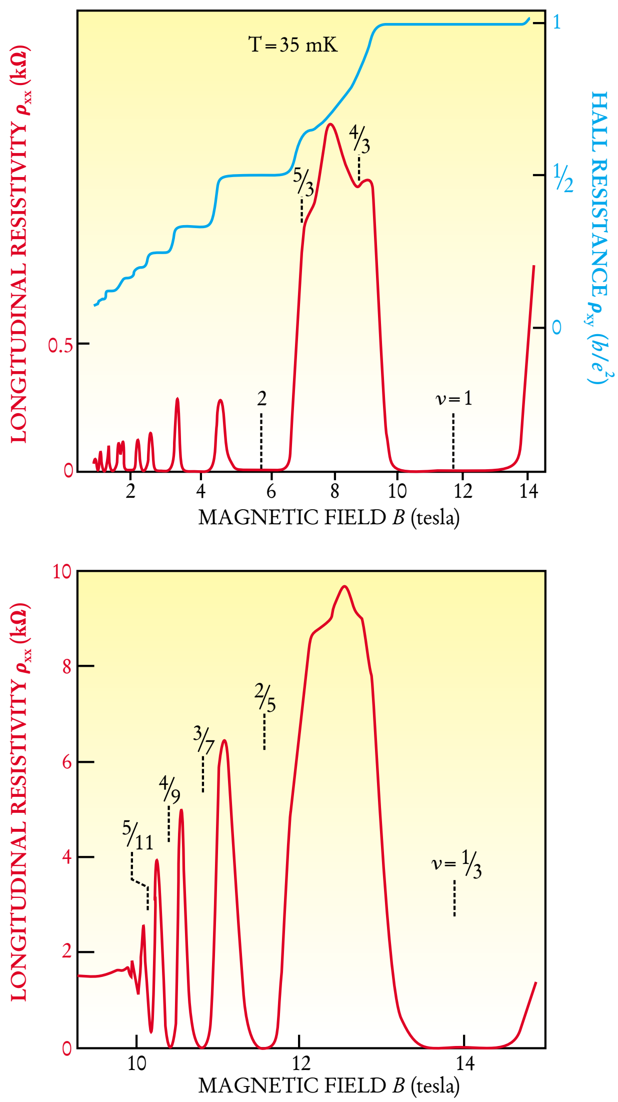

FIGURE 3. COMPARING INTEGRAL AND FRACTIONAL quantum Hall regimes, top and bottom panels, respectively.[16, 3] Blue curve (shown only in the top panel) is the Hall resistance with quantum plateaus. Red curves are ordinary longitudinal resistivity with a dip, labeled by its electron filling factor ν, for each plateau. The filling factor $\nu = n/(2n + 1)$ corresponds to a composite-fermion filling factor $\nu^* = n$. Thus the two red curves, despite their very different electron filling factors, are remarkably similar.

