Homework Assignment #3

This assignment is due on Thursday, March 27, at the start of class or earlier.

1. Consider a model with three periods, 0, 1, 2, and an infinite number of \textit{ex ante} identical agents (to make life simple think of the agents as individual points on the continuum from [0, 1] and normalize the entire set of agents as equal to 1). Agents learn whether they are patient or impatient in period 1. Let $\lambda$ be the probability that an agent is impatient, and they choose $x$ and $y$ in period 0 to maximize their expected utility. If they are impatient they consume $c_1$ in period 1, and if they are impatient they consume $c_2$ in period 2. Agents are endowed with one unit of the good which they can use to purchase, in period 0, a long-lived asset ($x$) or a short-lived asset ($y$), thus, $y + x \leq 1$. The long-lived asset pays a return, $R > 1$. The short-lived asset returns one unit for one unit. In period 1 there is a market where the long asset can be sold. The price of $x$ in period 1 is $P$.

(a) Write the budget constraint for a patient and an impatient agent. Write the expression for expected utility. Show that in equilibrium $P = 1$ [Hint: Show that supply cannot equal demand if $P \neq 1$]. Given that $P = 1$, what is the market allocation? Show that this is better than what is achievable under autarky.

(b) Given an infinite number of agents a social planner can treat $\lambda$ as the proportion of impatient agents in the economy. The planner wants to maximize social welfare. Write down the planners’ resource constraints and expected welfare. Write down optimal choices of $c_1$ and $c_2$. If agents are risk averse will the planners’ solution coincide with the market solution? Explain. [you can use graphs here]

(c) Define notional consumption as $C = c_1 + c_2$. Show that $C_{\text{market}}$ in the market solution is greater than $C_{\text{efficient}}$ in the efficient solution when agents are risk averse. How can this result hold if the efficient solution is preferred to the market solution? Explain.

(d) Explain how a bank could offer a demand deposit contract that coincides with the efficient solution.

(e) Suppose that the bank can liquidate the long asset in period one with return, $r \leq 1 < R$. Show that a bank run can be an equilibrium in this model.

(f) Why is the sequential service constraint important to generate a bank run?

2. Suppose we create an asset backed security (ABS) with five mortgages. These mortgages either pay off or default, and the probability of a default is .1. Defaults are independent across the mortgages. Now suppose that we create five tranches (\textit{senior1}, \textit{senior2}, \textit{senior3}, \textit{mezzanine}, and \textit{equity}). The \textit{senior1} tranche defaults only if all five mortgages default, and the equity tranche defaults if any mortgage defaults.
(a) Calculate the probability of default for each of the five tranches. How does the likelihood of a tranche defaulting compare with the risk of the underlying mortgages? [Note that you need to calculate the probability that, say, any two (or three, or four, etc.) of five mortgages default. This requires use of the binomial distribution, and you could use Excel or a similar program to aid your computation.]. What does this say about the risk of senior tranches?

(b) Suppose that each mortgage was worth $100,000, so the total pool is $500,000. If the price of a tranche is equal to its expected value, price the senior1 tranche and the equity tranche.

(c) Suppose we now form a new security made up of mezzanine tranches. That is, we combine five securities with the same probability of default you calculated for the mezzanine tranche in part a. Call this a CDO. Again tranche this new security into five parts with the same pattern of seniority. Calculate the probability of default of the various tranches of the CDO.

(d) Suppose that the probability of default of the underlying mortgages is really .15. How does this change the probability of the default of the tranches of the CDO? How much riskier (say, in percentage terms) does the mezzanine tranche of the CDO get given this 50% increase in the default probability?

(e) What if there were 100 mortgages, 10 to a tranche, and the probability of default of the underlying loans is .05. Consider tranche10, which defaults if 10 or more mortgages default. What is the default probability of that tranche? What of the CDO made up of tranche10 securities? What happens if the underlying probability of default rises to .06?