Homework Assignment #3: Answer Sheet

1. Consider the problem of uncertain stock size. Suppose that at time $t_0$ we know that the total stock of reserves can be either of two values, $S_1$ or $S_2$ ($S_1 < S_2$). Further, we know that at some future date $T$ we will be able to sink a well and determine for certain whether remaining reserves are $S_1$ or $S_2$. Everyone agrees at date $t_0$ that the probability of finding out that there are additional reserves, $S_2 - S_1$, is $\Pi$ (with probability $1 - \Pi$ reserves will be just $S_1$). Let the amount of reserves used up by date $T$ be $S$.

(a) Suppose $T$ is chosen optimally. Will $S \geq S_1$? Explain. What does the price path look like from $t_0$ to the optimal $T^*$?

**brief answer** Obviously $S < S_1$ since $1 - \Pi > 0$, and if this occurs $S_1 - S$ is all that is left to consume from $T^*$ till exhaustion.

(b) What happens to the price at $T^*$? Draw a picture of what the price path looks like from $t_0$ to $T^*$ and then to exhaustion.

**brief answer** See figure 1. Prior to $T^*$ prices rise at the Hotelling rate as usual. At $T^*$ we find out about future reserves. Prices either fall to $P_2(T^*)$ if we find more reserves, or they rise to $P_1(T^*)$ if we do not. Clearly, if we find more reserves the time to exhaustion must increase since prices must rise at the Hotelling rate.

(c) Suppose $\Pi$ was close to zero, would $p_0$ be higher or lower than if $\Pi = .5$? Explain.

**brief answer** If $\Pi$ is close to 0 then $p_0$ would be higher. One way to think about this is to note that if $\Pi = 0$ then there would be no jump in price at $T^*$, since there would be no news. So by inspection of figure 1 it is evident that the first part of the price path would have to shift up. Another way to see this is to note that at the instant prior to $T^*$ the optimal price is equal to $(1 - \Pi)p_1(T^*) + \Pi p_2(T^*) \equiv p(T^*)$, and as
\[ p_0 = p(T^*)e^{-rT^*} \] (that is, the initial price is the present value of the expected price once uncertainty is resolved), hence the smaller is the expectation of future reserves the higher is the initial price. This makes perfect sense. The less likely we are to find reserves the higher the initial price should be in the model with certainty.

(d) Suppose we implement the optimal price path you have derived. Suppose further, that it is possible for private agents to expend resources and learn about the actual size of reserves before date \( T^* \). Would a private agent spend money to find out? Why? If so, will the market be able to sustain the optimal production program? Explain.

**brief answer** Yes a private agent would spend. If you could find out early whether further reserves will be found you could make lots of money. Suppose you invest \( K > 0 \) and find out at \( T_0 < T^* \) that there are no additional reserves \( (S_0 = S_1) \). This is my private information. I could then offer to deliver \( X \) barrels at date \( t > T^* \) condition on further deposits being discovered. What would I get for this? The price must be the expected value, \( \Pi p_0X \). Of course, I will not have to fulfill this contract since (only) I know that oil will not be found. Similarly, if I find out that \( (S_0 = S_2) \) that oil will be found I offer to deliver \( X \) barrels at date \( t > T^* \) condition on reserves being barren. I receive \( (1 - \Pi) p_0X \). One of these outcomes I will certainly receive, and I have expended only \( K \) to find out. So by making \( X \) large enough I will make enormous sum. But this cannot be an equilibrium. If I can do it, so can you, and then this will change prices. It will drive prices away from the optimal path. Indeed, note that my actions will allow others to infer that I know something. This will further drive the price away from the optimal path.

(e) In the Adelman model the value of a barrel of reserves in the ground, \( V \), is worth less than the net price of a barrel produced, \( P \)? Why is this the case?

**brief answer** First, we note that in the Adelman model the depletion rate is given by the level of development investment. We assume that \( a \approx \frac{Q}{R} \), and the initial level of output, \( Q \), depends on investment. The growth rate of prices is given, there is no reason to expect it to equal the interest rate. In fact, we know that for long periods, the real price of oil was roughly constant. We cannot produce all the reserve right away, so its present value is less than the price of the barrel that we can sell. One could point to development costs, which are important, but that cannot explain why the net price is not equal to \( V \).

2. What happens to \( \frac{V}{P} \) if the depletion rate is higher? Why?

(a) **brief answer** It rises. Faster depletion means greater present value of revenues.

(b) Under what condition would \( V = P \).

