Homework Assignment #2

This assignment is due on Thursday, February 27, at the start of class or earlier.

1. Consider a world with two assets, one is a safe asset in variable supply, the other a risky asset in fixed supply. There are two periods. The payoffs are given by:

\[
\begin{array}{c|c|c}
\text{t = 0} & \text{t = 1} \\
\hline
\text{Safe Asset} & 1 & 1.5 \\
\text{Risky Asset} & P & 6 \text{ with prob .25} \\
\end{array}
\]

where \( P \) is the price that the asset will cost in period 0 and is to be determined, and suppose all investors are risk neutral. Suppose investors have one unit of wealth and invest their own money. In this case investors should equate marginal returns [I don’t think you need a calculator for this question, but it could make your life a bit easier].

(a) What is the expected value of the risky asset in period 0? The marginal return on the safe asset is \( \frac{1.5}{1} \). Given that marginal returns should be equal for the two assets (recall risk neutral investors), what should \( P \) be? Call this \( P_F \), the fundamental value of the risky asset.

(b) Now suppose that investors have no wealth of their own. They can borrow at date 0 and repay 1.33 in date 1 if they are able to. Further suppose that the lenders cannot observe how loans are used. What is the return to investing in the safe asset? Can \( P_F \) still equal 1.5 if investors are using borrowed funds?

(c) What will happen to \( P \) given that the supply of this risky asset is fixed? What will \( P \) be?

(d) Given the new equilibrium price \( P \) that you calculated, how do you interpret this price? Why is the price of the risky asset higher when investors borrow than when they invest their own wealth? Explain.

(e) Suppose we changed the probabilities and payoffs to make the riskier asset riskier. Consider

\[
\begin{array}{c|c|c}
\text{t = 0} & \text{t = 1} \\
\hline
\text{Safe Asset} & 1 & 1.5 \\
\text{Risky Asset} & P & 13.5 \text{ with prob .1} \\
\end{array}
\]

What happens to the fundamental price when there is no lending in this case? What happens to the ratio of \( \frac{P}{P_F} \) if there is lending? How does this ratio compare with your calculation in part d?
2. Suppose the fundamental price of an asset is constant and equal to 100 and that the interest rate is constant and equal to 3%. Suppose further that the actual price of the asset is growing 5% per period. Can this be a rational bubble? [for this question a calculator or a program like excel may make your life easier]

(a) Can you calculate the probability that the bubble will continue for another period? Call this \( q \) and calculate it. How many periods would you expect the bubble to persist before the likelihood that it bursts equals .25? Explain.

(b) Suppose that the interest rate is 5% and that \( q \) is the same number you calculated in part a. At what rate must the bubble price grow for this to be a rational bubble? Explain.

(c) Suppose the interest rate was 4%. What happens to the number of periods that you calculated it would take before the likelihood that the bubble bursts reaches .25?

3. Suppose that dividends grow at some constant rate, \( g \), and that the return on equity is some constant rate, \( k \). Let \( D_0 \) be the current level of dividends. What should the price of a share of stock be? Assume that \( k > g \), then the following useful fact will help you: if \( x > y \) then

\[
\sum_{j=1}^{\infty} \left( \frac{1+y}{1+x} \right)^j = \frac{1+y}{x-y}.
\]

[for this question a calculator or a program like excel may make your life easier]

(a) Given your expression for the price of a stock, suppose that \( D_0 = 20 \) and that \( k = 9.2\% \) and dividends grow at 8%. What should \( P_0 \) be? Suppose that dividends are now expected to grow at 7.5%.

(b) What happens to the value of the stock? Does this seem like a small or large change given the magnitude of the change in dividend growth?

(c) Suppose that \( g \) was unchanged but that stocks became riskier so \( k \) increased to 9.9%. What happens to the price of the stock now?

(d) How does this relate to bubbles, if at all?