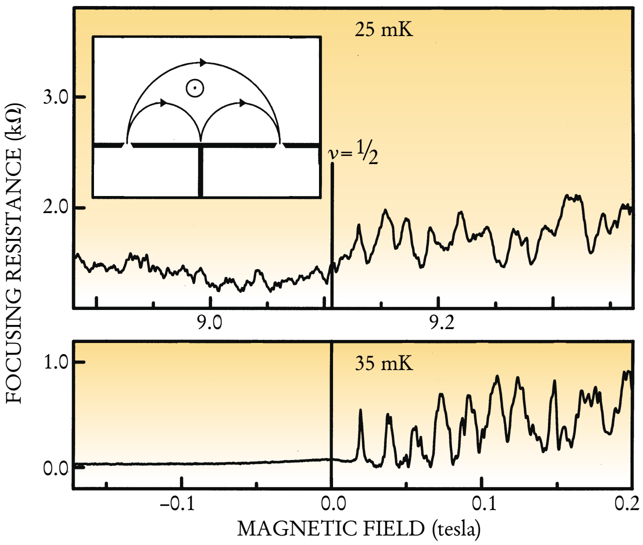

Figure 4. Measuring the effective magnetic field B^* felt by composite fermions, by magnetically focusing them—injecting them into one constriction and collecting them at another. The lower panel shows the focusing peaks *for electrons* at discrete values of B (near $B = 0$) corresponding to different numbers of bounces (see insert.) The upper panel shows the corresponding peaks for composite fermions near $B^* = 0$. The two sets of peaks (superimposed over mesoscopic resistance fluctuations due to disorder) align when one scales B^* by a factor of $\sqrt{2}$, to account for the fact that the composite-fermion Fermi sea, unlike the electron Fermi sea, is spin polarized. (Adapted from ref. 7.)

Figure 3 shows us its striking similarity to the magnetoresistance of electrons at low B (where they are weakly interacting), plotted as a function of $1/\nu$. This is direct evidence that the strongly correlated liquid of interacting electrons at filling factor ν behaves like a weakly interacting gas of composite fermions at ν^*.

The quantum Hall effect, evident in figure 3, is one of the most fascinating phenomena exhibited by two-dimensional electron systems in a magnetic field.[4] One sees plateaus of the Hall resistance R_H with quantized values $h/(fe^2)$ centered around $\nu = f$, where f is an integer or a simple fraction. The *integral* quantum Hall effect is understood straightforwardly,[4] in terms of independent electrons, as a consequence of the quantization of the single-electron energy into Landau levels, which produces a non-degenerate many-particle ground state whenever ν is an integer n. The analogous integral quantum Hall effect for composite fermions corresponds to $\nu^* = n$. These states occur at fractional *electron* filling factors given by

$$\nu = \frac{n}{2pn \pm 1}, \tag{5}$$

which turn out to be precisely the observed "magic" fractions at which the fractional quantum Hall effect is observed to be particularly prominent. (See Physics Today, July 1993, page 17.) There is, at present, evidence for more than 30 fractional quantum Hall states. The equation dictates only odd-denominator fractions, which, with only one exception, is what the experimenters find.

The fractional quantum Hall effect for electrons is thus interpreted as an integral quantum Hall effect of composite fermions—in effect, an observation of composite-fermion Landau levels. This simple explanation of the fractional quantum Hall effect not only obtains all the observed fractional plateaus in a single step; it also unifies the fractional and the integral quantum Hall effects.

The observation of the fractional quantum Hall effect serves as a macroscopic confirmation of some of the fundamental postulates of quantum mechanics. The principle governing the surprising precision of the quantization of macroscopic Hall resistance is the single-valuedness of the microscopic many-electron wavefunction, which requires that the vorticity of a composite fermion (the exponent $2p$ in equation 4) be precisely an integer. The empirical odd-denominator rule follows, because the $2p$ must be even so that the many-particle wavefunction have the exchange antisymmetry required for fermions.

Robert Laughlin's original theory of $\nu = 1/(2p+1)$ states, a subset of the observed fractions, falls naturally within the composite-fermion theory. At $\nu^* = 1$, putting into equation 4 the explicit form of the non-interacting Slater-determinant wavefunction $\Phi_{\nu^*=1}$ yields for the ground state at $\nu = 1/(2p+1)$

$$\Psi_{1/(2p+1)} = \prod_{j<k} (z_j - z_k)^{2p+1} \exp\Big[-\frac{eB}{4\hbar} \sum_l |z_l|^2\Big], \tag{6}$$

which is precisely the wavefunction formulated by Laughlin in 1983 to explain the first observed fractional quantum Hall state ($\nu = 1/3$). It represents one filled composite-fermion Landau level.

An early and influential approach, pioneered by Steven Girvin and Allan MacDonald,[5] regards the Laughlin wavefunction as a Bose condensate, with the role of the boson played by the bound state of an electron and $2p+1$ flux quanta.

