Does pairwise independence imply independence?

In the book on page 28 it has been shown that mutual independence of $A$ and $B$ together with mutual independence of $B$ and $C$ does not imply mutual independence of $A$ and $C$. More generally, consider a sequence $A_j$ of events such that for any pair $i \neq j$, $A_i$ and $A_j$ are mutually independent. This is called pairwise independence. The question is now: Does pairwise independence imply independence?

The answer is No! As a counter example, consider the following three subsets of the unit square $[0, 1] \times [0, 1]$, indicated by the shaded area.

The probabilities involved are the shaded areas themselves. Thus

$$P(A) = P(B) = P(C) = 1/2.$$  

Moreover, note that

$$P(A \cap B) = P(A \cap C) = P(B \cap C) = 1/4.$$  

Therefore,

$$P(A \cap B) = P(A)P(B)$$
$$P(A \cap C) = P(A)P(C)$$
$$P(B \cap C) = P(B)P(C)$$

However, $P(A \cap B \cap C) = P(A \cap B) = 1/4$ whereas $P(A)P(B)P(C) = 1/8$, hence, $A$, $B$ and $C$ are not independent.