Sub-cooled Boiling and a Cell Centered Conduction Method

1. Note that when boiling is induced by heat transport from a hot wall to liquid a point often exists where the liquid is sub-cooled, but the wall temperature is above T_{sat} resulting in the production of vapor in the presense of sub-cooled liquid conditions.

2. We must add a subcooled boiling term to the heat conduction limited model to include this physical process in our two-phase flow equations.

$$\Gamma = \frac{-(q_{ig} + q_{i\ell}) + h'_{\ell}(T_w - T_{sat})}{h'_{fg}} \quad \textbf{(1)}$$

$$q_{ig} = h_{ig}\, a_i\, (T_i - T_g) \quad \textbf{(2)}$$

$$q_{i\,\ell} = h_{i\,\ell}\, a_i\, (T_i - T_\ell) \quad \textbf{(3)}$$

3. The term h'_{ℓ} is just that portion of the liquid heat transfer coefficient that is directly associated with boiling. Terms related to convection processes are not included.
4. The contribution from the wall may be reasonable, but it is important to note that the net boiling is a dominantly a balance between the wall boiling and the condensation due to the liquid to interface heat transfer. For most current codes, this later term is not computed with any knowledge of what is going on at the wall and also generally lacks the subcell resolution of the wall terms. Hence, the physical basis of any net vapor production from this model is tenuous at best. What future safety codes for improvements in this situation.

5. Review my past demonstration of the TRACE conduction scheme, noting the special considerations at the edge of the metal and at rod centerlines. For centerline consider a radial only calculation. The radial Laplacian with azimuthal symetry can be written as:

$$\frac{1}{r}\frac{d}{dr}r\frac{dT}{dr} = \frac{d^2T}{dx^2} + \frac{d^2T}{dy^2}$$

At the centerline the numerical evaluation of the Laplacian follows from the realization that $\Delta x=\Delta y=\Delta r$ and the values of the temperatures at the first x,y mesh points away from the point x=y=0 are all equal to the temperature at $r=\Delta r$. If T_1 is the temperature at the centerline, and T_2 is the temperature at the first node out from the center, then the numerical Laplacian is $4(T_2-T_1)/\Delta r^2$ at the center (r=0) . This result can also be obtained by fitting a quadratic to the values of temperature at r=0 and

r=Δr (T_1 and T_2) and the condition that the radial derivative of temperature is zero at r=0.

A Slightly Different approach to Numerical Solution of the Conduction Equation.

The conduction schemes used in codes like TRACE and RELAP5 are no where near state of the art, and can be improved with minimal effort. One simple improvement is to go to a scheme where Temperatures are directly calculated at volume centered points halfway between the evaluation points currently used in TRACE. This is a useful set in making the coupling between conduction and flow equation more implicit. The temperature and all material properties are evaluated at the center of each computational volume, and the heat flux is evaluated at each of the sides of the volume. As is the case with the standard version of TRACE, no equations are solved to properly model cell size changes as the metal expands and contracts. This effectively results in small violations of energy conservation in heated structures.

The temperature distribution within any heat structure is obtained by solving the conduction equation,

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + \dot{q}'''.$$

This equation is solved by dividing the heat structure into logically rectangular finite volumes. Quantities associated with each volume are indexed with subscripts. The first subscript (generally i) is associated with the radial coordinate. The second (j) represents the axial or azimuthal coordinate. If one of the subscripts incudes a term + 1/2 then the subscripted quantity refers to a value at the cell edge. The heat fluxes at the cell edges are

$$F_{i,j+1/2} = A_{i,j+1/2} k_{ij} \frac{(T_{i,j+1/2} - T_{ij})}{\Delta z_j / 2}$$

$$F_{i,j-1/2} = A_{i,j-1/2} k_{ij} \frac{(T_{ij} - T_{i,j-1/2})}{\Delta z_j / 2}$$

$$F_{i+1/2,j} = A_{i+1/2,j} k_{ij} \frac{(T_{i+1/2,j} - T_{ij})}{\Delta r_i / 2},$$

and

$$F_{i-1/2,j} = A_{i-1/2,j} k_{ij} \frac{(T_{ij} - T_{i-1/2,j})}{\Delta r_i/2}$$

Continuity requires that fluxes match at cell edges leading to the equations relating cell edge temperatures to the cell center values. For the radial edges

$$T_{i+1/2,j} = \frac{\beta_{ij} T_{ij} + \beta_{i+1,j} T_{i+1,j}}{\beta_{ij} + \beta_{i+1,j}}$$

where

$$\beta_{ij} = \frac{k_{ij}}{\Delta r_i/2},$$

At a heat structure contacting a two-phase fluid, flux matching implies

$$T_{i+1/2,j} = \frac{\beta_{ij} T_{ij} + h_{\omega l} T_l + h_{\omega g} T_g}{\beta_{ij} + h_{\omega l} + h_{\omega g}}$$

Similar equations hold in the axial (or azimuthal direction), except that the end surface fluxes are usually forced to zero. In practice the cell edge temperatures are only directly evaluated and stored at the surfaces of a heat structure.

The ADI difference equations for the full conduction equation are

$$(\rho_{ij}^n c_p^n{}_{ij} \frac{\tilde{T}_{ij}^{n+1} - T_{ij}^n}{\Delta t} - \dot{q}'''_{ij}) V_{ij} = F_{i,j-1/2}^n + \tilde{F}_{i-1/2,j}^{n+1} - F_{i,j+1/2}^n - \tilde{F}_{i+1/2,j}^{n+1},$$

and

$$(\rho_{ij}^n c_p^n \frac{T_{ij}^{n+1} - T_{ij}^n}{\Delta t} - \dot{q}'''_{ij}) v_{ij} = F_{i,j-1/2}^{n+1} + \tilde{F}_{1-1/2,j}^{n+1} - F_{1,j+1/2}^{n+1} - \tilde{F}_{i+1/2,j}^{n+1}$$

When evaluating the cell edge flux terms, the advanced time superscript (n + 1) actually only applies to the temperatures. All other metal properties are evaluated at the old time level. When a simple radial conduction calculation is requested, the axial fluxes are set to zero and the second step of the ADI is bypassed.

The automatic rezoning capability for the axial (or azimuthal) mesh can be preserved with necessary modifications for the new cell centered temperature grid. The basic rule for rezoning is that the original coarse mesh cell edges must always exist as a cell edge. Cells are split until the input specifications are met for maximum temperature difference between cells (ΔT_{max}), the minimum cell length, or maximum number of cells. Cells are combined when the edge between them is not an original coarse mesh cell edge and the difference between the cell centered temperatures is less than ΔT_{max} divided by 2.1.