What about the fractional quantum Hall effect's celebrated fractional charge. (See Physics Today, November 1997, page 17.) It appears as what is called the "local charge" of an excited composite fermion, defined as the sum of its intrinsic charge ($-e$) and the charge of the screening cloud around it. Its value at $\nu = n/(2pn \pm 1)$ can be shown by a simple counting argument to be $-e/(2pn \pm 1)$. This fractional charge is a manifestation of a quantized screening by the quantum fluid of composite fermions.

Do composite fermions have a life outside the fractional quantum Hall effect? An important application of the concept concerns the metallic state at $\nu = 1/2$, where no fractional quantum Hall state is seen. If composite fermions exist at that filling factor, they would experience no effective magnetic field ($B^* = 0$). Thus a mean-field picture suggests a Fermi sea of composite fermions. (Once again, see Physics Today, July 1993, page 17.)

In an influential theoretical work, Bertrand Halperin, Patrick Lee, and Nicholas Read argued that many features of the Fermi surface of composite fermions survive when fluctuations beyond the mean-field theory are taken into account.[6] At ν values near $1/2$, the composite fermions, experiencing a very weak magnetic field, would execute classical cyclotron orbits of radius R^* orders of magnitude larger than any electronic length scale appropriate to B. Experiments by three different groups at Bell Labs and Stony Brook in 1993–94, and several experiments since then, have confirmed that R^* is indeed the cyclotron radius of the charge carriers.[3] Two of these experimental results are shown in figures 4 and 5. Farther away from $\nu = 1/2$, the semiclassical orbits of the quantum composite-fermion particles are quantized to produce composite-fermion Landau levels, first exhibiting Shubnikov–de Haas oscillations and then the quantum Hall effect.

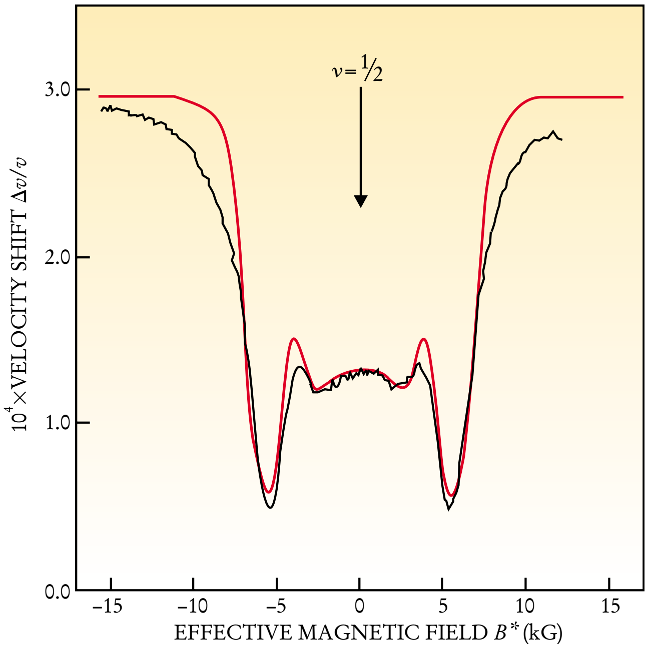

FIGURE 5. SURFACE ACOUSTIC WAVE STUDY of the composite-fermion state in the vicinity of $\nu = {}^1/_2$. The velocity shift of surface acoustic waves exhibits resonances near $B^* = \pm 5$ kG and ± 2.5 kG, as expected from theory (red curve).[6] Adapted from ref. 8.

The observation of composite fermions in the region around $\nu = {}^1/_2$, where there is neither a quantum Hall effect nor any other sort of excitation energy gap, was a watershed for the composite-fermion concept. It was an explicit demonstration that the composite fermion is more general than its manifestation in the fractional quantum Hall effect, where it forms Landau levels. The observation of the composite-fermion's Fermi sea explicitly verified its Fermi statistics, and the measurement of its cyclotron radius confirmed that it carries a charge $-e$.