**brief answer** Suppose that development costs are zero so net and gross price are the same. Then we can show that \( V = P \frac{a}{a + r - g} \). This follows from calculating the present value of the production from the reserve. Now suppose that the interest rate equals the growth rate of prices, \( i = g \). Then \( V = \frac{2}{a}P = P \). Thus if the growth rate of prices happens to equal the interest rate we are back in the Hotelling world.

(c) Why is the latter condition consistent with the Hotelling model? Explain.
**brief answer** In the Hotelling model oil in the ground is an asset that we deplete so as to eliminate arbitrage profits. The only reason we keep a barrel in the ground is that we believe that the value of future production would be greater than current production. Absence of arbitrage then means that the price and the value in the ground must be equal.

3. Suppose the interest rate unexpectedly, and suddenly, jumps up. What happens to oil production and the price of oil in the current period in the Hotelling model? What happens to oil production and the price of oil in the Adelman model? What explains the differences, if any?

**brief answer** In the Hotelling model we know that a higher interest rate means faster depletion. The time profile of production steepens. Since next period’s net price is now worth less in present value it would be better to produce today. But this must depress the current price. So the current price drops. In the Adelman model the change in the rate of interest has two offsetting effects. A higher interest rate lowers the present value of future production which would induce us to produce more today. But to produce more today requires more development investment and this is more costly with a higher interest rate. So, to a first approximation, the two effects offset, so production and the current price ought not to change.

4. What are the key problems facing a cartel like OPEC? How effective has OPEC been as a cartel? What have been OPEC’s most effective instruments for generating high oil prices? What have been its least effective instruments?

**brief answer** A cartel must prevent entry, coordinate production and punish free riders. OPEC has been relatively effective in that it has survived more than 50 years, which is success in itself. OPEC has not always been able to control price, OPEC countries have frequently cheated on their quotas. It has been much more effective in limiting entry. OPEC has the least-cost deposits in the world, and with nationalized production companies in many OPEC countries that limit development investment, it has been effective in controlling long run supply. OPEC (with 20 times the reserves) invests much less in development expenditure than oil super-majors.

5. Consider the following investment strategy. Buy 1 barrel of oil today at spot price \( P_t \) and sell the oil next year at some future price, the expected value of which today is \( E_t(P_{t+1}) \). Suppose that the interest rate is \( r \), and that the storage cost of oil per barrel is \( C > 0 \).

(a) If there is arbitrage and sufficient storage capacity, and if investors are risk neutral, what is the equilibrium expression for the current spot price? Explain.

**brief answer** \( P_t = \beta E_t(P_{t+1}) - C \), where \( \beta = \frac{1}{1+r} \). If we purchase a barrel and store it, we expend \( P_t + C \), so this must equal the present value of a barrel next period.

(b) What does your expression imply about the predictability of oil price changes if agents are rational?
brief answer If agents form expectations rationally then expected future prices differ from current prices (ignoring $\beta$) only by a forecast error (and the constant, $C$). So if expectations are rational these forecast errors should be unpredictable, so prices should be unpredictable.

(c) What if agents are risk averse? How does your expression from part (a) change?

brief answer Suppose agents require a risk premium, $\lambda$, to compensate them for holding risky assets. Then $E_t(P_{t+1}) = (P_t + C + \lambda)(1 + r)$, or $P_t = \beta E_t(P_{t+1}) - C - \lambda$. Risk aversion produces a wedge between present and expected future prices similar to the cost of storage. If people are more risky they will demand a higher expected profit to engage in arbitrage.

(d) Suppose that it is possible to purchase oil with a forward contract, paying $F_t$ today for delivery next year. If investors are risk neutral what should the equilibrium relationship be between the forward price and $E_t(P_{t+1})$? What does this imply about the relationship between the current spot price of oil and the forward price of oil?

brief answer We would expect $F_t = E_t(P_{t+1})$, since if this did not hold then people would have profit making opportunities. But this implies $P_t = \beta F_t + C_t$.

6. In 1950 the ratio of petroleum reserves to annual consumption was approximately 22 years. By 1990 this ratio was 45 years. How is this possible? Explain.

brief answer The fundamental limit to oil production is knowledge. With more investment we learn more about the deposits, and with improvements in technology we learn that more oil can be recovered from the deposits. It is also true that as prices rise more of the deposit is recoverable, so this also causes reserves to rise. Oil that was not recoverable at low costs is feasible at higher costs (think of the tar sands or any heavy oil, for example). In 1950 the share of US production from offshore was essentially zero, now it is more than one-third. This is a combination of technology and higher price. This would indicate that the oil industry is not well-characterized by the exhaustible resource model. The ultimate amount of oil in the ground is of no economic consequence. Better to think about oil reserves as an inventory problem. The decision is how to maintain inventories at the lowest cost: either by developing existing deposits or exploring and developing new ones.