What about its spin? The spin degree of freedom is frozen in strong magnetic fields at low temperature, when the Zeeman energy is large compared to the interaction strength and thermal agitation. But in relatively weak fields, several spin polarizations become possible. These differently spin-polarized states, as well as transitions between them, have been observed, and they are well described in terms of Landau levels of free, spin-${}^1/_2$ composite fermions.[3,9] The composite-fermion g factor deduced from these experiments is close to that of the electron.

What, then, is its mass? This question is somewhat subtle, because the composite fermion's entire mass is generated dynamically from interactions. There is, after all, no mass parameter in the Hamiltonian of equation 1. The mass is most straightforwardly determined by measuring the excitation gap at a given filling of composite fermions, and equating it to the cyclotron energy of a composite fermion in the B^* field.[1,2] But it can also be deduced from an analysis of the temperature dependence of the Shubnikov–de Haas oscillations,[3] or by ascertaining at what Zeeman energy the composite-fermion Fermi sea becomes fully polarized.[9] For typical experimental parameters, the composite-fermion masses obtained from these various methods are on the order of the free electron mass, and much larger than the electron band mass in, say, GaAs, but unrelated to either.

The rich phenomenology of the lowest Landau-level quantum liquid thus follows succinctly and coherently from the concept of composite fermions, without the need for a microscopic theory. But the simplicity of this explanation should not obscure the non-trivial nature of the underlying physics: Each strongly interacting electron, with no kinetic-energy degree of freedom, captures $2p$ quantum-mechanical vortices and is thus magically transformed into a nearly free, massive composite fermion. This composite fermion experiences a magnetic field drastically different from the external one, and its kinetic energy manifests itself through a Fermi sea and Landau levels.

Computer experiments

Fortunately, one can diagonalize the Hamiltonian of equation 1 numerically for a finite system, to obtain the exact eigenfunctions and eigenenergies. That provides further opportunity for rigorous, unbiased, and detailed testing of the composite-fermion theory. Some typical energy spectra are shown in figure 6. All the structure in these spectra is a consequence of the Coulomb interaction, in the absence of which all lowest-Landau-level states would be strictly degenerate.

The central prediction of the composite-fermion theory is that the low-energy physics of strongly interacting electrons in an external magnetic field B resembles that of nearly independent composite fermions in the residual field B^*. This prediction has been extensively confirmed in computer experiments that establish a one-to-one correspondence between the quantum numbers of the low-energy states of the two systems.

In particular, one expects a gap at the ν values of equation 5, which correspond to $\nu^* = n$. Sure enough, at those values the Coulomb interaction removes the enormous degeneracy of the non-interacting electron system to produce a non-degenerate ground state, as illustrated in figure 6. The ground state, highlighted in red, represents n filled Landau levels of composite fermions. The excited states highlighted in yellow in the figure are to be interpreted as different configurations of the composite-fermion exciton. (See also figure 1f.)

Besides giving the quantum numbers of the low-energy states, the composite-fermion theory also yields their wavefunctions (equation 4), which are now projected into the lowest electronic Landau level, as appropriate for the large-B limit under consideration. Extensive studies have shown that these functions have a nearly perfect overlap with the corresponding exact eigenfunctions, and that they typically predict the energies to within 0.1% or better. Some representative results are shown in figure 6 and the table on page 45.

To appreciate the significance of these comparisons, one should note that, for filled composite-fermion Landau levels or their excitons, Ψ involves no adjustable parameter whatsoever. Furthermore, the actual eigenstates are linear superpositions of a large number of distinct basis states, as indicated by the towers of excitation levels in figure 6. That rules out any possibility of accidental agreement. It is rare that such a simple, zero-parameter theory for a strongly correlated many-body state has such predictive power. These comparisons also demonstrate that the wavefunctions Ψ go beyond the simple mean-field picture that motivated them. They encode the physics of the residual interaction between composite fermions.

Ana Lopez and Eduardo Fradkin have initiated another approach for dealing with corrections to the mean-field description, in terms of a Chern–Simons field theory. That approach has been developed further by several groups.[6,10]

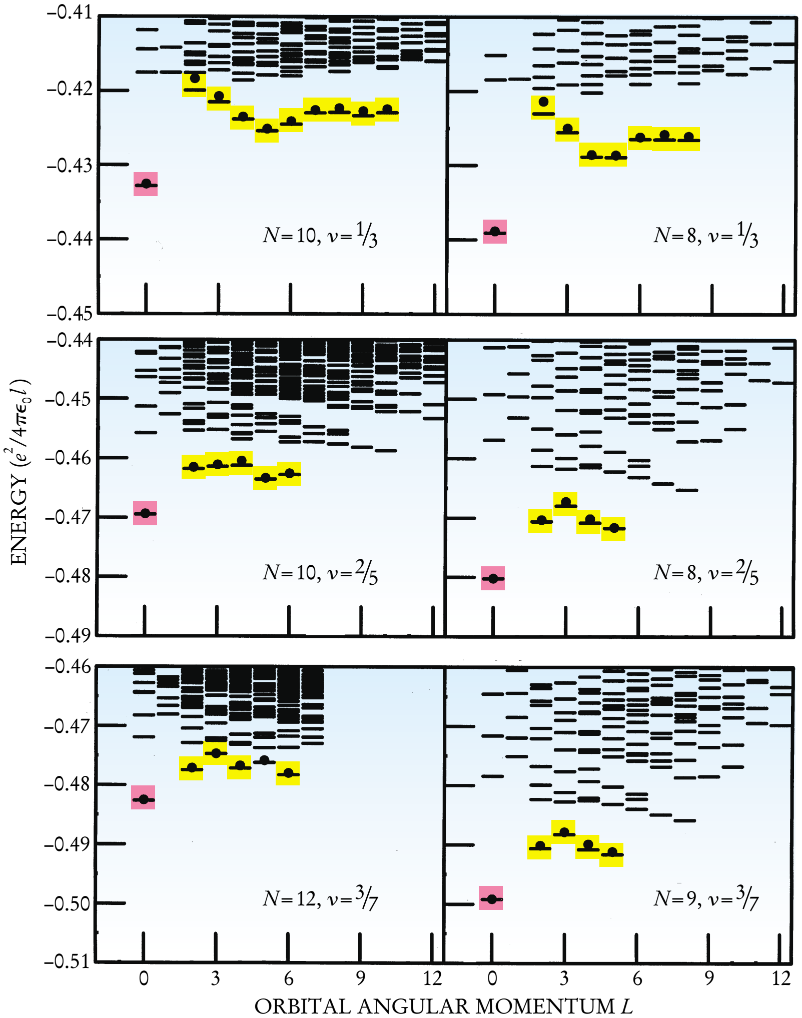

FIGURE 6. COMPARISON OF EXACT eigenenergies per particle (dashes) from numerical diagonalization against the energies predicted by composite fermion theory (dots) with no adjustable parameters, for the low-energy states of a model system of N = 8 to 12 interacting electrons on the surface of a sphere pierced by a radial magnetic field characterized by filling factor ν. L is the system's total orbital angular momentum. (Also see the table on page 45.) The ground state (L = 0) is highlighted in red, and the well-defined branch of low lying excited states (highlighted in yellow) represents the composite-fermion exciton in various possible configurations. The l in the energy unit is the magnetic length $\sqrt{\hbar/eB}$. The towers of excited states extend well beyond the figure's high-energy cutoffs. Adapted from ref.12.

A quantitative comparison with laboratory experiments requires a consideration of Landau-level mixing, transverse thickness of the electron wavefunction, and disorder—all of which are conveniently set to zero in the computer experiments. These realities do not affect the qualitative physics nor the quantizations, but they do introduce various parameters that can only be handled approximately. After incorporating some of these effects in various approximations, we find that theory and experiment typically agree, at present, within 10–30% on the energy of the *neutral* composite-fermion exciton[11] and the spin-related physics,[3,9] and within a factor of two for the charged-excitation gap.

Are there other phases?

Do the composite fermions exhibit any other phases? If they were strictly non-interacting, our discussion of their quantum-Hall and Fermi-sea effects would be the end of the story. But there is a residual interaction between composite fermions. By definition, it's whatever is left after most of the Coulomb interaction is used up in giving the composite fermion its mass. The residual interaction is often weak enough to be neglected. That is what we have done above. But it might, in certain circumstances, be responsible for creating fascinating new phases. After all, the interaction between electrons—the fermions we know best—generates numerous phases, for example, the BCS superconductor, the Wigner crystal, and Bloch's spontaneously polarized Fermi liquid. There are indications that all of these phases are feasible also for composite fermions.

At sufficiently small $\nu (\leq 1/9)$, the composite-fermion liquid becomes unstable against spontaneous generation of excitons, making way for the formation of a Wigner crystal.[12,13] In a range of filling factors prior to this ($\nu \leq 1/4$), the composite fermion liquid is predicted to exhibit the Bloch instability, namely a magnetically ordered broken-symmetry phase, even in the absence of Zeeman coupling.[14]

Another fascinating state is the BCS-like p-wave paired state of fully polarized composite fermions, increasingly believed to be the source of the fractional quantum-Hall effect at $\nu = 5/2$, the sole exception to the odd-denominator rule.[15] Here, even though the underlying interaction is purely repulsive, a capture of the vortices during the creation of composite fermions presumably overscreens the Coulomb interaction, producing a weak, effectively attractive interaction between them.

Finally, mixed states containing two different flavors of composite fermions (with different numbers of attached flux quanta) would also produce quantum-Hall fractions other than the principal fractions of equation 5. Preliminary evidence now exists for such additional fractions.

It remains to clarify the physics of these new phases, the precise nature of the Fermi liquid at $\nu = 1/(2p)$, and the role of disorder in these systems. Another poorly understood issue is how, as the temperature is raised, do composite fermions gradually ionize by shedding their vortices and turning back into electrons.

A quantum particle

Even though the composite fermion behaves, to a great extent, like an ordinary fermion, we must not forget that it is a most unusual particle. First of all, it is a truly collective, many-body entity. The definition of a single composite fermion inherently involves all the particles in the system. Moreover, the composite fermion is a quantum particle. Of course quantum mechanics describes all particles, but it participates in the very definition of the composite fermion, whose creation is the union of an electron and quantum mechanical phases (vortices). The compos-

EXACT ENERGIES PER PARTICLE for the the ground states of figure 6, compared with those calculated from the composite-fermion theory. Also given are results for the excited state in which a composite-fermion particle and its hole are farthest apart. (From ref. 12.)

ν	N	Ground state		Excited state	
		Composite fermion	Exact	Composite fermion	Exact
1/3	8	−0.4389	−0.4391	−0.4261	−0.4266
	10	−0.4326	−0.4328	−0.4224	−0.4229
2/5	8	−0.4802	−0.4802	−0.4714	−0.4717
	10	−0.4693	−0.4694	−0.4625	−0.4627
3/7	9	−0.4991	−0.4992	−0.4915	−0.4916
	12	−0.4825	−0.4826	−0.4782	−0.4783

ite fermion could not exist in a purely classical world. Futhermore, the orbits of composite fermions are quantized to produce a quantum fluid of quantum particles.

Among the remarkable features associated with the physics of composite fermions are the dynamical generation of a mass where there was none to begin with, the quantum-mechanical renormalization of the magnetic field, pairing due to purely repulsive interactions, and fractional charge generated by the quantization of screening. It is irresistible to wonder which of these concepts finds wider applications in nature.

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